Frontispiece. DOUBLE OUTSIDE SLIP ROADS *Photo No. 8*

RAILWAY PERMANENT WAY.

DIMENSIONAL THEORY AND PRACTICE.

A Manual for Engineers, Inspectors, Foremen &c.
with Tables,

BY

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PREFACE.

THIS Hand-Book which has been prepared for the use of all concerned in the construction and maintenance of the Permanent Way of Railways, supplies ready and accurate means of dealing with the problems of a dimensional nature which arise in the planning and laying of track in correct alignment, and with the proper use of the manufactured parts of which it is comprised.

Whilst aiming to provide an essentially practical work for the use of the trained Civil Engineer, his Assistants, and the Engineering Student, the authors have had also in mind the needs of those engaged in the actual laying and upkeep of the track—the Permanent Way Inspector, the Foreman, and that large class of manual workers in whose ranks are many thoughtful and intelligent men who take a real interest in their work. The book is intended to strengthen these men at their weakest point, as they have not usually had the opportunity of gaining a knowledge of the mathematical side of their work. It is trusted that they will number largely amongst the readers. Though it has been necessary to use mathematics with which they may be unfamiliar, they are asked not to be discouraged if at first they fail to understand certain parts, since, as with any subject such as this, a complete grasp can only be attained by study, which should, where possible, be supplemented by a certain amount of oral instruction.

With regard to the last point it is hoped that the book will fulfil a long-felt want as a text book suitable for use with any scheme of lectures on the subject, such as may wisely be organised on the lines of the already existing Schools of Signalling, and similar courses of instruction dealing with the technical side of railway work.

The mathematics are kept as simple as practicable, but it must be recognised that scope in the treatment of such a subject would be seriously restricted unless Geometry, Algebra, and Trigonometry were employed. Simple explanations of these subjects have therefore been included to enable the previously uninformed, but enterprising student, to follow the derivation of the rules, and to put them to use without being confined to the laborious methods of elementary arithmetic.

It must be emphasised, however, that in practically all cases the rules are converted into such a form that they may be employed by the use of arithmetic alone.

The practice on British Railways of Standard Gauge has been the first consideration but, in principle, the views expressed apply to railways in general, and the rules are stated in such a way that they may be applied to any gauge or unit of measurement in use.

It is not only as a text book that the work is intended, but also as a book of reference. In this respect every effort has been made to facilitate ready reference and universal application. To meet the different conditions obtaining on various railways, blank columns have been provided where it is necessary, so that the user of the book may insert dimensions applying to the practice of the particular railway with which he is concerned.

The treatment of curve problems, and their application to points and crossing problems by the methods of drawing to scale or lining out, is treated thoroughly and, it is believed, in an original manner.

The calculation of dimensions required in points and crossings is based upon a turnout curve tangential to the switch at its heel, and not on the obsolete "tangential to main line method." The rules given will, it is hoped, stabilise the practice and do away with the confusion and inaccuracies which have hitherto existed.

The Centre Line Method of measurement of the crossing angle has been adopted as being practically universal on the railways of this country.

Nearly all the text and diagrams and many of the tables are published for the first time, and contain many new ideas and fresh methods of calculation. They are the results of long experience and careful working, and have been tested in actual practice. Pains have been taken to avoid errors; but it is possible that such exist, and, should any be discovered, the authors would be glad to be notified of them, so that they may be rectified in a future edition.

Any suggestions for increasing the usefulness of the work will be gladly received and acknowledged.

The importance of the subject of track alignment has been kept carefully in view, and the principles have been systematically set forth so as to facilitate the thorough study which is necessary by all concerned.

It is a desideratum that our Railways should have the safest and easiest running roads possible under the varying circumstances. The principles explained in this volume will tend to this and also to economy in making the best of the costly materials used in forming and maintaining a modern railway track.

WM. HEPWORTH.
J. THOS. LEE.

MANCHESTER,
May, 1922.

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CHAPTER I.

INTRODUCTORY.

The purpose of this volume is briefly explained in the Preface. It is now proposed to take a general survey of the procedure and methods of undertaking railway track dimensional work, and notes and suggestions will be given as to the study and practical application of the contents of the book.

General Procedure.

This may best be explained by taking the instance of an intended extension or alteration of track arrangements. After the accommodation and facilities necessary have been decided upon, a plan to a comparatively small scale, such as 60 or 40 feet to an inch, is drawn in the Engineer's Office. The purpose of this plan, which is usually named the "Scheme Plan," is to discover and decide upon the best arrangement or "lay-out" that can be made in the space available. The Inspector, or other person instructed to carry out the work, may in some cases be provided with nothing more than this scheme plan.

In other cases he may also be supplied with a "Working Detail Plan," to a scale of 8, 10, or 20 feet to an inch, showing every dimension necessary to enable him to order the material, and lay it in its correct position. In either event, the important points in the lay-out, according to circumstances, may or may not, be pegged out on the ground by an Engineer.

For intricate and important work, the latter procedure should always be employed, with the proviso that the Inspector should be present when detail setting out is proceeding, so that he may call attention to practical platelaying considerations, and be familiar with the positions arrived at.

The former procedure however, is frequently adopted, and, therefore, it must be assumed that it is reasonable to expect the practical man to work from the scheme plan alone. This means that he must be able, from fixed lengths, widths, radii of curves, etc., which may be given

on the plan, or are otherwise determinable, to derive such other dimensions as are necessary to enable him to order the requisite material, and afterwards lay it in its correct position.

These "Detail Dimensions" will include suitable lengths of switches, rails and timbers, proper types of chairs, correct angles of crossings, leads of turn-outs, distances apart of crossings, and such lateral dimensions as will give the track the required clearances to adjacent track and structures. He must be able to set out minor curves, apart from the general main centre lines located by the Engineer, and to give curves their correct super-elevation, increased gauge, and other adjustments. In addition to working from a plan, he should know how to obtain particulars for the laying of track arrangements of a fairly simple nature from verbal or written instructions.

With regard to an existing "Lay-out," he should know how to record its particulars, such as radii of curves, angles of crossings, etc., and be able to critically examine it and find where improvement can be made when opportunity affords.

This summary includes work which in some cases would be dealt with by an Engineer, but there appears no reason why the aspirant to the position of Inspector or Foreman should not study to equip himself with all the knowledge that he can. Occasions may, and indeed often do, arise in the exigencies of railway working, for him to use it. On many occasions a man has been brought into prominence by his ability, when an accident has occurred, to quickly obtain exact particulars of the materials required, and without delay determine the positions for the reinstatement of important tracks.

Methods of Working.

In obtaining the "Detail Dimensions," more particularly as regards those of points and crossing work, there are three main Methods of Working:—

1. By **Calculation** based upon dimensions given from headquarters, or such as can be directly obtained on the site. Under this method is included the use of Tables of Dimensions applicable to the work.
2. By **Drawing** the new work, or portions of it, upon paper to a scale large enough to measure the required dimensions with sufficient accuracy by scaling.

3. By **Lining out** the new work upon the ground with strings, so as to obtain the required dimensions directly on the site.

Method 1 (Calculation): should be applied as far as possible. It calls for little or no assistance. Instruments are not required. The results are usually very satisfactory, as a high degree of accuracy may be obtained.

Unless, however, the lay-out is fairly simple and with regular outlines, there may be parts of the work which will be difficult to solve by calculation alone. Recourse should then be made to one of the other methods

Method 2 (Drawing): Requires some aptitude or practice in geometrical drawing, and the provision of the necessary drawing instruments, including a set of curved rulers. The chief difficulty, for those not trained in surveying, may occur in the drawing of the position of existing work by which the new work will be affected This difficulty, however, may often be overcome by measurements of a simple character, as explained in Chapter XXV.

Method 3 (Lining Cut): The method most generally in use by Inspectors when the method of calculation will not serve.

It has the advantage of showing by the strings the appearance of the new work in its actual position, and the various crossing points, etc., may be marked forthwith. The disadvantages, however, are serious. The method involves delay, as the lining out must await favourable weather and traffic conditions. A fair amount of assistance is required, and accuracy may be interfered with by carelessness on the part of an assistant, or by the wind deflecting the lines. The chief value of the method is to check and establish measurements arrived at by the other methods.

In some cases it may serve to solve some problem with regard to a portion of the work, which would need elaborate calculation or surveying to solve otherwise.

There need be no hard and fast rule adopted as to the use of any of the three methods. Any one, or even all of them, may be brought into service upon the same work;

the aim to be kept in view being the attainment of the most satisfactory result under the circumstances. In the following chapters will be found the information necessary to put these methods into practice.

Résumé of Contents.

The Miscellaneous Dimensions which must be observed in laying trackwork are given in Chapter II., and, to avoid misunderstanding, Definitions of the technical terms used are included where necessary.

Chapter III. will serve as a reference to the particulars of the Component Parts of the track. The information given will enable a selection to be made from the available rails, chairs, etc., such as will most efficiently and economically fulfil the purpose required. It will also form a guide to the spaces necessary for chairing, timbering, etc.

In these two chapters, and elsewhere when necessary, the dimensions usually adopted are given, but blank spaces are provided, so that the dimensions obtaining in the practice of the particular railway the reader may be concerned with can be inserted.

Chapter IV. explains, in brief, the elementary principles of Mathematics, and Chapter V. those of Geometry with its general theorems and problems, upon which latter so much of the theory of the subject is based. The range of knowledge required to understand these is only that of the ordinary school education of to-day, and anyone thus equipped will be able to follow the derivation of, and put to use, all the rules in the book. Those who are not prepared to use more than simple arithmetic will necessarily be obliged to accept the rules in the later chapters as correct, and use them as converted to their simplest and plainest form. To suit such readers, the rules most frequently in use are stated not only by the usual mathematical formulæ, but also in words. This, however, is only done where the rule is stated for the first time. In the worked out examples the rule will be quoted as a formula, its reference number being given, so that its equivalent in words may be referred to if required.

Usually the working out of the examples will be shown by other than Trigonometrical Rules, it being assumed that those who choose to use these will easily be able to do so without examples.

The reader who will make himself familiar with the symbols used, and learn to read a rule expressed by those symbols in the shape of a formula, will find his study of this and other works, together with his calculations in every-day work, greatly facilitated. It must be noted that all the dimensions in any rule are in the same unit of measurement, so as to avoid confusion.

Curves, with the problems and adjustments appertaining thereto, are dealt with in Chapters VI. to X. The reason for this extensive treatment is, that not only does the procedure given apply to curves in plain line, but it enables problems in points and crossings to be correctly dealt with by the methods of "Drawing to scale" and "Lining out."

Points and Crossings are dealt with in the later chapters. Each chapter upon any particular contrivance will deal separately with:—the general aspect, the practical details, and the calculations. To understand the calculations applying to any track arrangement, such as the Turnout, it will be necessary to first peruse the important chapters dealing with the Switch, the Crossing, and the General Rules.

The System of Factors, mentioned in Chapter XIII., is more fully explained, with examples, in Chapter XXIV. By this system required dimensions in points and crossing problems may, in many cases, be found with sufficient accuracy, by simply multiplying a given dimension by the Factor.

Special notes upon the Method of Drawing to Scale are given in Chapter XXV. Much of the procedure, however, is dependent upon principles explained in the earlier chapters.

System of Symbols and Reference Numbers.

In order that easy reference to the Rules, Diagrams, etc., may be made, the following system has been adopted:

Symbols denoting a definite dimension from one point to another, and used in a rule or formula, are indicated by an upright capital or small letter, thus:—"R" for Radius, "L" for Lead, "d" for some special distance, etc.

Points upon the diagrams, such as the ends of a line, the centre of a circle, etc., are indicated by sloping letters, thus:—"*A B*." These two letters in the text would denote a line from point "*A*" to point "*B*." There is an exception to this rule in the case of Chapter V.

The General Rules for Circular Curves, Chapter VI. and Tables 23 and 24, are numbered and marked with a "C" preceding the number of the rule.

The numbers of the General Rules for Switches and Crossings, Tables 25, 26, and 27, are followed by a small letter "a," "b," or "c," according to the type of rule, trigonometrical or otherwise.

Miscellaneous Rules and Tables occurring throughout the work, the Diagrams, and the Problems are numbered thus:—"XV.—5"; the Roman numeral indicating the chapter in which the rule is first given, or in the case of Tables, Diagrams, and Problems, the chapter in which they are included.

When a Table is referred to thus:—"Table 25," it must be understood that it is one of the collection of General Tables at the end of the book.

Tables.

Of the 51 Tables, only 15 are confined to the 4ft. $8\frac{1}{2}$ins. gauge. By the employment of the rules given in the book, it will be a comparatively easy matter to construct tables applying to other gauges.

Attention is called to Table 20, which, in cases where dimensions are proportional to the gauge, enables several of the other tables to be applied to different gauges.

CHAPTER II.

MISCELLANEOUS DIMENSIONS AND DEFINITIONS.

Dimensions of a miscellaneous character applying to railway track and structures in connection therewith, are tabulated with explanatory notes in this chapter. The usual dimensions adopted on British Railways of Standard Gauge will be given, but in some cases it will be necessary for the reader to make alterations and additions, so that the figures may conform to the practice of the railway he is concerned with

Opportunity will be taken to define (as printed in italics), the terms used, because as is well known, there is a regrettable lack of uniformity in railway technical terms and their application.

In America, the Railway Association has recommended certain standard definitions for adoption in that country, but the only steps taken with regard to British practice would appear to be the definitions with regard to Tramway Track, laid down by the British Engineering Standards Association, in their Report No. 79, 1919, and the efforts recently made by the Institution of Railway Signal Engineers in respect to matters coming within their province.

Dimensions such as Spaces, Clearances, Distances in connection with points and signals, etc., are influenced by the Board of Trade Requirements in regard to Railway Construction and Operation, dated 1908. The present authority in this respect, is the Ministry of Transport, who have revised Requirements under consideration. When these are issued, certain alterations may be necessary to the Board of Trade dimensions given in these notes.

GEOMETRICAL TERMS.

Simple Curve.

A line which is a portion of the circumference of a circle, i.e., the arc of a circle. This is the curve universally used either for the whole or the main part of a railway

curve. Curve problems are dealt with in Chapters V., VI., and VII., whilst the question of Limiting Radii and other kindred points are discussed in Chapter IX.

Contra Flexure.

A term used to denote that two connected curves bend in opposite directions.

Similar Flexure.

A term used to denote that two connected curves bend in the same direction.

Tangent.

In Geometry, a straight line touching but not cutting a curve. In some countries the straight portions of a railway track are known as "Tangents." The term "Tangent" is also used in Trigonometry.

Tangent Point or Springing of Curve.

The point from which a curve springs, that is where it touches a line or other curve, the two being said to be "Tangential" at the point.

Compound Curve.

A curve formed by two or more simple curves tangential to each other and of similar flexure. See Chapter VIII.

Reverse Curve.

A curve formed by two simple curves tangential to each other but of opposite flexure. See Chapter VIII.

Transition Curve.

A curve making a gradual change from a straight line to a circular curve or from one circular curve to another. The American term is "Easement Curve." See Chapter X.

GAUGES, SPACES, AND CLEARANCES.

Gauge.

The least distance between the rail heads of a track where they are touched by the flanges of the wheels.

Table of principal Railway Gauges in use.

ft.	ins.	
4	8½	The Standard Gauge of Great Britain, there being less than 200 miles of statutory railways of other gauges. Any gauge other than this needs special authorisation. It is also the Standard Gauge in U.S.A., Canada, and on certain railways in Europe, Australia, and South America.
4	9½	1·45 metres, the Gauge in France, Germany, and many countries of Europe.
5	6	The Broad Gauge in India and some countries of South America. In Spain and Portugal the gauge is 5ft. 5⅔ins.
5	3	Irish Broad Gauge. Also used in South Australia and Victoria.
5	0	Russia.
3	6	South Africa, Queensland, West Australia, New Zealand, Japan.
3	3⅜	The Metre Gauge used for Light Railways in France, India, Malay, etc.
3	0	Light Railway Gauge in Ireland, Isle of Man.
2	6	One of the Light Railway Gauges in India and used in several of the British Colonies. Recommended by the Light Railways Investigation Committee (M. of T.) in 1921, to be regarded as the standard for narrow gauge lines.
2	0	South African Light Railways and some Indian lines.
1	11⅝	60 Centimetre Gauge. Light Railways on the Continent. Festiniog Railway and others in North Wales. British Military Light Railways.
1	6	A common Workshop Railway Gauge.
1	3	The smallest gauge in practical use. Eskdale Railway.

Space.

The distance between adjacent tracks.

The Space is usually measured between the outside edges of the rail heads, *i.e.*, "clear." In drawing plans or making calculations in which the gauge lines are indicated or used, the space must be taken to inside edges of rails. It will then be termed "Space (gauge lines)," and equals, "Space (clear)" plus width of two rail heads.

On curves, spaces must be wider than on the straight to allow for the overthrow of vehicles, unless the standard space is wide enough to give the necessary safe clearance between vehicles. This subject is dealt with in Chapter IX.

SPACES (clear).	Usual.	.... Ry.
	ft. ins.	
Standard between a pair (Up and Down) of Main Tracks.........	6 0*	
Between a pair of Main Tracks and one or a pair of additional Running Tracks	10 0*	
Between a Running Track and a Siding	10 0	
Between a Running Track and a Catch Siding.....................	6 0	
Between Goods Yard Sidings......	6 0	
Between Goods Yard Sidings for Cartways	24 6	
Between Coal Sidings	6 0	
Between Marshalling or Sorting Sidings	7 0	
Between Carriage Sidings	8 0	
Space to allow for Lamp Posts between Sidings, etc.	8 0 to 10 0	
Space to allow for Large Main Line Signal Post	10 6 to 11 0	
Space to allow for Ordinary Main Line Signal Post	10 0	
Space for Siding Signal Post	9 8	

* Board of Trade Requirements.

Fouling Point.

The point at which the space between two converging tracks becomes insufficient to allow vehicles on them to safely clear each other. Where a running or passenger track is concerned, the fouling point must be taken where

the space is 6ft. In computing standing wagon room in goods sidings, the fouling point is often taken at a 5ft. space.

Clearance.

The horizontal distance from outer edge of rail to a structure or object of any kind.

According to Board of Trade Rules no structure other than a passenger platform, may be nearer to the side of a carriage than 2ft. 4ins. at any point between the level of 3ft. above rail level and the top of carriage doors.

On the 4ft. 8½ins. gauge, with rails 2¾ins. wide, and carriages 9ft. wide, the minimum clearance must therefore be 4ft. 3ins. on a straight road.

To allow a man to stand between a structure and an open carriage door, the clearance should be about 5ft. 6ins.

On curves, clearances should be increased to allow for the overthrow of vehicles, and the extra overhang on the inside of the curves, due to super-elevation. See Chapter IX.

	Usual.	..........Rly.
Absolute minimum clearance to structures above platform height, on passenger lines...	4′ 6″	
Desirable ditto. ...	5′ 0″ to 7′ 0″	
Minimum clearance to structures in Goods Yards, such as to sides of Warehouse doorways	4′ 3″	
Desirable ditto ...	4′ 6″ to 7′ 0″	
On curves add to above the versed sine on a chord of ..	40 ft.	
To the inside of curves add the super-elevation multiplied by	2	
Add for oscillation on curves ...	1″	

Other clearances are dealt with under "Structures."

Headway.

The clear height from rail level, i.e., top of rail, to the underside of a structure over the track.

	Usual.	..............Rly.
Absolute Minimum Headway on Passenger Lines	ft. ins. 14 6	
Desirable ditto ditto	15 0	
Absolute Minimum Headway in Goods Yards..........................	14 0	

Loading Gauge.

The outline with respect to a base line across the top of the rails, beyond which no part of a vehicle or its load may project.

Diagrams of the various loading gauges in use on British Railways will be found in the Railway Year Book (1921) and particulars are usually included in the "Appendix to Working Time Tables."

The usual width of British Loading Gauges is 9ft., though four railways have a width of 9ft. 3ins., and the G.W.R. width is 9ft. 8ins. for certain lines. These widths decrease at the lower part

English Loading Gauge heights above rail level vary from 13ft. 9ins. (three railways) to 13ft. (G.E.R.). The height is less on Scottish Railways, the lowest being 12ft. 9⅝ins. on the Highland Railway.

The "Berne Gauge" has been generally adopted on the Continent of Europe. Its height is 14ft. 0¼ins., and width 10ft. 2ins.

Structure or Construction Gauge.

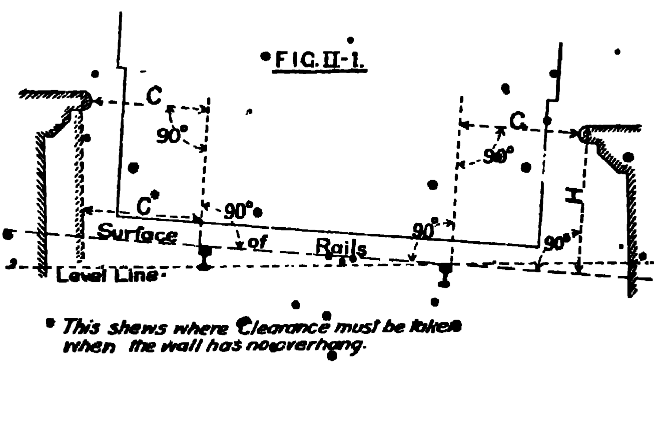
The outline with respect to a base line across the top of the rails, within which no part of a structure or object of any kind may encroach.

	Usual.	..........Rly.
Margin or clearance between Loading and Structure Gauges:—		
At top	9"	
At side, above "set off" for platform	2' 4" to 2' 7"	
At side, below "set off" for platform	4"	
Height of bottom line of structure gauge above rail......	Nil *	

* Electric conductor rails and water troughs are generally excepted.

STRUCTURES.

FIG. II-1.



* *This shews where Clearance must be taken when the wall has no overhang.*

	Usual.	.... Ry.
Passenger Platforms. (Fig. II.-1)-		
Height (H on Fig. II.-1) of Platform above rail, measured at right angles to a line across the tops of the two rails of the track	3′ 0″	
Clearance (C Fig. II.-1) from outer edge of rail, to nose of platform coping, on straight road	2′ 1″	
Ditto on curves, add to ′ - ″ † the versed sine on a chord of... (This clearance to be measured to a line at right angles to the surface of the track rails.)	40′ 0″	
Overhang of coping from wall face	1′ 0″*	
Cross fall, away from track, of coping	1 in 72	
Ditto of platform behind coping	1 in 36	
Nearest distance of pillars, etc., from edge of platform	6′ 0″*	
Width of platform, small stations	6′ 0″*	
Do. Do. large Do.	12′ 0″*	
Inclination of ramped ends	1 in 8*	

* B. of T. Requirements. † Usual about 1 ft. 11 ins.

When a line at a platform changes directly from the straight to a sharp curve, the extra clearance necessary to the curved portion, should be obtained by moving the wall line gradually away from the rail on the straight. Care will be necessary to get an "eyeable" line on the coping and still retain the correct alignment of the rails. A transition curve on the rails avoids this difficulty.

	Usual.	.... Ry.
Goods and Mineral Loading Stages.		
Height above Rail of :—		
Stage for loading and unloading general goods ...	3′ 6″	
Cattle Stage	3′ 6″	
Horse Stage	3′ 6″	
Stage for loading high side wagons with minerals etc. from carts by tipping......	8′ 0″ to 9′ 6″	
Stage for coaling Loco's......	5′ 0″	
The side clearances from rail are usually made 1 inch more than as required for passenger platforms.		
Carriage Loading Docks.		
For transferring road vehicles to and from railway stock over the end of the latter.		
Height above rail	4′ 0″	
Cart Weighing Machines.		*
Length of table	14′ 0″	20′ 0″
Width of table	8′ 0″	8′ 0″
Tonnage	15tons	20 tons
Wagon Weighing Machines.		
Length of table	14′ 0″	
Tonnage	20 tons	
Distance apart of Weighing and "Dead" Rails	1′ 0″	
Water Column or Water Crane.		
Space required when column is between tracks	11′ 0″	
Distance from rail to centre of column, when column is to outside of tracks ...	6′ 6″	
Distance of Water Column behind signal of line on which engine takes water	60′ 0″	

* For Road Motor Vehicles.

	Usual.	.... Ry.
Water Troughs.		
Height of rim above rail...	3″	
Height of water above rail...	2″	
Gradient of track towards ends of troughs	1 in 300	
Gradient of trough at ends	1 in 192	
Engine Turntables.		
Present standard diameter	65′ 0″	
Other diameters of tables in use		
Space between rim of table and an adjacent track ...	9′ 0″	
Wagon Turntables.		
Present standard diameter	13′ 0″	
Other diameters in use		
Engine Pits.		
Width between side walls...	3′ 9″	
Depth of Pit floor below rail level. "Inside" pit......	2′ 2″ to 2′ 5″	
Depth of Pit floor below rail level. "Outside" pit ...	3′ 0″ to 3′ 3″	
Carriage Pits.		
Width between side walls...	3′ 9″	
Depth of pit floor below rail level. Steam Stock	2′ 6″ to 3′ 0″	
Depth of pit floor below rail level. Electric Stock ...	3′ 4″	
Overbridges.		
Minimum clear span or opening for single line ...	13′ 8″ ‡	
Desirable clear span or opening for single line ...	15′ 6″ ¶	

† All tracks approaching a turntable should be laid straight for a length adjacent to the table, equal to the diameter of the table. This straight must be in line with the table rails when turned towards it.

‡ 4′ 3″ clearance; should not be used on new work.

¶ 6′ 2″ clearance.

	Usual.	.... Ry.
Overbridges—*con.*		
Minimum clear span or opening for double line ...	24′ 10″ ‡	
Desirable clear span or opening for double line ...	26′ 6″ §	
Minimum clear span or opening for four lines with one 10′ 0″ space ...	51′ 2″ ‡	
Desirable clear span or opening for four lines with one 10′ 6″ space ...	53′ 6″	
Minimum headway	14′ 6″	
Desirable headway	15′ 6″	
Width between parapets, Turnpike road	35′ 0″ ‖	
Width between parapets, Public carriage road	25′ 0″ ‖	
Width between parapets, Private road	12′ 0″ ‖	
Height of parapets	4′ 0″ ‖	
Underbridges.		
Headway.—Turnpike road, for centre 12 ft. of arch	16′ 0″ ‖	
Public carriage road, for centre 10 ft. of arch	15′ 0″ ‖	
Private road, for centre 9 ft. of arch	14′ 0″ ‖	
Widths of roadway	As for overbridges ‖	
Height of parapets	4′ 6″ ‖	
Desirable width between parapets, for single line...	15′ 6″ ¶	
Desirable width between parapets, for double line...	26′ 6″ §	

‡ 4′ 3″ clearance; should not be used on new work.

§ 5′ 1″ clearance.

‖ Minimum under Railway Clauses Consolidation Act, 1845, and subject to objection by Road Authorities.

The bridge openings given, are for straight tracks.

¶ 5′ 2″ clearance.

TRACK.

For definitions and particulars of the various track arrangements in which points and crossings are used, reference should be made to the chapters dealing specially with them, and similarly with regard to the switch and the crossing themselves.

The component parts of the track, such as rails, fastenings, etc., are dealt with in Chapter III.

Flying Junction.

A Junction where the Down Road of a Branch Line crosses the Up Road of a Main, or vice versa, overhead, to avoid them crossing on the level.

Burrowing Junction.

As above, but where the branch line passes underneath the main.

Loop.

A track connected at both ends to the same main road, which it is used to relieve. Loops may be classified as Passenger Running Loops, Goods Running Loops, and Refuge Loops. Goods, but not Passenger Loops* must be provided with Safety Points, see below.

Space for a main road signal between the main and the loop, must be provided at the exit of the loop. In some cases space may also be required for a water column.

Refuge, Lay-by, Pass-by, or Relief Siding.

A siding for the reception of trains, usually goods, whilst other trains pass by on the main line.

The length of such sidings is determined by the maximum number of wagons in any train using the line, plus engine or engines and brake van.

Catch or Runaway Points.

Trailing switches inserted in a main road to derail vehicles accidentally running back on an incline. See "Gradients."

Safety or Trap Points and Sidings. See Chapter XXIII.

*Except in certain cases as may be required by the Ministry of Transport.

ELECTRIFIED TRACK.

	Usual.*	....
Gauge measured between centre of horizontal contact surface of conductor, positive, third, or live rail, and gauge line of nearest rail of track	ft. ins. 1 4	
Vertical height of contact surface above plane of top of running rails:—		
(a) For top contact rails........	0 3	
(b) „ under „ „	0 1½	
Overhead Clearances:—		
Between underside of live wire and loading gauge:—		
(a) In the open	3 0	
(b) Through tunnels & bridges	0 10	
Between any part of a structure and a live wire	0 6	

BUFFER STOPS, ETC.

Buffer Stop or Buffer Block.

A structure designed to stop vehicles, the buffer plungers of which come into contact with it.

	Usual.	...........Rly.
	ft. ins.	
Length of buffer beam	7 6	
Height of centre of beam above rail.	3 6	

***These dimensions, and others, are as recommended for adoption by the Advisory Committee upon the Electrification of Railways, in their Report to the Ministry of Transport, dated June 30, 1921.**

Stop Block.

A fixed block to stop vehicles by contact with their wheels.

	ft.	ins.
Height above rail	1	0

Skotch Block.

A pivoted or hinged block which can be placed across a rail to prevent vehicles passing a certain point.

	ft.	ins.
Height	0	9
Clearance from rail when open ...	2	1

Skotch or Skid.

A loose block as above. The Hand Shunting Skid is designed for stopping wagons in gravitation yards, etc. See "Railway Gazette," April 22nd, 1921.

Sand Drag.

A device for providing a layer of sand on the top of a rail, so that vehicles will be more or less gradually brought to a stand. These are sometimes provided in connection with Runaway Points where break-aways are frequent. A short Sand Drag or a bank of sand is sometimes used instead of a Buffer Stop; it has been found effective, and less damaging to rolling stock. See "Railway Gazette," October 15th, 1920.

Derailer.

A block which may be moved on to the rail by a lever worked by the signalmen, so that vehicles will be derailed if attempting to pass without authority. See Chapter XXIII.

Chain Drag.

A device by which a hook attached to a heavy chain is arranged to catch the axle of a moving wagon and so bring it to a stand. First used at Edge Hill (L. & N. W. R.).

SIGNALLING ARRANGEMENTS.

General Notes.

The designer of a lay-out should be conversant with the primary signalling requirements, so that the necessary spaces between tracks to receive signal posts, and also

suitable lengths at points for locking bars, etc., may be provided. A few notes for guidance will be given, but consultation with the Signalling Department will always be necessary.

As a general rule, signals should be placed adjacent to the road to which they refer, and on the engine driver's left hand side as he approaches them.

On approaching any Block Post, *i.e.* area worked from a Signal Box, the first Stop Signal for the main road is required as soon as any other track commences to join it.

This is normally the Home Signal. It may, if necessary, be placed at the fouling point of the two roads, which is taken where they are 6ft. apart. Usually a safety margin is allowed, and the signal placed say, 30ft. before the fouling point.

The next signal, in a simple arrangement, will be the Starting Signal, often placed an engine length past the end of a station platform. Other signals which may be required are, the Inner and Outer Homes and the Advanced Starting, the latter being placed a train length beyond the last set of points.

The Distant Signal, placed at various distances up to 1,000 yards from the box, will seldom need special arrangements for its reception

Each track joining a main must have a signal to control trains passing on to the main. Where the joining line is not a main or passenger line, a Ground or Disc Signal may be used for this purpose, though Siding and Goods Loop exits are often provided with ordinary signals of a small size.

For the spaces required for various signals, see under "Spaces." Special spaces need not be provided for Disc Signals as they are so constructed as to be clear when placed in a 6ft. space.

Facing Points.

Points on running roads which are used to divert trains travelling in their normal direction.

The Board of Trade Rules provide that Facing Points must be avoided wherever possible, and be placed as near

as practicable to the levers working them, the limiting distance being 200 yards, though 300 yards is sanctioned for power worked points, and 250 yards under certain conditions, for manual worked points.

Where Facing Points occur, a signal is needed with arms for each direction. By Board of Trade Rules a locking bar or track circuit, at least equal in length to the longest distance between centres of wheels of any vehicle on the line, must be provided. The signal in such cases must be placed so as to stop a train clear of the bar. If a crossover occurs just before reaching the facing points, the signal will generally be fixed at the commencement of the crossover.

A minimum distance between facing and trailing points should not be encroached upon in order that the standard locking bar may be fixed clear of the switches. This distance should allow for a stock length of rail between the stock rail joints. Locking bars may be fixed on the switches but this should be avoided wherever possible.

	Usual	............Rly.
	ft. ins.	
Length of locking bar.................	40 0	
Minimum distance between facing and trailing points	45 6	

Trailing Points.

Points on running lines other than Facing Points. The Board of Trade limit of distance from the levers is 300 yards, including points of Safety Traps to Goods Sidings.

Ground Points.

Points not worked from a Signal Cabin or Lever Frame, and therefore not situated on the running lines or giving access thereto. Such points are worked by a Ground Point Lever, also known as a Hand or Dummy Lever. According to their mechanism, the levers may be designated as Tumbler Levers or Spring Levers.

Ground Frame.

A frame of point and signal levers with no signalman in regular attendance. These frames are used to work points and signals which are beyond the Board of Trade distance from a Signal Box.

GRADIENTS AND VERTICAL MEASUREMENTS.

Datum.

A level line or level plane to which all heights or depths under consideration are referred.

Levels in Britain are usually given in feet and decimals above Ordnance Datum (O.D.). Thus, if two spots are marked on a plan as 298·54 and 286·46, the latter spot will be 12·08ft., or 12ft. 1in. below the former.

Gradient.

The rate at which the surface of a railway or road ascends or descends in its length.

In British railway practice, a gradient of 1 in 150, means that there is a difference in the level of the railway of one unit in a horizontal distance of 150 of the same units, as for example 1 foot in 150 feet.

In some other countries the gradient is referred to as so much per cent., or sometimes as so many feet per mile.

The importance of avoiding steep gradients is evident, especially when we remember that in making the ascent, the fraction of the weight of the train which is added to the resistance the engine has to overcome is the same as the rate of the gradient.

The resistance to be overcome on the level varies, but if we take it for example, as 12 lbs. per ton, or $\frac{1}{187}$ of the train weight, we see that a gradient of 1 in 187 will double the pulling force required.

It has been assumed that when a train ascending a gradient steeper than 1 in 260 is brought to a stand, there is a danger of it running back. Hence the Board of Trade Rule that no station shall be constructed or siding join a passenger line on a gradient steeper than this, unless it is unavoidably necessary. Where this cannot be avoided,

a catch siding or points must be provided, at a distance greater than the longest train length of any train standing with its engine outside of the home signal.

Gravitation Gradients.

In Marshalling and Sorting Yards, and sometimes in other cases, such as in Shunting Necks or Reception Sidings feeding a Goods Yard, wagons are desired to move by gravity on the brakes being released. The necessary *Gravitation Gradient* is a matter for careful consideration in each case. Factors to be taken into account are the length of the gradient, the point the wagons must reach, the type of axle boxes, the prevailing winds, etc. Usually, wagons of all kinds will run on a gradient of 1 in 100 on a straight road without acquiring too great a speed

On curves, the gradient must be steeper to allow for the resistance of the curves (see below).

Where wagons are pushed over a "Hump" or "Kip" the same remarks apply.

The vertical curves (see below), should generally be short at the top of gravitation gradients, and long at the bottom, so that wagons may start quickly and come to rest slowly.

At the top of a hump, a short stretch of level track is sometimes provided, to receive the engine after the last wagon is released

Compensation on Gradients for Curvature.

It requires more force to pull a train on a curve than on the straight, and when planning new railways it is usual to *compensate gradients for curvature*.

This broadly, means that the gradient is reduced where curves occur upon it so that the engine pull required will be the same on the curve as on the straight.

In America the practice is to reduce the gradient by ·03 to ·05 % for each degree of the curve, but in this country there is little opportunity for such refinement.

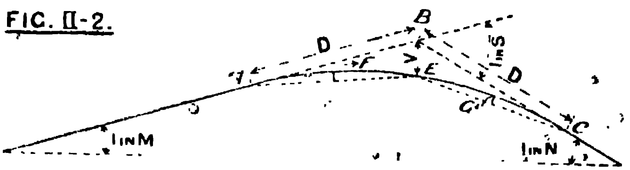
In compensation problems, it will be advisable to transpose the Curve and Gradient into the American methods of measurement. The Degree of the curve may be obtained as shown in Chapter IX. or from Table 11. The gradient is expressed as so much per cent., thus:—1 in 80 would be $100 \div 80 = 1{\cdot}25\%$.

In gravitation yards, compensation must be allowed for curves in a reverse manner, that is, the percentage of compensation must be added to the ruling gravitation gradient (expressed as a percentage) for the straight roads.

At Lostock Sidings (L. & N. W. R.), the gradients are 1 in 100 on the straight, and 1 in 65 on the 10 Chain curve.

Vertical Curve or Transition Gradient.

FIG. II-2.



A curve in the vertical plane to render the change between two gradients gradual.

Scientific methods of determining such transition gradients have been proposed, but in practice it is usually sufficient to round off the angle formed by the intersecting gradient lines with a circular curve. The following method of procedure may be adopted, which however anticipates several rules given in later chapters.

Referring to Figs. II.—2 and 3:—

1st. Mark off the intersection point *B* of the two gradients 1 in M and 1 in N.

2nd. Choose the depth V which may be conveniently rounded off, and mark point *E*.

3rd. Obtain the angle 1 in S, which is the sum of angles 1 in M and 1 in N.

$$S = \frac{M \times N^*}{M + N} \text{ as per Rule XII.—24.}$$

4th. Obtain distances D, from intersection point *B* to ends of vertical curve, *A* and *C*.

$$D = 2V \times 2S \quad \text{(Approx.)}$$

$$\text{or } \underline{D = 4 \times V \times S} \qquad \textbf{Rule II.—1.}$$

* When both gradients fall in the same direction, the flatter gradient being 1 in M, then $S = M \times N \div (M - N)$. When the flatter gradient becomes level, $S = N$.

Intermediate points in the curve may be set by quartering thus:—String or sight a line AE and at its centre set the point F at $\frac{1}{4}$ of V above the line. Similarly fix point G.

Otherwise we may fix pegs at $A\ F\ B\ G$ and C, all at the gradient line, and state that the rails must be V″ below peg B and $\frac{1}{4}$ V′ below pegs F and G.

This applies where the gradients form a "summit" as in Fig. 2. Where they form a "sag" as in Fig. 3, the words "above" and "below," in the explanation, must be reversed.

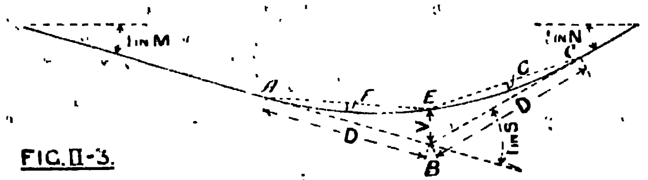


FIG. II-3.

EXAMPLE.—A gradient of 1 in 50 meets a gradient of 1 in 100 at a summit. An underbridge at the summit will allow the "peak" to be lowered 6ins.; it is required to set 5 points in the vertical curve.

1st. $S = \dfrac{50 \times 100}{50 + 100}$

$= \dfrac{5000}{150}$

$= 33\frac{1}{3}$

2nd. $D = 4 \times 6'' \times 33\frac{1}{3}$

$= 66\frac{2}{3}'$

3rd.* At 66ft. 8ins. on either side of the peak B, set pegs A and C on the gradient line. At the peak set a peg 6ins. below gradient level, and 33ft. 4ins. on either side of the peak set pegs $\frac{1}{4}$ of 6ins. $= 1\frac{1}{2}$ins., below gradient line.

*Super-elevation. See Chapter IX.

ROLLING STOCK.

A knowledge of certain dimensions of the vehicles using the tracks is often necessary to the permanent way worker. Below are given definitions and dimensions which will be found useful:—

TOTAL WHEEL BASE ...	Length between centre of first and last wheels of a vehicle.
RIGID WHEEL BASE ...	Length between centres of the first and last wheels of a set which are fixed with their axles at right angles to a rigid frame.
BOGIE TRUCK	A frame with four or six wheels which is pivoted in the centre by a "King Bolt" to the main frame of a vehicle, so that it need not lie in the same line as the main frame in taking curves. The Bogie, in addition to the pivotal movement, has sometimes a lateral movement bodily
PONY TRUCK	Sometimes called a two-wheeled truck. A frame with two wheels which is pivoted at the end of a bar, the pivotal point not lying on the axle line.
CENTRES OF BOGIES ...	Length between centres of two bogie trucks of a vehicle
RADIAL AXLE BOXES ...	Axle boxes which allow the ends of an axle to move so that it need not lie at right angles to the frame when on a curved road.
HEADSTOCK	The member across the end of the frame of a vehicle, which receives the longitudinal framing and "sole-bars," and which takes the "Buffing and Draw Gear."

	Locos. (Steam).	Coaching Stock (Steam).	Coaching Stock (Electric).	Wagon Stock.
	Usual	Usual	Usual	Usual
	ft. in.	ft. in.	ft. in.	ft. in.
Length to be allowed in computing standing room	60 0	37 0 6 whe'l 61 0 Bogie	65 0	21 0
Height of floor above rail level (unloaded)	4 1	4 3	[illegible] 3	4 1½
Height of centre of buffers above rail level (unloaded)	3 6	3 6	3 6	3 5
Width, centre to centre of buffers	5 9	5 8	5 8	5 8
Size of buffer face (dia. or oval)	1 1	1 1 6 whe'l 18"×12" Bogie	18"×12"	0 12
Diameter of wheels on tread (when new)	Vary	3 6	3 6	3 1
Distance back to back of tyres	4 5⅜	4 5½	4 5½	4 5½
Width overall of tyre	0 5½	0 5	0 5	0 5
Depth of flange from centre of tread (when new)	0 1⅛	0 1 3/32	0 1 3/32	0 1 3/32
Depth of flange from centre of tread (when worn)	0 1 5/16	0 1 9/32	0 1 9/32	0 1 9/32
Inclination of tyre tread with the horizontal	1 in 20	1 in 20	1 in 20	1 in 20

Other notes concerning Rolling Stock are included in Chapter IX.

CHAPTER III.

DIMENSIONS OF COMPONENT PARTS OF PERMANENT WAY.

This chapter is intended for reference as to the particulars of the component parts of the track. The Switch and the Crossing will not be included, these being dealt with separately in other chapters.

The usual dimensions adopted are given where possible, blank spaces being provided so that dimensions obtaining in the particular practice the reader is concerned with may be inserted. When the information is thus completed, it will enable a selection to be made from the available rails, chairs, etc., such as will most efficiently and economically fulfil various purposes. It will also form a guide to the spaces required in timbering, chairing, etc.

	Usual.	.... Ry.
EARTHWORKS.		
Width of cutting at 2′ 0″ below rail level for two main lines of way with 6 ft. space	30′ 0″	
Width of embankment at 2′ 0″ below rail level for two main lines of way, with 6 ft. space	30′ 0″	
Inclination of slopes for ordinary ground..............	1½ to 1	
Depth of formation at centre, below rail level	2′ 0″	
Depth of formation at side, below rail level	2′ 6″	
Depth of formation required for crossing timbers.............	2′ 3″	

	Usual.	.... Ry.
BALLAST.		
Bottom ballast, thickness	0′ 9″	
Bottom ballast, width for one line	11′ 0″	
Bottom ballast, cubic yards required per lineal yard of single track	·92	
Top ballast, level of surface ...	Top of sleeper	
Top ballast, distance of top edge from rail........................	3′ 3″	
Top ballast, cubic yards required per lineal yard of single track (deducting sleepers)...	1·11	
DRAINS.		
Distance of centre of pipe side drain from rail, where width allows	5′ 0″	
Depth of invert of pipe side drain below rail, where gradient of drain may be made same as that of railway	3′ 9″	
Distance apart of catch pits	50 yards *	
* This allows of cleaning by rods.		
Inside size of catch pits	3′ × 2′	
SLEEPERS.		
Sleepers, ordinary, size	†9′ × 10″ × 5″	
Do. joint, Do.	†9′ × 12″ × 5″	

† Some railways now use sleepers 8′ 6″ long.

	Usual.	.... Ry.
SLEEPERS—*continued.*		
Distance apart of joint sleepers, centre to centre	2′ 0″	
Joint sleeper to next sleeper, centre to centre	2′ 3¾″—2′ 5″	
Between next two sleepers ...	2′ 6½″—2′ 7″	
Normal distance apart of sleepers, centre to centre	2′ 6½″—2′ 9″	
Normal number of sleepers to a 30 ft. rail	11 to 12	
Normal number of sleepers to a 45 ft. rail	17 to 18	
Normal number of sleepers to a 60 ft. rail	23 to 24	
Number of extra sleepers to a ......ft. rail on sharp curves	45 ft. rails :— 2 to 3	
Do. Do. in tunnels	2 to 3	
Do. Do. at water troughs	2 to 3	
Crossing timbers, breadth and depth	14″ × 7″	
Crossing timbers, to switches, length	9′ increasing to 10′	
Crossing timbers, to carry ground lever, length	12′ 6″	
Crossing timbers, under nose of crossing in ordinary turnout, length	14′ 0″	
Crossing timbers, to carry two roads with 6 ft. space, length	20′ 6″	
Longitudinal sleepers, breadth and depth	1′ 6″ × 8″	
Longitudinal sleepers, breadth and depth, to receive check chairs	1′ 9″ × 8″	

Principles in Timbering Points and Crossings.

In the timbering of points and crossing work the desiderata are as follows:—

1. No rail bearing must seriously exceed that allowed by the normal spacing of sleepers in the plain line. In difficult cases objection need not usually be taken to a rail bearing of 3ft. (centre to centre of chairs), nor to one of 2ft. 6ins. at a joint.
2. Space should be allowed between timbers for packing ballast.
3. Opposite chairs under rails of the same track should rest on the same timber.
4. All chair fastenings should be accessible.
5. Exceptionally long timbers should be avoided, as they tend to difficulty in repairs.
6. Through channels should be allowed where required for point rods.

CHAIRS. (Figs III, 1 to 17.)

The following table, with the diagrams, when completed in accordance with the practice concerned, will show the types of chairs provided, their purposes, and the spaces occupied by them.

The dotted outlines show the nearest possible positions of adjacent rails to the chairs. Such positions should be avoided wherever possible, the chairs being selected, and the alignment arranged, so that all fastenings are accessible.

L = Length of base of chair, over all.

B = Breadth of base of chair, over all.

U, V, W, X, Y, Z = Widths from gauge line of rail, occupied by chairs under various conditions

F = Flange-way between two rails.

E = Elevation of check rail.

S = Spread between gauge lines of a crossing at the centre of the chair. This dimension is dealt with in Chapter XII.

With regard to any particular chair, only those dimensions marked upon its diagram need be inserted in the table.

Cant of Rails with the vertical:—

	Usual.	............Rly.
Running rails	1 in 20	...
Stock rails and switches	,,	...
Crossing legs	,,	...
Wing rails alongside crossing	,,	...
Check rails	Vertical	...

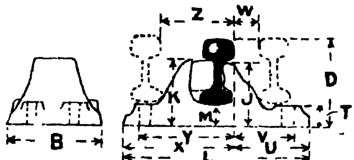


Fig III-1

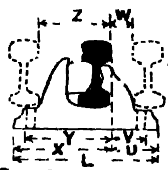


Fig 2

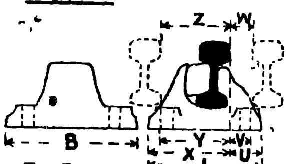


Fig. 3.

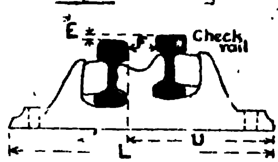


Fig. 4.

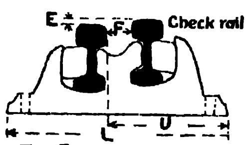


Fig. 5.

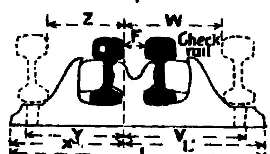


Fig. 6.

Fig. 7.

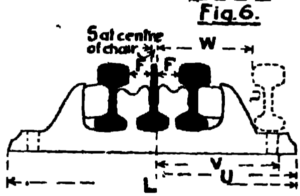
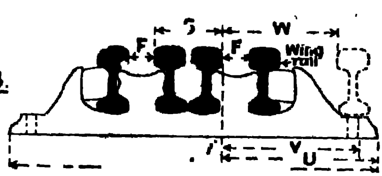


Fig. 8.



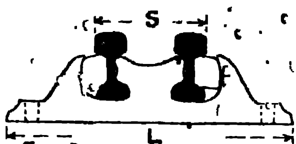


Fig. 9.

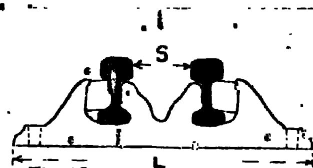


Fig. 10.

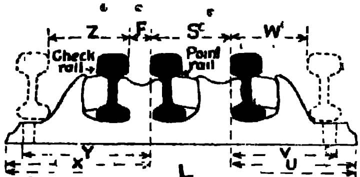


Fig. 11.

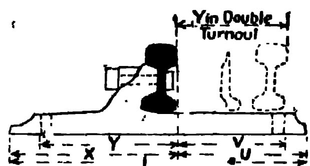


Fig. 12.

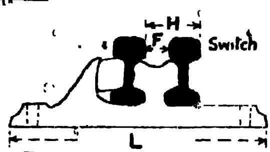


Fig. 13.

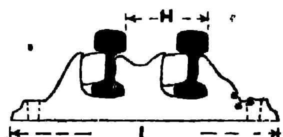


Fig. 14.

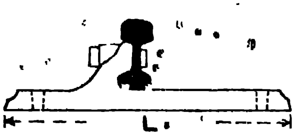


Fig. 15.

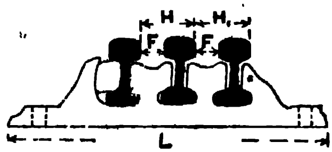


Fig. 16.

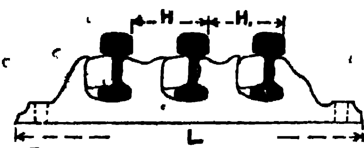


Fig. 17.

General distinguishing marks for Rail Section, Date, etc.

Usual name and purpose of chair.	Rail Section	Type Marks	Weight lbs.	Inches. L	B	U	V	W	X	Y	Z
Ordinary. Fig. 1 .... To be used wherever there is room for it, and it will serve the purpose.											
Joint. Similar to Fig. 1 .... For use at joints in plain line, and where room for it in points and crossings.											
Narrow Ordinary. Similar to Fig. 1 .... For use where two chairs are required to lie side by side on the same timber.											

HEIGHTS.

Section.	D	T	K	J	M

Usual name and purpose of chair.	Rail Section	Type Marks	Weight lbs.	Inches.								
				L	B	U	V	W	X	Y	Z	F
Bridge Chair. Fig. 2												
For use on longitudinal sleepers, and on crossing timbers where there is not room for the length of an ordinary chair.												
Small Bridge Chair. Fig 3												
For use where there is not room for bridge chair.												
Check No. 1. Fig. 4												
For checking plain line with cross sleepers, on straight roads, as on viaducts, etc.												
For checking curves over...........radius												
Check No. 2, as Fig. 4												
For checking curves...........to...........radius.												
Bridge Check No. 1. Fig. 5												
For checking straight roads on longitudinal sleepers												

Usual name and purpose of chair.	Rail Section	Type Marks	Weight lbs.	L	B	U	V	W	X	Y	Z	F
Bridge Check No. 2, as Fig. 5 .... For checking curves on longitudinal sleepers.												
Crossing Check No. 1. Fig. 6 .... For checking crossings with normal gap between rails.												
Crossing Check No. 2, as Fig. 6 .... For checking crossings at splayed end of check rails.												
Crossing Check No. 3, as Fig. 6 .... As above.												
Bridge Crossing Check No. 1 .... For checking crossings where there is not room for usual chair.												
Crossing No. 1 or Point.* Fig. 7 .... Under nose of Vee crossing.												
Crossing No. 2. Fig. 8 .... Next chair to above supporting crossing legs and wing rails.												

Inches: L to F.

For further particulars and positions of Crossing Chairs see Chapter XII. * Also used as Diamond No. 1 on............Rly.

Usual name and purpose of chair.	Rail Section	Type Marks	Weight lbs.	Inches. L	B.	U	V	W	X	Y	Z	F
†**Crossing No. 2.** Fig. 8, for two parallel wing rails												
†**Ditto,** for right hand parallel wing rail												
†**Ditto,** for left hand parallel wing rail												
Crossing No. 3, similar to Fig. 8 Next chair to above and supporting legs and wings in flat crossings.												
†**Ditto,** for two parallel wing rails												
†**Ditto,** for right hand parallel wing rail												
†**Ditto,** for left hand parallel wing rail												
Crossing Collar No. 1. Fig. 9 Supporting legs of crossing but clear of wing rails.												
Crossing Collar No. 2, as Fig. 9 Next chair to above for flat crossings												
Ditto, No. 3 As above.												

For further particulars and positions of Diamond Chairs see Chapter XX. † When usual chair is for splayed wings.

Usual name and purpose of chair.	Rail Section	Type Marks	Weight lbs.	L	B	U	V	W	X	Y	Z	F
Crossing Neck No. 1. Fig. 10 .... Supporting wing rails in front of crossing.												
Crossing Neck No. 2, as Fig. 10 .... Next chair to above for flat crossings.												
Diamond No. 1 or Point,** similar to Fig. 7 .... Under "dead" point of obtuse crossing.												
Diamond No. 2.† Fig. 11 .... Next chair to above.												
Diamond Elbow or Neck. Similar to Fig. 6 .... Under elbow in flat crossings.												
Switch Slide Chair No. 1. Fig. 12 .... Under toe of switch.												
Switch Slide Chair No. 2, as Fig. 12 .... Under switch.												

For further particulars and positions of Diamond Chairs see Chapter XX. ** Crossing (Vee) No. 1 Chair is used on........................Rly. † Crossing Check and Narrow Ordinary are used on........................Rly.

Usual name and purpose of chair.	Rail Section	Type Marks	Weight lbs.	Inches. L	B	U	V	X	Y	F	H	H_1
Switch Heel Chair. Fig. 13 .... With ordinary switches, in front of heel joint. With spring switches, last chair on which switch slides.												
Turn-out Chair No. 1. Fig. 14 .... Supporting both rails of turn-out behind switch heel.												
Turn-out Chair No. 2 .... Next chair to above.												
Turn-out Chair No. 3 .... Next chair to above.												
Three-throw Slide Chair. Fig. 15 .... Under both switches of three-throws.												
Three-throw Heel Chair. Fig. 16												
Three-throw Turn-out Chair No. 1. Fig. 17												
Three-throw Turn-out Chair No. 2. Fig. 17												

For positions of Switch Chairs see Chapter XI. For positions of Three-throw Chairs see Chapter XVIII.

CHAIR FASTENINGS.

	Wood treenail.	Iron spike.	Screw spike.	Chair bolt.
Length over all				
Diameter				
Diameter of hole to be drilled in sleeper				

CHAIR KEYS.

	Plain line.		Crossings.	
Rail Section	...... lbs.	...... lbs.	...... lbs.	...... lbs.
Length				
Thickness thin end, for new work				
Thickness thick end, for new work				
Thickness thin end, for repairs				
Thickness thick end, for repairs				

RAILS.

The leading dimensions of Railway Rail Sections, as standardised by the British Engineering Standards Association, are given in Tables 49, 50, and 51.

RAIL SECTIONS IN USE.

		Date introduced	Section. lbs. per yd.
Rail Section for important main lines ...			
Ditto	secondary ditto ...		
Ditto	sidings, where new material is required		
Ditto	tunnels		
Ditto	water troughs		
Ditto	viaducts with longitudinal sleepers..................		
Ditto	engine pits		
Ditto	carriage pits		
Ditto	travelling cranes		
Ditto	electric positive, third, or live rail....................		
Ditto	electric negative, fourth, or return rail..................		

LENGTHS OF RAILS.

	Rail Sections.			
Standard length				
Other stock lengths				
Short lengths for inner rail on curves				
Long lengths for crossing bridges				

The B.E. Standards Association recommend the following standard normal lengths of rails: 30, 36, 45, or 60 feet.

The Measurement of Cross Section of Rails in the Road.

This is often necessary to give information as to the wear, etc. For a description of various appliances for the purpose, reference may be made to a paper by A. E. Langley in the "Permanent Way Institution Journal," Vol. 38, Part II.

The method of taking reliable cross-sections of the rails is preferable to the cumbersome and expensive practice of weighing the rails, provided that notes are made as to "chair gall" and indentation by fishplates, etc.

RAIL JOINTS AND FISH PLATES.

Expansion Spaces.

Expansion spaces, depending in amount upon the temperature of rails when being laid, must be left at the joints, so that when the rails expand owing to extremely hot weather, there may still be a small space between the rail ends, such as $\frac{1}{16}$in. for 30ft. or 45ft. rails, and $\frac{1}{8}$in for 60ft rails.

A rail free to expand and contract will alter $\frac{1}{150000}$ part of its length with each degree Fahrenheit difference in temperature, thus a 30ft. rail would alter $\frac{1}{4}$in. under a range of temperature of 100 degrees. As there are practical considerations, one being that the rails are partially fixed, tables of expansion spaces in use are not quite in keeping with these figures. A table which has been recommended for use is given below.

On some railways a thermometer, which is inserted into a piece of rail, is provided. This is laid on the ground, and indicates the temperature and the expansion space to be allowed on the particular work in progress.

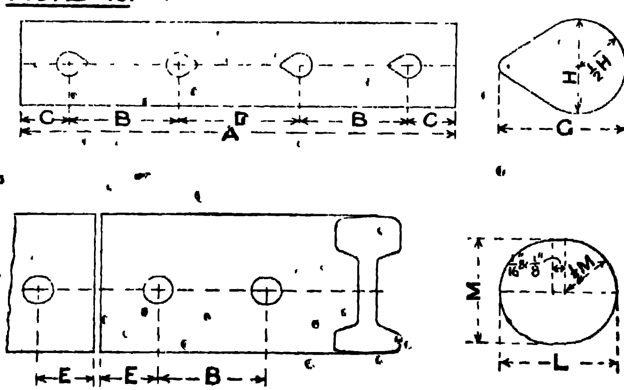
When rails are being laid, iron expansion gauges of the correct thickness should be placed in the joints, and kept in position until the fishplates are bolted up.

In tunnels, where the range of temperature is small, about one-half of the usual expansion space may be allowed.

Table of Expansion Spaces.

Kind of Day.	Temperature Fahrenheit.	Expansion space.		
		30 ft. rail.	45 ft. rail.	60 ft. rail.
		Ins.	Ins.	Ins.
Hot summer day with the sun shining on the rail	119° to 127°	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$
Spring or autumn day with the sun shining on the rail	97° to 101	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$
Hot summer day but without sunshine	75° to 84°	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{3}{8}$
Spring or autumn day but without sunshine	49° to 58	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{1}{2}$
About freezing point.........	32° to 36	$\frac{9}{32}$	$\frac{3}{8}$	$\frac{9}{16}$
Hard frost	10° to 19°	$\frac{11}{32}$	$\frac{7}{16}$	$\frac{11}{16}$

FIG. III-18.



Fishplate Dimensions, Bull-Head Rails.

	See Fig. 1	British Standard Fishplates, Report No. 47, June 1919. Rail Section. 70 & 75 lbs.	80 to 100 lbs.
		ins.	ins.
Length	A	18	18
Depth, over all	..	$3\frac{7}{32}$	$3\frac{1}{2}$§
Centres of bolt holes in plates ...	B	$4\frac{1}{2}$	$4\frac{1}{2}$
Ditto ditto ...	C	2	2
Ditto ditto ...	D	5	5
Centre of bolt holes in rails	B	$4\frac{1}{2}$	$4\frac{1}{2}$
Ditto ditto	E	$2\frac{3}{8}$*	$2\frac{3}{8}$*
Ditto ditto	F	$2\frac{1}{32}$†	$2\frac{1}{32}$†
Size of hole in plates	H	$\frac{15}{16}$	1
Ditto rails	L	$1\frac{1}{16}$*	$1\frac{1}{8}$*
Ditto ditto	L	$1\frac{1}{8}$†	$1\frac{3}{16}$†
Ditto ditto	M	1	$1\frac{1}{16}$
Diameter of drill for holes made on site..................................		—	—
Diameter of fishbolt		$\frac{7}{8}$	$\frac{15}{16}$
Length of fishbolt, not including head		—	—
Total allowance for expansion		$\frac{1}{2}$*	$\frac{1}{2}$*
Ditto ditto		$\frac{5}{8}$†	$\frac{5}{8}$†
Weight per pair in lbs.		$26\frac{1}{4}$	$32\frac{1}{4}$‡

* For rails up to 45 ft. long.

† For rails above 45 ft. long.

‡ Excepting 80 lbs which are $29\frac{3}{4}$ lbs. per pair.

§ Ditto. ditto. $3\frac{7}{16}$ ins.

Fishplates, Dimensions, Flat-Bottom Rails.

	British Standard 4-hole Fishplates.* Report, June, 1919. Rail Section. 25 to 40 lbs.	45 to 75 lbs.	80 to 100lbs.
See Fig. 18	ins.	ins.	ins.
Length .... A	14	16	18
Centre of bolt holes in plates and rails .... B	$3\frac{1}{2}$	4	$4\frac{1}{2}$
Centre of holes in plates ... C	$1\frac{3}{4}$	2	$2\frac{1}{4}$
Centre of holes in rails ...... E	$1\frac{3}{8}$	$1\frac{7}{8}$	$2\frac{1}{8}$
Diameter of fishbolt	$\frac{1}{2}$ to $\frac{11}{16}$	$\frac{3}{4}$ to $\frac{7}{8}$	$\frac{7}{8}$ to 1

* There are also B.S. 6-hole Fishplates.

The information in the above two tables is extracted by permission of the British Engineering Standards Association from their Report No. 47, 1919, Steel Fishplates for Bull-head and Flat-bottom Railway Rails; official copies of which can be obtained from the Secretary of the Association, 28, Victoria Street, Westminster, S.W. 1. Price 1s. 2d. post free.

CRANKED FISHPLATES.

Cranked fishplates are made for connecting worn to new rails, also rails of different sections. If the rail heads are of different widths, side cranking is necessary, and the plates must be in pairs of one right and one left hand. Insulated fishplates are used at the extremities of a track circuit.

CHAPTER IV.

MATHEMATICAL CALCULATION.

A simple explanation will now be given of the mathematical knowledge necessary to understand the derivation and employment of the rules set forth in the following chapters. The explanation will serve to indicate to the student the range of mathematical knowledge required, and at the same time refresh the memory of those whose studies have previously covered this subject.

Elementary students should supplement the notes by the study of a text book on "Practical Mathematics," preferably one of a modern type.

ARITHMETIC.

Signs.

$9+6$ is read as "9 plus 6" and means "to 9 add 6."

$9-6$ is read as "9 minus 6" and means "from 9 deduct 6."

9×6 is read as "9 multiplied by 6."

$9\div 6$ is read as "9 divided by 6" and also as "9 over 6."

$=$ is read as "equals."

$\therefore$ is read as "therefore."

$\frac{9}{6}=\frac{3}{2}$ is read as "9 over 6 equals 3 over 2" and is a way of expressing proportion.

$9:6::3:2$ is read as "9 is to 6 as 3 is to 2" and is another way of expressing proportion.

9^2 signifies "9 squared" and equals $9\times 9=81$.

$\sqrt{9}$ signifies the square root of 9.

Brackets thus ( ), mean that quantities within them must be taken together, and worked out before adding to or multiplying by, etc., quantities outside of the brackets, thus:—

$$7\times(4+6-2)=7\times 8=56.$$

$$8-(4+6-2)=8-8=0.$$

$$\sqrt{(12+7-3)}=\sqrt{16}=4.$$

The last, however, is generally written thus:—

$$\sqrt{12+7-3}$$

When there are no brackets the operations of squaring and extracting square roots must be done first, then the multiplying and dividing, and lastly the addition and subtraction, thus:—

$$\begin{aligned} &7 \times \sqrt{9} + 6^2 \div 2 \\ &= 7 \times 3 + 36 \div 2 \\ &= \ 21 \ + \ 18 \\ &= \quad 39. \end{aligned}$$

Fractions.

$\frac{5}{8}$ is a Vulgar Fraction, the top figure (5) being the Numerator, and the bottom figure (8), the Denominator.

$\frac{13}{3}$ and $\frac{8}{1}$ are Improper Fractions, because they are not less than 1, their values being $4\frac{1}{3}$ and 8. The latter example showing how a whole number may be expressed as a fraction.

$\frac{12}{16}$ is a fraction which is not in its lowest terms, because the 12 and the 16 will "cancel" by dividing by the "greatest common measure" of 12 and 16 which is 4. It then becomes $\frac{3}{4}$.

To add or subtract fractions: bring them all to the same denominator by finding the "least common multiple," and add or subtract the new numerators thus:—

$$\frac{2}{3} + \frac{3}{4} - \frac{1}{6} = \frac{8+9-2}{12} = \frac{15}{12} = \frac{5}{4} = 1\tfrac{1}{4}$$

To multiply fractions: multiply the numerators together and multiply the denominators together, previously cancelling if any numerator and demoninator will divide by the same number, thus:—

$$\frac{\overset{3}{\cancel{15}}}{4} \times \frac{3}{\underset{2}{\cancel{10}}} \times \frac{\overset{1}{\cancel{2}}}{7} = \frac{9}{28}$$

To divide one fraction by another: invert the divisor and proceed as in multiplication, thus:—

$$\frac{3}{5} \div \frac{2}{3} = \frac{3}{5} \times \frac{3}{2} = \frac{9}{10}$$

To find the square root of a fraction: take the square root of numerator, and place it over the square root of denominator:—

$$\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$$

Otherwise the fraction may be reduced to a decimal, and the square root of this decimal extracted, or taken from tables, as referred to later.

Proportion.

To find the "unknown" of four proportionals, the algebraic method is simplest. x being the "unknown," the proportion may be written thus:—

$$\frac{2}{3} = \frac{x}{8}$$

Then, noting that any value taken from a denominator to the other side of the equation must be placed in the numerator and *vice versa*, we get:—

$$\frac{2 \times 8}{3} = x$$

$$\frac{16}{3} = x$$

$$5\tfrac{1}{3} = x$$

The advantage of this method is that the terms have not to be set down in one certain order, for instance the above may be written:—

$$\frac{3}{2} = \frac{8}{x}$$

$$\text{then } \frac{x}{1} = \frac{8 \times 2}{3}$$

$$\text{or } x = \frac{16}{3} = 5\tfrac{1}{3}$$

Decimals and Conversions.

In multiplying or dividing lengths by numbers or other lengths, the best general way of working is to convert the lengths into the same unit of measurement, usually to feet and decimals thereof.

From the tables, to obtain the square of a number containing decimals: Put down the square of the number as if there were no decimals, then give it twice the number of decimal places as in the original number.

To obtain the square of a number beyond the reach of the tables: Divide the number by 2 and multiply the square of the result by 4. Any other numbers may be used instead of 2 and 4, so long as the latter is the square of the former, such as 10 and 100.

When the number is not divisible, and exactness is required, we may proceed as follows:—

Required, the square of 1837.

1st. From Table 4	$183^2 = 33489$
2nd. Adding one 0 to the number adds two 0's to the square, thus	$1830^2 = 3348900$
3rd Calling 1830 = N, take the diff. for 7 from Table 4a, thus, 14 times N plus 49	$= 25669$
	$1837^2 = 3374569$

Square Root, the use of Tables.

Space does not allow for an explanation of the process of extracting the square root of a number, but the use of "Four Figure Tables" will be explained

To obtain the square root of a number beyond the reach of the tables: Divide the number by 4 and multiply the square root of the result by 2.

To obtain the square root of a number containing decimals:—

For convenience, we will call the following numbers "odd figures," viz., numbers with 1, 3, or 5, etc. integers, and fractions with 1, 3, or 5, etc., ciphers following the decimal point, thus:—

15230, 152·3, 1·523 and ·0523 will be "odd."
and 1523, 15·23, ·1523 and ·00523 will be "even."

EXAMPLES.—(A.) Required $\sqrt{15 \cdot 23}$

This is an "even figure," so from the tables take the square root of a similar "even figure," thus:—

$$\sqrt{1523} = 39 \cdot 026$$

Two less digits in the integer means one less in the root, so:—

$$\sqrt{15 \cdot 23} = \underline{\underline{3 \cdot 9026}}$$

(B.) Required $\sqrt{1 \cdot 523}$

This is an "odd figure," and a similar odd figure (15230) is beyond the reach of tables, so we may take:—

1st. $\sqrt{152} = 12 \cdot 33$

and therefore $\sqrt{1 \cdot 52} = 1 \cdot 233$

Next look up what number 1233 is the square of,

$$1233^2 = 1520289$$

$$\text{but } 1234^2 = 1522756$$

and we see that the latter number is nearer our 1523, and our root will be $\underline{\underline{1 \cdot 234}}$

It will be useful to remember that:—

Roots of numbers of 100 and under 10,000 have two integers.

Roots of numbers of 1 and under 100 have one integer.

Roots of numbers of ·01 and under 1 are decimals with no cyphers following the point.

Roots of numbers of ·0001 and under ·01 are decimals with one cypher following the point.

Reciprocals.

The reciprocal of a number is the result of dividing that number into 1. In other words, the reciprocal is the decimal fraction corresponding to a vulgar fraction whose denominator is the number and the numerator is 1, thus:—

The reciprocal of 16 is $\frac{1}{16}$ or ·0625.

A table of reciprocals such as Table 4 is useful:—

A.—In converting vulgar fractions to decimals.

EXAMPLE.—

$$\frac{1}{64} = \underline{\underline{\cdot 015625}}$$

$$\text{and } \frac{5}{64} = 5 \times \cdot 015625 = \underline{\underline{\cdot 078125}}$$

B.—In avoiding division by a large number, as we may instead, multiply by the reciprocal.

C.—In obtaining a result when the product of two numbers has to be divided by their sum or difference of which a number of cases occur in the subject in Hand.

To find the reciprocal of a number containing decimals, such as 12·36: First take the reciprocal, neglecting the decimal, thus:—

Reciprocal of 1236 = ·000809.

Then each decimal place moved to the left in the number moves the decimal a place to the right in the reciprocal, and *vice versa*, thus:—

Reciprocal of 12·36 = ·0809.

General Remarks.

The reader, by experience, will soon see what degree of accuracy is required in calculation; for instance, when calculating a Lead of a Turnout, the length to the nearest inch will be sufficiently accurate. In the case of a crossing angle, the nearest ½ will be near enough as a result, except for angles of small number, say under 1 in 4; but if the Number of the Crossing as found, is to be used to find some other dimension, such as a Radius, it will be necessary to state its value to two or three decimal places. Generally it is more important to keep small multipliers and divisors accurate than large ones.

A rough check should always be made on a calculation, such as by neglecting decimals or mentally converting decimals to fractions. Such a check will often lead to the discovery that the decimal point has been wrongly placed, or some factor forgotten.

There are other simplified methods of arithmetical calculation such as the use of Logarithms and of the Slide Rule. Space forbids a detailed explanation of these, but it may be stated, that whilst the Slide Rule is of little service in the type of calculation required, the use of Logarithms will be found a great advantage where much calculating is undertaken.

ALGEBRA AND THE USE AND CONVERSION OF FORMULÆ.

Though it is intended that most of the Rules in this book may be put to use by those who have only a knowledge of ordinary arithmetic, the derivations of the rules

cannot well be understood without a knowledge of the elementary operations of Algebra. Without this knowledge the student cannot expect to make progress in investigations on his own account, nor can he read any but the most rudimentary treatises on the subject.

Unless the student understands a rule or statement expressed as a formula, all the rules by which he is to obtain his results must be put down in lengthy wording. Again, he will have difficulty in transposing his rule when another dimension to that found by his first rule is required; or, in other words, in turning his rule round.

To elucidate these remarks we may take a rule, given later, for finding the Lead of a Turnout when the Number of the crossing and the particulars of the Switch are known.

This rule in words is:—From the gauge deduct the divergence at the switch heel. Multiply twice this dimension by the switch angle number, the result by the crossing number, and divide the product by the sum of the switch and crossing angle numbers.

The rule expressed as a formula, with symbols given in Chapter XIII. to represent the various dimensions, becomes:—

$$L = \frac{W \times 2 \times M \times N}{M + N}$$

The multiplication signs, however, are generally omitted.

Now suppose instead of the Number of crossing being given, the Lead is given, and we wish to calculate the Number (N); it would be almost impossible without using elementary algebra to discover that

$$N = \frac{L \times M}{(2 \times M \times W) - L}$$

It must also be noted that in mechanically following the operations of the worded rule, opportunities of shortening the work by cancelling out, etc., will be missed.

In Algebra, values such as lengths, weights, etc., are represented by symbols or letters, and in some branches of engineering, such as Electricity and Reinforced Concrete Construction, the symbols have been standardised, thus rendering it easy to compare rules given by different writers.

There appears to be no generally accepted notation which will cover the dimensions we are here concerned with, and therefore the symbols given in Chapter XIII. and elsewhere have been chosen; an endeavour being made to keep to those largely used by other writers, and also to render them suggestive of their meanings.

Signs, etc.

In Algebra the signs $+$, $-$, $\div$, and $=$ are used as in Arithmetic. The following statements will show other ways of indicating operations to be performed on numbers and symbols:—

$a \times b$, that is the product of two letters, is generally written ab, as also is the product of a number and a letter thus, $5 \times a$ is written $5a$. In this case 5 being a number, is known as the coefficient of a.

Brackets thus ( ) indicate that the quantities within them must be taken altogether. When there is no sign between two sets of brackets, the results of the quantities within the brackets must be multiplied together, thus:—

$(a-b)\ (c-d) =$ a less b, multiplied by c less d.

$7\ (a-b) =$ a less b, multiplied by 7.

$(a \times b) + (c \times d) =$ the product of a and b, plus the product of c and d.

$(a-b) \div (c-d) =$ the difference of a and b, divided by the difference of c and d.

$7 + (a-b) = 7 + a - b.$

$7 - (a - b + c) = 7 - a + b - c.$

The last exemplifies the rule that a minus sign outside a bracket changes all the signs inside, on the bracket being removed.

$a \div b$ that is, a divided by b, is generally indicated as a fraction thus, $\frac{a}{b}$ and $(a+b) \div (a-b)$, thus $\frac{a+b.}{a-b.}$

That a quantity "a" is to be squared, is indicated thus, a^2, which $= a \times a$, and $(a+b)^2 = (a+b) \times (a+b)$.

It must be noted that:—

$$ab^2 = a \times b^2.$$

whilst $(ab)^2$ = the square of ab or $a^2 \times b^2$

also $\frac{a^2}{b} = a^2 \div b$

and $\left(\frac{a}{b}\right)^2 = a^2 \div b^2$

The small "2" which indicates the square or "second power" of a, is named an "index." An index is distinct from a "suffix," which is a small number, letter, or dash at the foot of a symbol to give a special meaning, for example, N_2 may mean "the Number of the second crossing."

The sign $\sqrt{}$ or $\sqrt{}$ means that the square root of the quantity is to be taken, thus:—

$$\sqrt{9} = 3$$

$$\sqrt{9 + 7} = \sqrt{16} = 4$$

$$\sqrt{(a + b)} \text{ or } \sqrt{a + b} = \text{square root of } (a + b)$$

$$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Proportion.

Proportion, instead of being expressed as in arithmetic, thus:—a : b : c : d, is generally expressed $\frac{a}{b} = \frac{c}{d}$ which is a more useful form.

A collection of algebraical quantities such as, 4a + 6x − 3b, is known as an "expression," and the separate quantities 4a and 6x are called "plus terms," whilst 3b is a "minus term."

If we know the values of a, x, and b in the above expression, to obtain its ultimate value the operations of multiplication must be done first and the addition and subtraction afterwards.

Thus suppose a = 3, x = 2, and b = 4, then

$$\begin{aligned} &4a + 6x - 3b \\ &= (4 \times 3) + (6 \times 2) - (3 \times 4) \\ &= 12 + 12 - 12 \\ &= \underline{\underline{12}} \end{aligned}$$

When the symbols of two or more terms do not differ and are of the same "power," the terms are named "like," otherwise they are "unlike," thus, 4a, $-3a$ and $\frac{a}{3}$ the last being $\frac{1}{3}a$, are like terms, whilst $4a^2$, 4a, $\sqrt{a}$, and 4ab, are all unlike terms.

We can add or subtract like terms to make one term, thus, $4a+3a=7a$, and $5\sqrt{a}-3\sqrt{a}=2\sqrt{a}$; but this is not so with unlike terms.

Addition.

In adding several expressions, place the like terms under each other and add their coefficients thus:—

$$\begin{array}{r} 3x^2+7x-3a+1 \\ 2x^2+4x+3a-8 \\ -2x-6a+2 \\ \hline 5x^2+9x-6a-5 \end{array} = \text{Sum.}$$

Subtraction.

Subtraction is performed as an addition, but in the expression to be deducted, all the plus signs must be read as minus signs, and *vice versa*, thus:—

$$\begin{array}{rr} & 4x^2+3x-a-4 \\ \text{Less} & 2x^2-5x-2a+6 \\ \hline & =2x^2+8x+a-10 \end{array}$$

Multiplication.

In multiplying two expressions, multiply every term of one expression by each term of the other. When any two terms being multiplied have the same sign, prefix a plus sign to their product, and when they have different signs prefix a minus sign.

Also note that:—

$$a \times a = a^2$$
$$-a \times -a = a^2$$
$$-a \times a = -a^2$$
$$a^2 \times a = a^3$$
$$4a \times 3b = 12ab$$
$$4a^2 \times 3b = 12a^2b$$
$$(a+b)\times(a+b)=a^2+2ab+b^2$$
$$(a+b)\times(a-b)=a^2-b^2$$
$$(a-b)\times(a-b)=a^2-2ab+b^2$$

Division.

The rules for dividing one expression by another are simple, but rather lengthy. We shall usually in this work, only have to divide an expression by a single term. In this case the rule is to divide each term of the expression by the divisor, thus:—

$$2a \,\big|\, \underline{2a^2 + 4ab - b^2 + 6}$$
$$= a + 2b - \frac{b^2}{2a} + \frac{3}{a}$$

Note that:—

$$a \div a = 1$$
$$a^2 \div a = a$$
$$a \div b = \frac{a}{b}$$
$$ab \div a = b$$
$$4a \div 3a = \tfrac{4}{3}$$

Simple Equations.

An equation is a statement that two expressions are equal.

An equation is generally made up in order to find some unknown quantity, which process is known as "solving the equation."

To solve an equation we may employ any one of the following artifices:—

1. Add the same quantity to each side of the equation.
2. Deduct the same quantity from each side of the equation.
3. Multiply or divide both sides by the same quantity.
4. Square or extract the square root of each side.

In accordance with 1 and 2, a quantity may be removed from one side of the equation to the other by taking it over and changing its sign. This is known as "transposing."

EXAMPLES:—

Required M when,

$$4M + 6 = 26$$
Transpose $\quad 4M = 26 - 6$
that is $\quad 4M = 20$
$\div$ by 4 $\quad \underline{\underline{M = 5}}$

Required N when

$$\frac{N}{4} - 5 = 9$$

Transpose $\frac{N}{4} = 9 + 5$

$$\frac{N}{4} = 14$$

$\times$ by 4 $N = 56$

The general rules for simple equations are :—

1. Clear of fractions by multiplying throughout by the least common multiple of the denominators of the fractions.
2. Transpose all the terms with the "unknown" in them to the left hand side, and the "knowns" to the other side.
3. Divide by the coefficient of the "unknown," and the "unknown" will be obtained.

In accordance with Artifice No. 3, we may note that if fractions completely form the sides of an equation, a numerator may be taken to the other side and there put into the denominator, and *vice versa*. A useful variety of using this rule is known as "cross multiplying" thus :—

If $\frac{a}{b} = \frac{c}{d}$

then $ad = bc$.

Examples of equations will occur in the treatment of the various problems.

A Quadratic Equation is one in which the square of an unknown quantity is involved; an example occurs in Chapter XIV. (Rule 4d).

Formulæ.

Broadly speaking, a formula is a statement which shows the relation existing between a certain set of quantities or measurements. For instance :—

$$L = \frac{2\ W\ M\ N}{M + N}$$

is a formula showing the relation between L, W, M, and N.

In this particular form it is suited for finding L when W, M, and N are known. By the operations of algebra the formula may be manipulated either to give any one of

the other values when the remainder are known, or to show the relation between two of the values when the others are known.

In the course of the work the rules will generally be given in the shape of formulæ, the simpler formulæ will be translated into words, and an example of the arithmetical working out of the more frequently used formulæ will be given.

TRIGONOMETRY.

As the name implies, Trigonometry deals with the measurement of triangles.

A simple explanation of the elements of the subject is called for, so that the presence of Sines, Cosines, etc., in rules and their derivations may present no difficulty to the reader previously unacquainted with their use.

The explanation will only extend to the solution of the Right Angled Triangle, and it must be understood that such a triangle is referred to, unless otherwise stated.

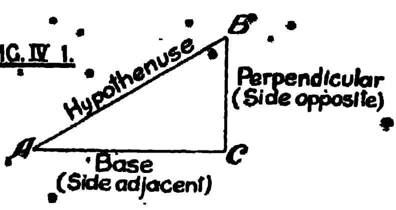
In the course of the calculations we shall often find that we know the length of only one of the sides of a triangle. Now if we also know the measure of one of its acute angles in degrees, see Chapter V., or other method of measurement, we can by Trigonometry, obtain either of the other sides. This we do by multiplying the given side by a *number* depending upon the measure of the angle. This *number* is known as a Trigonometrical Ratio, and is simply the ratio of the required side to the known side.

Ratio means the number of times a first quantity contains a second, therefore a ratio can always be expressed as a fraction, for example, four ways of expressing the ratio of 3 to 4 are:—

$$3 : 4, \quad 3 \div 4, \quad \tfrac{3}{4} \text{ and } \cdot 75.$$

Fig. IV.-1 shows the names usually given to the sides of a triangle with respect to the angle A, the names in brackets being preferred to the others by some writers.

FIG. IV 1.



The Trigonometrical Ratios, sometimes called Functions, of the angle A are the ratios of the sides one to another, thus:—

The ratio $\frac{\text{perp}}{\text{hyp}}$ = The Sine or Sin A

,, ,, $\frac{\text{base}}{\text{hyp}}$ = ,, Cosine ,, Cos A.

,, ,, $\frac{\text{perp}}{\text{base}}$ = ,, Tangent ,, Tan A.

,, ,, $\frac{\text{hyp}}{\text{perp}}$ = ,, Cosecant ,, Cosec A.

,, ,, $\frac{\text{hyp}}{\text{base}}$ = ,, Secant ,, Sec A.

,, ,, $\frac{\text{base}}{\text{perp}}$ = ,, Cotangent ,, Cot A.

The three latter ratios are the inverse of the three former ratios.

Two other ratios or functions are:—

Versed Sine of A or Vers A = 1 − Cos A.
Coversed Sine ,, Covers A = 1 − Sin A.

The Sines, Cosines, etc., of any acute angle, the value of which is given in degrees and minutes, may be extracted direct from mathematical tables, where they are given as decimal fractions.

If the angle contains seconds, a difference proportional to the number of seconds must be added or subtracted as the case may be.

EXAMPLE.—

Required the Sine and Cosine of 7° 9′ 8″.

The Sine of 7° 9′ from the tables = ·1244674

The difference for 1′ more = plus 2886

,, ,, 8″ ,, = $\frac{8}{60}$ of 2886 = plus ·0000384

∴ Sine 7° 9′ 8″ = ·1245058

The Cosine of 7° 9′ from the tables = ·9922237

The difference for 1′ more = minus 363

∴ ,, ,, 8″ ,, = ,, $\frac{8}{60}$ of 363 = minus ·0000048

∴ Cos of 7° 9′ 8″ = ·9922189

In using tables it is helpful to remember the following points:—

The Co-functions alone, decrease as the angle increases.

The Sine and Cosine are always less than 1, and therefore make a quantity less if multiplied by them, and greater if divided by them.

The Secant and Cosecant are always greater than 1.

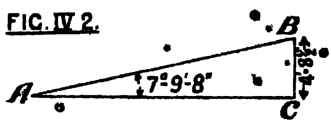
The Tangent and Cotangent may be of any value.

Solution of Right Angle Triangles.

(1) TO FIND THE SIDES FROM THE ANGLE A AND A GIVEN SIDE.

EXAMPLE —In the triangle $A\,B\,C$ Fig. IV.—2 the angle at A is 7°.9′ 8″., the perp $B\,C$ is 4′ 8½″ = 4·708′.

FIG. IV 2.



Required the hyp. $A\,B$ and the base $A\,C$.

To find $A\,B$,

$$\frac{A\,B}{B\,C} = \frac{\text{hyp}}{\text{perp}} = \frac{\text{Cosec A}}{1}$$

$$\therefore\ \text{hyp} = \text{perp} \times \text{Cosec A}$$

$$\text{i.e. } A\,B = 4{\cdot}708' \times 8{\cdot}031$$

$$= \underline{37{\cdot}81'}$$

To find $A\,C$,

$$\frac{A\,C}{B\,C} = \frac{\text{base}}{\text{perp}} = \frac{\text{Cot A}}{1}$$

$$\therefore\ \text{base} = \text{perp} \times \text{Cot A}$$

$$\text{i.e. } A\,C = 4{\cdot}708 \times 7{\cdot}969$$

$$= \underline{37{\cdot}52'}$$

Another way to find $A B$ is,

$$\frac{B C}{A B} = \frac{\text{perp}}{\text{hyp}} = \frac{\text{Sin } A}{1}$$

$$\therefore \frac{\text{perp}}{\text{Sin A}} = \text{hyp.}$$

$$\text{i.e. } \frac{4{\cdot}708'}{{\cdot}1245} = A B$$

$$37{\cdot}81' = A B$$

In the above we first form a ratio between the required length and the given length and then state that this equals the Trig. Ratio between them. Next we get the required length as one side of the equation, remembering that a value above the line, when transferred to the other side goes below the line, and *vice versa*.

By this procedure we may obtain the rules given below:—

Per	= hyp	× Sin	or hyp	÷ Cosec
Per	= base	× Tan	„ base	÷ Cot
Base	= hyp	× Cos	„ hyp	÷ Sec
Base	= perp	× Cot	„ perp	÷ Tan
Hyp	= base	× Sec	„ base	÷ Cos
Hyp	= perp	× Cosec	„ perp	÷ Sin

(2) To FIND THE ANGLE A WHEN TWO SIDES OF THE TRIANGLE ARE GIVEN.

The procedure is to form a Trig. Ratio by dividing one side by the other, give this ratio its name, and look up in the tables what angle it is the ratio of.

EXAMPLE.—The base of a triangle is 8 feet and the perpendicular is 6 inches; required, the angle.

$$\begin{array}{rl} \text{Perp} \div \text{base} & = \text{Tan A} \\ \text{i.e. } {\cdot}5' \div 8' & = \text{Tan A} \\ \therefore \ {\cdot}0625000 & = \text{Tan A} \\ \text{From tables } {\cdot}0623306 & = \text{Tan } 3^\circ\ 34' \\ \text{difference} \quad 1694 & \end{array}$$

To find how many seconds this is the difference for, multiply it by 60 and divide by difference for 1 minute, *i.e.*, for 60 seconds, thus:—

$$\frac{1694 \times 60}{2920} = 35 \text{ seconds.}$$

The required angle is therefore $3^\circ\ 34'\ 35''$.

In all cases, when a second side of a triangle has been obtained, the third may be found by the rule as to the squares of the sides (Euclid 1—47), but if trigonometrical tables are to hand, it may just as well be found by using them.

If the third angle is required, it is found by deducting the angle A from 90°, because the three angles of any triangle = 180°.

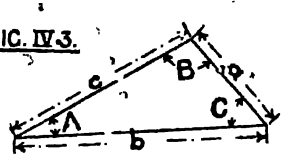
General Trigonometrical Formulæ.

There are formulæ for solving triangles which are not right-angled, and there are also certain relations between the ratios. In this work recourse to this knowledge will seldom be necessary, and the notes will be confined to the following brief statements:—

Formulæ for Solution of Triangles (which are not right-angled).

Given, three sides.

FIG. IV 3.



$$\text{Cos A} = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\text{Cos B} = \frac{c^2 + a^2 - b^2}{2ca}$$

Given, two sides and the included angle.

$$a^2 = b^2 + c^2 - 2bc \text{ Cos A}$$

$$\text{and Sin B} = \frac{b \text{ Sin A}}{a}$$

Given, two angles and a side.

$$b = \frac{a \text{ Sin B}}{\text{Sin A}} \text{ and } c = \frac{a \text{ Sin C}}{\text{Sin A}}$$

Given, two sides and an opposite angle.

$$\text{Sin B} = \frac{b}{a} \text{ Sin A and } c = \frac{a \text{ Sin C}}{\text{Sin A}}$$

NOTE.—*In the text books, additional formulæ are given for use when logarithms are employed.*

Relations between the Ratios.

$$\text{Sin A} = \frac{1}{\text{Cosec A}} = \sqrt{1-\text{Cos}^2 \text{A}} = \frac{\text{Tan A}}{\sqrt{1+\text{Tan}^2 \text{A}}}$$

$$\text{Cos A} = \frac{1}{\text{Sec A}} = \sqrt{1-\text{Sin}^2 \text{A}} = \frac{\text{Cot A}}{\sqrt{1+\text{Cot}^2 \text{A}}}$$

$$\text{Tan A} = \frac{1}{\text{Cot A}} = \frac{\text{Sin A}}{\text{Cos A}} = \sqrt{\text{Sec}^2 \text{A} - 1}$$

$$\text{Cosec A} = \frac{1}{\text{Sin A}} = \sqrt{1+\text{Cot}^2 \text{A}}$$

$$\text{Sec A} = \frac{1}{\text{Cos A}} = \sqrt{1+\text{Tan}^2 \text{A}}$$

$$\text{Cot A} = \frac{1}{\text{Tan A}} = \frac{\text{Cos A}}{\text{Sin A}} = \sqrt{\text{Cosec}^2 \text{A} - 1}$$

$\text{Sin A} = \text{Cos}\,(90^\circ - \text{A})$ $\qquad$ $\text{Cos A} = \text{Sin}\,(90^\circ - \text{A})$

$\text{Sin}\,(180^\circ - \text{A}) = \text{Sin. A}$ $\qquad$ $\text{Cos}\,(180^\circ - \text{A}) = -\text{Cos A}$

$\text{Tan}\,(180^\circ - \text{A}) = -\text{Tan A}$, etc.

$\text{Sin}\,(\text{A}+\text{B}) = \text{Sin A Cos B} + \text{Cos A Sin B}$

$\text{Sin}\,(\text{A}-\text{B}) =$ ditto $-$ ditto

$\text{Cos}\,(\text{A}+\text{B}) = \text{Cos A Cos B} - \text{Sin A Sin B}$

$\text{Cos}\,(\text{A}-\text{B}) =$ ditto $+$ ditto

$$\text{Tan}\,(\text{A}+\text{B}) = \frac{\text{Tan A} + \text{Tan B}}{1 - \text{Tan A Tan B}}$$

$$\text{Cot}\,(\text{A}+\text{B}) = \frac{\text{Cot A Cot B} - 1}{\text{Cot B} + \text{Cot A}}$$

$$\text{Cot 2 A} = \frac{\text{Cot}^2\text{A} - 1}{2\,\text{Cot A}} \qquad \text{Sin 2 A} = \frac{2\,\text{Tan A}}{1+\text{Tan}^2\text{A}}.$$

$2\,\text{Sin A Cos B} = \text{Sin}\,(\text{A}+\text{B}) + \text{Sin}\,(\text{A}-\text{B})$

$2\,\text{Cos A Sin B} =$ ditto $-$ ditto

$2\,\text{Cos A Cos B} = \text{Cos}\,(\text{A}+\text{B}) + \text{Cos}\,(\text{A}-\text{B})$

$2\,\text{Sin A Sin B} = \text{Cos}\,(\text{A}-\text{B}) - \text{Cos}\,(\text{A}+\text{B})$

etc.

CHAPTER V.

GENERAL GEOMETRICAL THEOREMS AND PROBLEMS.

The solution of all problems in connection with the alignment of railway track depends initially upon the facts of Elementary Geometry. There is nothing remarkable about this statement, when it is remembered that this branch of mathematics may be said to be independent of any other, its truths being built up by plain reasoning from a basis of self-evident facts, or "axioms."

It will be sufficient for the purpose if we state the Geometrical "Theorems," or proved facts, which are useful in solving the kind of problems arising in trackwork, leaving the student to study their proofs in the text books.

A number preceding a statement refers to the Book and Proposition of Euclid from which it is derived, this ancient author being still the universal standard of reference.

Certain geometrical definitions are given in Chapter II., but a knowledge on the reader's part of the elementary definitions and more obvious facts of geometry will be assumed.

GEOMETRICAL THEOREMS OR FACTS.

Triangles.

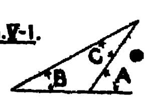
I., 5 and 6. In a triangle, equal sides have equal angles opposite to them, and *vice versa.*

I., 4, 8, and 26. If a triangle has three of its six measurements, which are three sides and three angles, equal to three similar measurements, each to each, in another triangle, then, provided one of the measurements is a side, the two triangles are equal in every respect.

In certain cases, known as ambiguous cases, which are easily recognisable and need not here be discussed, this rule does not apply.

I., 32 (Fig. V.—1). If the side of a triangle be produced, the exterior angle is equal to the sum of the two interior and opposite angles,

FIG. V-1.



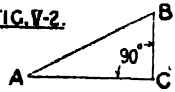
i.e., Angle A = Angle B + Angle C.

I., 32. The three angles of any triangle are equal to two right angles, or 180 degrees.

I., 37. Triangles on equal bases and between the same parallels, that is of the same height, are equal in area (The area = base × height.)

I., 47 (Fig. 2). In any right-angled triangle the square on the hypothenuse, or side opposite to the right angle, is equal to the sum of the squares on the sides containing the right angle.

FIG. V-2.



$$AB^2 = AC^2 + BC^2.$$

VI., 4. (Fig. 3). Similar triangles are those which have their angles equal each to each, and thus may be said to be of the same shape.

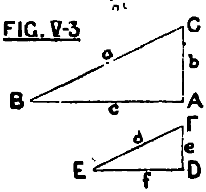
The sides of similar triangles are proportionals.

This fact is very important, and wide in its application, as the following examples will show:—

If the angles of the triangle A B C are equal to those of D E F, each to each, then:—

$$a : b :: d : e \quad \text{etc.}$$
$$\text{and } a : d :: b : e \quad \text{etc.}$$

FIG. V-3



VI., 2 (Fig. 4). If a line be drawn parallel to one side of a triangle, it cuts the other sides, or those sides produced, proportionally

Thus if FD or F_1D_1 be drawn parallel to the side CA, then :—

$$BF : FC :: BD : DA$$
$$BC : FC :: BA : DA$$
$$BC : CF_1 :: BA : AD_1 \quad \text{etc.}$$

It will be seen that the parallel lines form similar triangles to the original one. Figs. 4a and 4b show applications of the above to a switch angle and a crossing angle.

FIG. V 4.

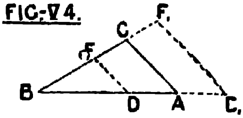


FIG. V-4a.

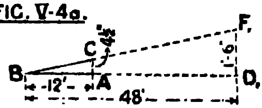
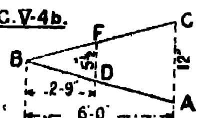


FIG. V-4b.



Circles.

III., 3 (Fig. 7). The centre of a circle lies in a line bisecting any chord at right angles.

In Fig. 7, the centre of the circle lies in line B D O, which bisects the chord A C at right angles.

III., 11. If two circles touch each other, the line joining their centres passes through the point of contact.

In Figs. 5 and 6 the line H O O_1, joining the centres O and O_1, passes through H.

III., 11a (Figs. 5 and 6). If a circle touches a second circle, and also a given line, then a line drawn through the points of contact will also pass through the end of a diameter of the second circle which is perpendicular to the given line.

FIG. V-5.

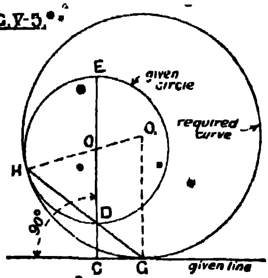
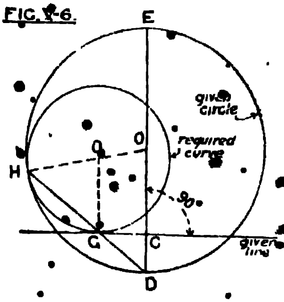


FIG. V-6.



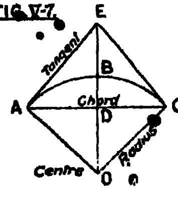
Referring to the diagrams, the value of this fact is seen, because if we know the point G at which a required curve touches a line, we can obtain the point H at which it also touches a given curve, and *vice versa*.

III., 18 (Fig 7). The centre of a circle lies in a line drawn at right angles to a tangent at its point of contact, *i.e.*, Radius is at right angles to tangent.

In Fig. 7 Radius A O is at right angles to tangent A E.

From a point outside of a circle only two tangents can be drawn to the circle, and these are equal in length. In Fig. 7 the tangents from point E, namely E A and E C, are equal.

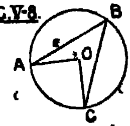
FIG. V-7.



III., 20 (Fig. 8). The angle at the centre of a circle is double the angle at the circumference on the same base or arc.

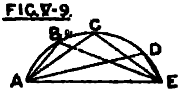
Angle A O C = Angle A B C × 2.

FIG. V-8.



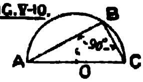
III., 21 (Fig. 9). All angles in the same segment are equal. Angles A B E, A C E, and A D E are equal. This forms the basis for the process occasionally employed of setting out curves by the use of two theodolites, which would be set up at A and E.

FIG. V-9.



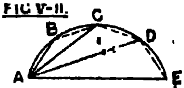
III., 31 (Fig. 10). The angle in a semi-circle is a right angle.

FIG. V-10.



III., 26 and 29 (Fig. 11). Equal chords are subtended by equal angles. The equal chords B C, C D, and D E are subtended by equal angles B A C, C A D, and D A E. This fact forms the basis for the ordinary process of setting out curves by the use of a theodolite, which would be set up at A.

FIG. V-11.

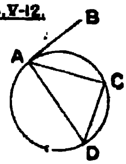


III., 22. The opposite angles of a four-sided figure in a circle are equal to two right angles.

III., 32 (Fig. 12). The angle which a tangent makes with a chord, is equal to the angle in the opposite segment.

Angle B A C = Angle A D C.

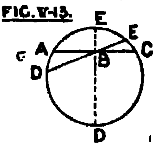
FIG. V-12.



III., 35 (Fig. 13). If two lines in a circle cut each other, the product of the segments of one is equal to the product of the segments of the other.

A B × B C = D B × B E.

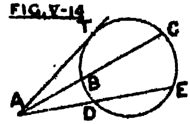
FIG. V-13.



III., 36 (Fig. 14). If from a point outside of a circle any line be drawn cutting the circle, the product of the whole line, which is called a secant, and its part without the circle is equal to the square of the tangent line from the point.

A T squared = A C × A B = A E × A D.

FIG. V-14.



III., 35 A (Fig. 15). From several of the above theorems a certain useful working rule may be derived which will be frequently applied throughout this work.

This rule is generally stated thus:—

The Rise of an Arc is equal to the square of the Chord of half the Arc, divided by twice the Radius.

The wording adopted, however, will be:—

The Versed Sine = Sub-chord squared ÷ twice Radius.

Or in Symbols to be explained later:—

$$V = C_s^2 \div 2R$$

It will be shown how this rule is derived, which will serve as a typical example of the manner in which the theorems may be applied in establishing useful working rules.

With regard to the Arc A B C, A C is the Chord, B D is the Versed Sine, and A B is the Sub-chord.

The Chord A C of the Arc is bisected at right angles by the line B D E.

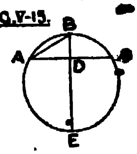
B E therefore passes through the centre of the Circle, and so is a diameter or twice the Radius (III., 3).

A D × D C = B D × D E (III., 35)

but A D = D C

∴ $AD^2 = BD \times DE$

FIG. V-15.



Also $AD^2 + BD^2 = AB^2$ (I., 47).

For AD^2 substitute B D × D E, then:—

$BD \times DE + BD^2 = AB^2$

that is $BD \times (DE + BD) = AB^2$

and $BD = AB^2 \div (DE + BD)$.

But D E + B D = twice Radius, and so the rule is proved.

GENERAL GEOMETRICAL PROBLEMS.

A problem in geometry consists of finding how to draw a line, a circle, or other figure, to fulfil certain conditions.

The text books of geometry give methods of solving such problems, but it is impracticable to deal with all the problems in railway work by geometrical methods alone. Drawing must be supplemented by calculation to give the necessary freedom and scope to the procedure.

This will be evident when it is remembered that we are not concerned with merely making small scale drawings, aided by compasses and angle measurers, etc., but with the setting out on the ground, and the obtaining of particular dimensions by calculation for reference tables, etc.

General problems in lines, angles, and some of the elements of the circle will now be dealt with, leaving to the next chapter the special problems in setting out circular curves.

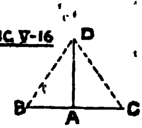
The proofs of the problems depend upon the foregoing theorems, and a reference to the theorem concerned will be given, except where the procedure is obvious.

The letters (A), (B), etc., show alternative methods of treating the problems.

Prob. 1. From a given point in a straight line, to set a line at right angles to the first line.

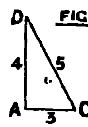
(A) (Fig. 16.) On either side of given point A measure off equal distances A B and A C. From B sweep out any suitable distance B D and cut it with the same distance from C. D A is then at right angles to B C.

FIG. V-16



(B) (Fig. 17.) Using the principle of Euclid, I., 47, measure off A C equal to any measurement multiplied by 3. Sweep out A D equal to the same measurement multiplied by 4, and cut it with C D, which make the same measurement multiplied by 5. This is known as the 3, 4, and 5 method.

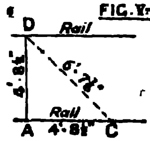
FIG. V-17.



(C) (Fig. 18.) Using the same principle, measure a base A C, and sweep a distance A D equal to the base, cut it with C D which make equal to the base multiplied by the square root of 2 or 1·414.

The measurements in the figure show how rail joints may be squared.

FIG. V-18.



(D) Referring to Fig. 20, if E is the given point in the given line B C, choose a centre such as D, and with a radius D E strike an arc which cuts B C at C.

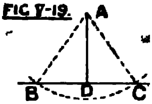
Draw a line through C and D, producing it to cut the other side of the arc at A.

A line AE will be at right angles to BC (Euclid III., 31).

It will be noticed that by this method nothing need be used beyond a pair of compasses in the office, or a piece of string on the ground.

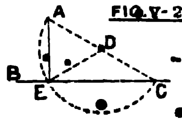
Prob. 2. From a given point outside of a given line, to set a line at right angles to the line.

FIG. V-19.



(A) (Fig. 19.) From the given point A cut the given line with equal distances AB and BC. Bisect BC at D. AD is the line required.

FIG. V-20



(B) (Fig. 20.) When the portion DB (Fig. 19) of the line is not available:--

Take any suitable point C in the given line BC, bisect AC at D, and from D cut BC with a distance DE equal to DC. AE is the line required. The angle AEC is in a semi-circle and is therefore a right angle (Euclid III., 31).

Prob. 3. To draw or set out an angle, the measurement of which is given in degrees, minutes, and seconds (° ′ ″).

We must first examine this, which is the scientific method of measuring an angle.

If we draw a circle and divide its circumference into 360 equal parts, and then draw two lines from the ends A and B of one of those parts to the centre C of the Circle, those lines will form an angle ACB of one Degree.

The Degree is sub-divided into 60 Minutes and the Minute into 60 Seconds.

The circumference of a Circle is 2×3·1416 times its Radius, so the portion AB, is:—

$$\frac{1}{360} \text{ of } 2 \times 3\cdot1416 \times \text{Radius}$$

$$\text{that is, } AB = \frac{1}{57\cdot30} \times \text{Radius.}$$

Now suppose we wish AB to be one unit, such as one foot, then:—

$$1 = \frac{1}{57\cdot30} \times \text{Radius}$$

so, Radius = 57·30.

Therefore if we draw a Circle with a Radius of 57·30ft., each foot on its circumference will represent a Degree; $\frac{1}{5}$ of an inch, a minute; and $\frac{1}{300}$ of an inch, a second.

The size of angles may also be estimated by the statements :—

1 Degree is 1 in 57·30.

1 Minute is 1 in 3438, or 1″ in 286′ 6″.

1 Second is 1 in 206280 or 1″ in 3¼ Miles approx.

Procedure of setting out an angle.

(A) By using the principle, that if a circle be drawn with a radius of 57·30 to any unit, degrees may be marked off on the circumference by using the same unit to represent degrees.

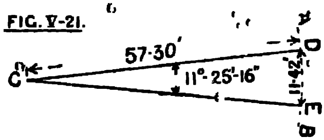
EXAMPLE.—(Fig. 21.) To set out an angle of 11° 25′ 16″.

First reduce the angle to degrees and decimals, thus:

$25' \ 16'' = 25 \times 60 + 16 = 1516$ seconds.

There are 3600″ in a degree, so $25' \ 16'' = \frac{1516}{3600}$ degree. Reducing this fraction to a decimal we get ·421 degrees. The whole angle is thus 11·421 degrees.

FIG. V-21.



From a centre C and with a radius of 57·30 feet, that is 57 feet, 3⅝ inches, sweep an arc A B.

Along this arc mark off a length D E equal to 11·421 feet, or 11 feet 5 inches.

Join D and E to C, and D C E will be the angle required.

Extending this principle, if it is wished to make $\frac{1}{16}$ inch represent exactly one minute on the arc, the radius must be 17 feet 10⅞ inches.

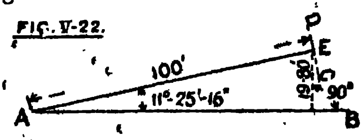
The advantage of this method is that no tables are required in its use.

(B) By using a trigonometrical ratio obtained from tables.*

The sine of the angle will give the greatest accuracy for setting out the small angles we are concerned with.

EXAMPLE.—(Fig. 22.) To draw a line, making an angle of 11° 25′ 16″ with the line A B.

FIG. V-22.



* At this stage, these ratios may be used without a knowledge of their meaning, which is explained in the notes on Trigonometry, Chap. IV.

From centre A and with a suitable radius, say 100 feet, sweep an arc C D.

Look up the Sine of 11° 25′ 16″ in a Mathematical Table. This will be found to be ·1980185.

Multiplying this by 100 feet we get 19·80 feet.

Mark off this distance at right angles to A B and cutting the arc at point E.

Join A E, and E A B is the angle required.

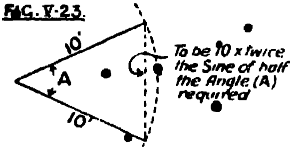
(C) By the use of a Protractor, which, however, will not usually give sufficient accuracy for the purpose.

(D) By a Table of Chords, which is useful, but special Tables are required.

(E) By using the Sine of half the angle, which does not entail any setting out of a right angle.

This method is briefly shown in Fig. 23.

FIG. V-23.



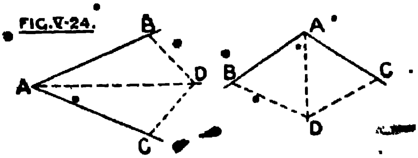
Prob. 4 (Fig. 24). **To bisect a given angle, B A C.**

Set out equal distances A B and A C from A.

From points B and C sweep out equal distances B D and C D intersecting at D. A line D A will bisect the angle.

The examples show both the bisection of an acute angle and of an obtuse angle.

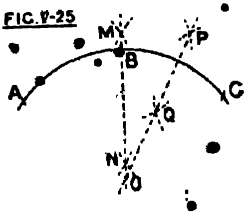
FIG. V-24.



Prob. 5 (Fig. 25). **To find the centre from which a given arc is struck.**

(A) Without calculation.

FIG. V-25

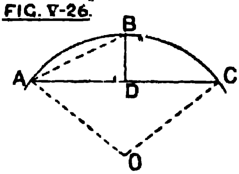


Take any two points A and C on the arc, and with centres A and C, at any convenient radius sweep out arcs at M and N. Repeat the process on any other portion of the arc, preferably B C, the arcs being at P and Q.

The centre required will be the point O, which is the intersection of lines through M N and P Q produced. (III. 3.)

(B) (Fig. 26.) By the aid of calculation.

FIG. V-26.



Take any two points A and C on the arc. Join A C, bisect it at D and draw D B at right angles. Measure A B, which is the chord of half the arc, and D B.

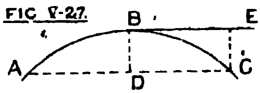
Then the radius = A B squared ÷ twice B D. (III. 35A.)

The centre O may then be found by striking out this radius from the points A and C.

Prob. 6. To draw a tangent to an arc at a given point on the arc.

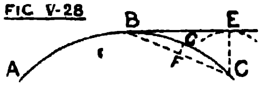
It is required to draw a tangent to the arc A B C touching it at point B.

FIG. V-27.



(A) (Fig. 27.) Fix two points A and C in the arc, equally distant from B. Join the chord A C, and at its centre measure the versed sine B D. From C set C E at right angles to the chord and equal to B D. Join E B which is the tangent required.

FIG. V-28



(B) (Fig. 28.) If the portion A B of the arc is not available.

Join B C and at its centre measure F G.

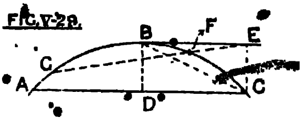
From centre C and with a radius C E, which make equal to four times F G, sweep an arc. A line drawn from B and touching this arc will be the tangent required. There is a very slight error in taking C E = four times F G, which will be referred to in Chapter VI.

(C) Referring to Fig. 25, obtain the line M N as in that problem, and through point B draw a line at right angles to M N. This will be the tangent required. (III. 18.)

Prob. 7. To draw a tangent to an arc from a point outside the arc. (Fig. 29.)

(A) From the given point E draw a line E B just touching the arc. To find the exact point of contact, B, draw any line A C parallel to B E; bisect A C at D and draw D B at right angles to A C. B will be the point required.

FIG. V-29.



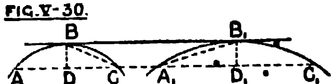
(B) If the portion F B is not available, but the radius of the arc is known, draw E B as before and E C at right angles thereto. Then chord B C = $\sqrt{\text{twice } E C \times \text{Radius}}$. (III.—35A).

(C) From E draw any line cutting the curve as E F G, measure E F and E G.

Then $E B = \sqrt{EG \times EF}$. (III.—36).

Prob. 8. To draw a tangent touching two arcs, A B C and $A_1B_1C_1$. (Fig. 30).

FIG. V-30.



Draw a line $B B_1$ touching the two arcs. To find the exact points of contact, draw a line $A C A_1 C_1$ parallel to $B B_1$.

Measure B D which $= B_1 D_1$.

Then chord $B C = \sqrt{2 \times BD \times \text{Radius of arc } A B C}$,

and chord $A_1 B_1 = \sqrt{2 \times B_1 D_1 \times \text{Radius of arc } A_1 B_1 C_1}$,

CHAPTER VI.

RULES FOR DRAWING AND SETTING OUT CIRCULAR CURVES.

GENERAL RULES FOR CIRCULAR CURVES.

Under this heading, rules will be explained which, supplemented by the theorems and problems of the last chapter, may be applied in dealing with circular curves by methods suitable to railway work.

Dimensions relating to a Circular Curve.

The dimensions which appertain to a circular arc will first be enumerated. A name for each dimension will be adopted, and the relations between the dimensions will be stated, in order that basic rules may be obtained.

FIG. VI-1.

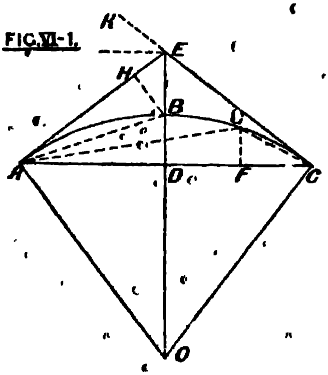
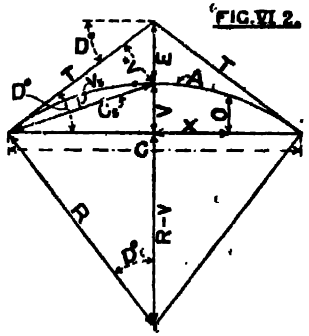


FIG. VI 2.



Referring to Fig. 1,—and it is important that the reader should draw this and all other figures step by step for himself,—with a centre O and a radius OA draw a portion of the circumference of a circle ABC. The curve ABC is named **an arc** of the circle. Join AC by a straight line; this is called the **chord of the arc.** Bisect AC at D and through OD draw the line $ODBE$; this line will be at right angles to AC. BD is variously known as the **middle ordinate**, the **rise of the arc**, or the **versed sine**,* the last name being adopted in this work.

* Sometimes written "versine."

A line such as $F G$, at right angles to the chord, is a **side ordinate.**

A line $A B$ is the **chord of half the arc,** but in this volume it will be named the "**sub-chord**" (of the arc $A B C$).

From point A draw a line $A E$ at right angles to the radius $A O$. $A E$ will be a **tangent** to the arc touching it at point A. From the point E only one other tangent can be drawn to touch the circle, namely $E C$, and this will be equal to $A E$. E is the **point of intersection** of the tangents, $A E K$ is the **angle of intersection,*** and the angle $A E C$ is the **apex angle.**

$E A D$ is the **tangential angle** between the chord $A C$ and the tangent.

$A O C$ is the angle subtended by the arc at the centre of the circle or shortly the **central angle.**

A further angle, which is used in curve ranging by the theodolite, is the **deflection angle.** This is the angle between a chord and the prolongation of a preceding chord of the same length. Thus, in Fig. 3, page 83, the angle $J B C$ is the deflection angle for the chord $B C$. This angle is twice the tangential angle for the same chord. It will not be necessary to refer again to this angle.

Elementary geometry shows that the **tangential angle** is equal to half the angle at the centre and also equal to half the angle of intersection.

The tangential angle for half the arc is equal to half the angle for the full arc., *i.e.*, in Fig. 1 angle $E A B$ = angle $B A D$.

Any line drawn from a point outside of the circle, such as the point E, and cutting the circle is named a **secant.** The line $E B O J$† is a secant through the centre of the circle. The portion $E B$ of this line is called the **external secant.**

The terms tangent, versed sine and secant, originate from an obsolete way of stating the trigonometrical functions, not used in the modern treatises.

---

* It is the usual English practice to call the angle $A E C$ the angle of intersection, but the American practice is adopted as being more convenient.

† The point J is at the other end of the diameter and not shewn in the figure.

Table of Dimensions relating to an Arc, with Symbols.

	Symbol (See Fig. 2)	Shewn on Fig. 1 by lines :—
Radius	R	*OA, OB & OC*
Chord	C	*AC*
Sub-chord	C_s	*AB*
Versed Sine	V	*BD*
Side ordinate at a point, which is at a distance x from the centre of the chord	O	*GF*
Tangent	T	*AE, EC*
External Secant	E	*EB*
Arc	A	*ABC*
Tangential Angle	D°	*EAD*

It is only necessary for the purpose to mention one of the angles in the above table because the other angles follow directly from it.

Provided that any two of the dimensions tabulated above are known, then all the remaining dimensions can be obtained by calculation.

The dimensions are nine in number, so that there are thirty-six ways of choosing any given two dimensions.

Now if any two dimensions are given, we may be asked to calculate any one of the remaining seven dimensions. Thus it is possible to be presented with problems two hundred and fifty-two in number. This, however, is only a matter of mathematical interest and we need not consider such an appalling number of problems, because, when any two dimensions are given, we may usually

calculate a third one, which may not be the one required, but which in combination with one of these first given, will enable us to find the one required.

It may also be noted that many of the problems will seldom, and others never, occur in practice.

Applying a knowledge of the elementary geometry given in Chapter V., rules may be derived enabling any of the problems mentioned to be solved.

Tables 23 and 24 give an extensive list of such rules, providing means for solving any problem likely to occur in practice.

For the present the reader may confine his attention to the following set of rules, which are chosen with a view to simplicity and general utility.

These rules bear the same reference numbers as they do in Tables 23 and 24. The rules will be quoted by these numbers throughout the work.

SUMMARY OF RULES FOR CIRCULAR CURVES.

For full list of Rules see Tables 23 and 24.

For symbols refer to Figs. 1 and 2.

Required.	Given.	Rule No.	Formula.	Rule in words.
Radius (R)	Vers. sine and chord	C 1	$\frac{(\frac{1}{2}C)^2 + V^2}{2 \times V}$	Sum of squares of Half Chord and Vers. sine divided by twice the Vers. sine.
Do.	Vers. sine and Sub-chord	C 2	$C_s^2 \div 2V$	Sub-chord squared divided by twice the Vers. sine.
Do.	Side ordinate and *AG* and *GC* (Fig. 1)	C 3	$\frac{AG \times GC}{2 \times O}$	Product of the two distances from top of ordinate to ends of Chord, divided by twice the Ordinate.

(continued)

SUMMARY OF RULES FOR CIRCULAR CURVES—*continued.*

Required.	Given.	Rule No	Formula.	Rule in words.
Vers. sine (V)	Radius and Chord	C 7	$R - \sqrt{R^2 - (\frac{1}{2}C)^2}$	From the square of Radius take the square of Half chord, extract square root of result and deduct it from the Radius.
Do.	Do.	C 8	$(\frac{1}{2}C)^2 \div 2R$ (approx.)	Square of Half chord divided by twice Radius.
Do.	Radius and Sub-chord	C 9	$C_s^2 \div 2R$	Sub-chord squared divided by twice Radius.
Side ordinate (O)	Radius, Vers. sine and X	C 13	$\sqrt{R^2 - X^2} - (R - V)$	From square of Radius take square of X, extract square root of result and from it deduct Radius decreased by Vers. sine.
Do.	Radius, Chord and AF & FC (Fig. 1)	C 12a	$\frac{AF \times FC}{2 \times R}$ (approx.)	Product of distances of ordinate from ends of chord divided by twice the Radius.
Chord (C)	Radius and Vers. sine	C 15	$2 \times \sqrt{(2 \times R \times V) - V^2}$	Multiply twice Radius by Vers. sine; deduct square of Vers. sine Extract square root and multiply by 2.
Sub-chord (C_s)	Radius and Vers. sine	C 17	$\sqrt{2 \times R \times V}$	Multiply twice the Radius by the Vers. sine and extract square root of result.
Second Vers. sine	First Vers. sine (V)	C 26	$V \div 4$ (approx.)	Second Vers. sine is ¼ of first Vers. sine.
Radius (R)	Angle of Intersection of two lines = 2 D° and Tangent length (T) from intersection	C 35	$T \div \text{Tan } D°$	Multiply the Tangent length by twice the "Centre Line Measure" * of the Angle of Intersection, or by the "Right Angle Measure" of half that angle.
Tangential Angle D°	Radius and Sub-chord	C 57	$C_s \div 2R = \text{Sine } \frac{1}{2} D°$	Angle is twice the Angle whose Sine is Sub-chord divided by twice Radius.

* See Chapter XII.

Examining the above rules, it will be seen that those connected with the **Sub-chord** have the advantage of simplicity combined with accuracy.

It is astonishing how far the simple rules:—

$$R = C_s^2 \div 2V$$
$$V = C_s^2 \div 2R$$
$$C_s = \sqrt{2 \times R \times V}$$
$$\text{and } V_s = V \div 4 \text{ (approx.)}$$

will serve in practice, and these rules should be memorised in preference to any others.

The curves, or arcs of circles, with which we shall often be concerned, will be flat, *i.e.*, their versed sine will be small compared with the chord and the radius.

Therefore the Half-chord and the Sub-chord will be nearly equal, and in many cases it will not matter which we take in working out the rules.

The amount of the error in taking the Half-chord instead of the Sub-chord is shown in Table 23.

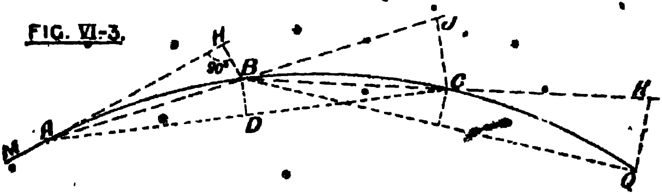
Examples of working out the rules will be given when curve problems are dealt with in detail.

Processes of setting out Curves.

Before proceeding to the solutions of definite problems in the next Chapter, several of the usual processes of setting out a circular curve without the use of angle measuring instruments will now be described.

METHOD A (FIG. 3). *By an offset from a tangent followed by offsets from secants, i.e., lines cutting the curve.*

FIG. VI-3.



As an example, let MA be a line from which the curve is to spring at point A. The curve is to be of 10 chains, = 660 feet Radius, and the pegs are to be 66 feet apart.

1st.—Calculate the Versed sine for a Sub-chord of 66 feet. Versed sine = Sub-chord squared divided by twice the Radius. (Rule C 9.)

$$= \frac{66 \times 66}{2 \times 660} = \frac{33}{10} = 3{\cdot}30 \text{ ft. or } \underline{\underline{3'\ 3\tfrac{5}{8}''}}$$

2nd.—Prolong the line MA a little over 66 feet beyond A., *i.e.*, to about point H; this is the tangent line. From point A measure a distance $AB = 66$ feet, and cut it with an offset, HB equal to the Versed sine of 3′ 3⅝″, measured square to the tangent line. Point B will be the first point in the curve.

3rd.—To continue the curve; from point B measure a distance BJ of 66 feet along the line through A and B prolonged to J. The line ABJ is called a secant line.

Also from B measure a distance BC of 66 feet and cut it with an offset JC = twice the Versed sine, or 6′ 7¼″.

C will be the second point in the curve.

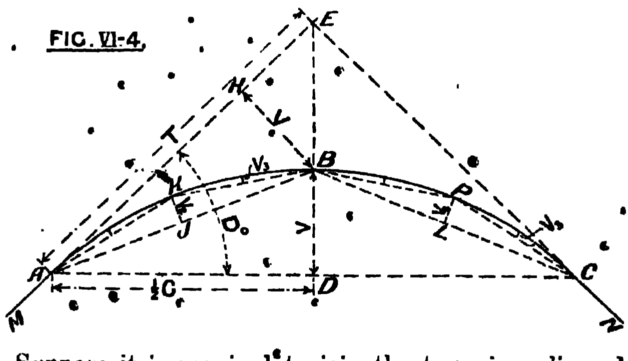
4th.—A third point Q is obtained in a similar manner by producing the line through B and C to K with an offset $KQ = JC$.

Following the dotted lines it will be seen that a curve has been set out tangential to the line MA and with Versed sines of 3′ 3⅝″ for Sub-chords of 66 feet.

METHOD B (FIG. 4). *By versed sines on consecutive chords.*

This method is commonly known as "Quartering," and is very convenient when the total Chord or Sub-Chord and the Versed sine are known or can be obtained, as shown in the following example:—

FIG. VI-4.



Suppose it is required to join the two given lines MA and NC by a curve commencing at point A, where it must spring from the line MA.

1st.—Prolong the two given lines until they meet in point E.

2nd.—Measure the tangent line AE and set off EC equal to it. Point C will be the other end of the curve, where it will be tangential to the line NC.

3rd.—Join A and C by a chord, and bisect it at D.

4th.—Measure AE, DE, and AD.

5th.—Obtain the Radius:—

$$R = T \div \text{Tan } D^\circ \quad \text{(Rule C 35.)}$$

$$\text{that is, } R = AE \div \frac{DE}{AD}$$

$$\text{or, } R = AE \times \frac{AD}{DE}$$

6th.—Calculate the Versed sine BD from Rule C 7 or C 8, according to the degree of accuracy required.

Alternatively V may be obtained without calculation thus:—

Bisect AC at D, and with the ring end at D take a tape along the line DBH. When the lengths DB and BH are equal and at right angles to AD and AE respectively, DB will be the versed sine required.

7th.—Apply the method of Quartering. After fixing point B stretch a line ABC; bisect AB and BC at J and L and set at right angles the distances V_2 or JK and LP, each equal to $\frac{1}{4}$ of V.

8th.—The string may be put round the points A, K, B, P, and C, and further Versed sines (V_3) set equal to $\frac{1}{4}$ of V_2.

9th.—The operation may be repeated until sufficient points have been obtained.

The arcs generally occurring in practice are flat, and with such arcs, in obtaining the first V, it will usually be near enough to make it $\frac{1}{2}$ of DE.

The secondary Versed sines (V_2), found by quartering, are slightly too small, but the error is very little. It is difficult to express it exactly, but practically it amounts to:—

(V cubed ÷ Sub-chord squared × 16).

The following shows the amount of the error in various instances:—

Main Chord (C).	Proportion of error to Second Vers. sine.
Equal to Radius	1·7% or 1 in 60
Ditto ½ Radius	·5% or 1 in 200
10 times first V	1% or 1 in 100
20 Do. Do.	·17% or 1 in 600

METHOD C (FIG. 5). *By a series of square offsets from a tangent line, the points being located at equal distances apart.*

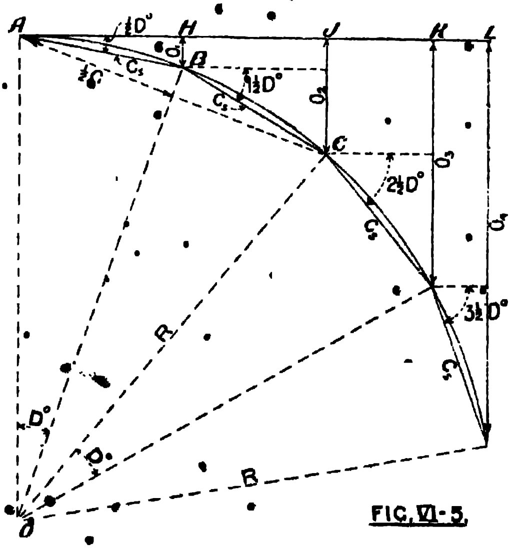


FIG. VI-5.

This method, though very accurate, does not generally prove as useful as those preceding.

The Radius (R) and the distance apart (C_s) of the pegs (*A*, *B*, *C*, etc.), must be given.

Referring to Fig. 5 we must calculate the offsets O_1, O_2, O_3, etc. from the tangent line *AL*. The calculation, except as regards O_1, O_2, O_4 and O_8, is complex, unless trigonometry is employed.

The truth of the rules may be seen by comparing Fig. 1 with the portion of Fig. 5 bearing the same reference letters, and noting the facts of Euclid III., 26 and 29.

Obtain the angle $\frac{1}{2}$ D°.

Sine $\frac{1}{2}$ D° = $C_s \div 2$ R. (Rule C 57.)

Then:—

$$O_1 = C_s \times \text{Sine } \tfrac{1}{2} \text{ D}$$
$$O_2 = O_1 + (C_s \times \text{Sine } 1\tfrac{1}{2} \text{ D})$$
$$O_3 = O_2 + (C_s \times \text{Sine } 2\tfrac{1}{2} \text{ D})$$
$$O_4 = O_3 + (C_s \times \text{Sine } 3\tfrac{1}{2} \text{ D}), \text{ etc.}$$

It is not necessary to know the distances *AH*, *HJ*, *JK*, *KL*, along the tangent, if we take care that the O's are measured square to *AL*, but if required they are:—

$$AH = \sqrt{C_s{}^2 - O_1{}^2} \qquad \text{(Euclid I.—47)}$$
$$HJ = \sqrt{C_s{}^2 - (O_2 - O_1)^2}, \text{ etc.}$$

When using Method C, the offsets may often be taken from tables such as those of Kennedy and Hackwood (Spon), which give offsets for curves from 5 to 80 chains radius advancing by 1 chain and from 80 to 240 chains by 5 chains, the pegs being 66 feet apart.

This may be a suitable place to call attention to the fact that offsets to a circular curve from a tangent line are approximately proportional to the square of their distances from the tangent point.

Thus if we take a practical example instead of the purposely exaggerated diagram (Fig. 5), the distances *AH*, *AJ*, *AK*, *AL* will be very nearly in the proportions 1, 2, 3, and 4.

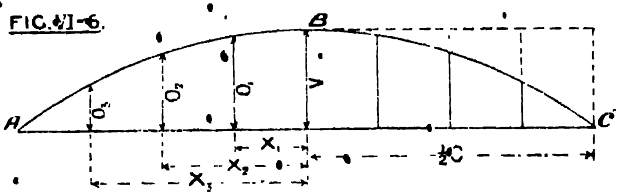
The squares of these numbers are 1, 4, 9, and 16, and the offsets will be found to be approximately:—

$$O_2 = O_1 \times 4$$
$$O_3 = O_1 \times 9$$
$$O_4 = O_1 \times 16$$

METHOD D (FIG. 6). *By a series of offsets or ordinates from a chord, located at equal distances along the chord.*

This is a method seldom used on the ground.

FIG. VI-6.



The Radius and Chord being given, the Versed sine may be calculated from Rule C 7, and the Ordinates (O) from Rule C 12a or C 13.

If the Half chord is divided into fourths, as in Fig. 6, the O's will approximately be:—

$$O_1 = \tfrac{15}{16} \text{ of } V$$
$$O_2 = \tfrac{3}{4} \text{ of } V$$
$$O_3 = \tfrac{7}{16} \text{ of } V$$

This is because the offsets from the tangent, as shown in dotted lines, increase nearly as the square of their distance from *B*.

CHAPTER VII.

PROBLEMS IN SETTING OUT AND DRAWING CIRCULAR CURVES.

In the setting out and drawing of circular curves to fulfil definite purposes, the geometrical facts, the curve rules, and the detail processes of setting out, discussed in the preceding chapters, will be largely brought into service.

Some of the problems may be considered to be of an advanced nature to the average reader, but he will soon recognise those problems which will prove useful to his requirements, and be within his capabilities. The more advanced problems are included to render the series complete, and it must be noted that all the problems included are liable to confront the designer in the preparation of plans for new work.

Conditions necessary to fix a Curve.

In order that a simple circular curve may be defined, so as to be capable of being set out on the ground or drawn on paper, there must be given three conditions, or requirements, which the curve must fulfil.

These *conditions*, any one of which may go towards defining the position of a curve, are as follows:—

P. The curve must pass through a given point.

L. ,, ,, ,, touch a given straight line.

C. ,, ,, ,, touch a given curve.

R. ,, ,, ,, be of a given radius.

It may be mentioned, that if *four* conditions are given which a curve must fulfil, it will be a *compound curve;* that is, two curves of different radii tangential to one another.

Each of the conditions will be represented by an abbreviation consisting of the letter preceding it in the above list.

Of the conditions, one, two, or all three, may be taken from either P. L. or C. For example, a curve will be defined if it is stated that it must:—

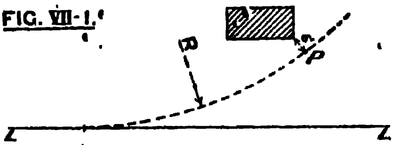
1. Pass through a given point.

2. Touch a given line.
3. Touch a given curve.

The problem of drawing such a curve will be shortly denoted thus, "P.L.C."

To give a practical illustration of one of the problems:—

FIG. VII-1.

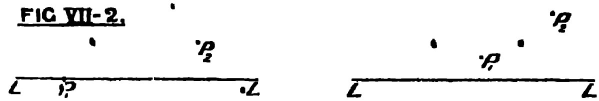


The line LL in Fig. 1 represents a straight main line. A curve of given Radius is required to be tangential to the line, that is, to spring from it or touch it, and also to pass through the point P, 6 feet away from the building shown.

Here we have a problem which by the notation adopted is represented by "P.L.R."

By taking three given conditions in every possible manner, problems 16 in number will arise. Of these, some will have two or three cases, because of variety in the relative position of the given points, lines, and curves. For instance, a given point may be either in or out of a given line or curve. As an example, the problem "P. P. L." may be represented by either of the sketches in Fig. 2, where a curve is required to pass through two points P_1 and P_2 and to spring from a line LL. In one case the point P_1 is in the line, and in the other case both points are out of the line.

FIG VII-2.



The full list of the three condition problems will be:—

Prob. No.		Prob. No.		Prob. No.		Prob. No.	
1	P L R	5	P P R	9	L L L	13	C C R
2	L L P	6	L L R	10	L C R	14	C C P
3	P P R	7	P C R	11	P P C	15	C C L
4	P P L	8	P L C	12	L L C	16	C C C

Solutions of many of these problems are included in the text books on Geometry, but usually the procedure therein depends upon the centre of the circles being available. This rarely occurs in railway work.

Some of the problems seldom, and others never, occur in practice.

Solutions will be given of Nos. 1 to 11, omitting the remainder as they are of an intricate nature, and are not likely to arise in actual work.

It will be noticed that usually, the required curves are shown to spring direct from a given line or curve. If a switch is to intervene, then the given line or curve must be taken, not as representing the main rail, but as a line parallel thereto.

The "shift" or distance of this parallel line from the rail may, for all practical purposes, be taken as half the heel divergence of the switch, or usually $2\frac{1}{4}$ inches.

If it is desired to allow for a transition curve, a parallel line must again be worked to, the "shift" in this case being in accordance with the rules for transition curves.

To avoid repetition, the following symbols and lettering will be used as far as possible:—

P_1, P_2 = Given points.

AB, CD = Given lines.

EFG, HJK = Given curves.

R = Radius of a required curve.

R_0 = Radius of a given curve.

d = The least distance between a curve or a point, and a line.

m, n, t = Other distances to be measured or calculated.

Given and required lines are in full.

Auxiliary lines are dashed.

Extra lines to show an extension of a problem are dotted.

The Rules and Theorems to which reference is made, are those of the preceding chapters. The propositions of Euclid are referred to thus, "(III., 36)."

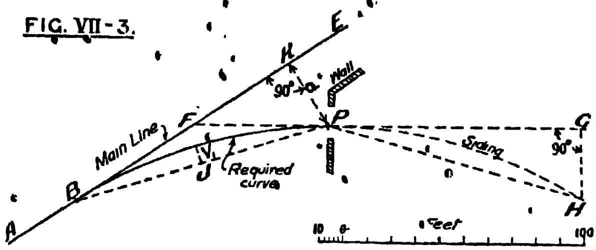
The Curve Rules of Chapter VI. and Tables 23 and 24 are denoted:— "(Rule C 7)."

Problem No. 1 (P. L. & R.). Case (1): P. in L.

This case is dealt with under Method A of Setting out Curves (Chapter VI.).

Problem No. 1 (P. L. & R.). Case (2): P. not in L. (Fig. 3.)

FIG. VII-3.



EXAMPLE.—A curve of 200 feet radius is required to spring from the given Line AB and to pass through the Point P.

PROCEDURE.—Produce the line AB to E, slightly past point P. Measure the least distance, PK=d, between the line and the point. Suppose d = 30 feet.

To find the point B, from which the curve will spring, calculate a length PB, using d and the given radius.

Referring to Fig. 3, d is a Versed sine for a Sub-chord $= PB$.

$$\text{Therefore, } PB = \sqrt{2 \times d \times R} \qquad \text{(From Rule C 17.)}$$
$$= \sqrt{2 \times 30 \times 200}$$
$$= \sqrt{12000}$$
$$= \underline{\underline{109{\cdot}54'}} \text{ or } \underline{\underline{109'\ 6\tfrac{1}{2}''}}$$

From P measure this distance to cut the line at point B.

It only remains to set out points in the curve between B and P. The most practical method will be to set the points by "quartering." String a line between B and P and make the first Versed sine (V_1) equal to $\frac{1}{4}$ of d, or 7′ 6″, and proceed in accordance with Method B (Chapter VI.).

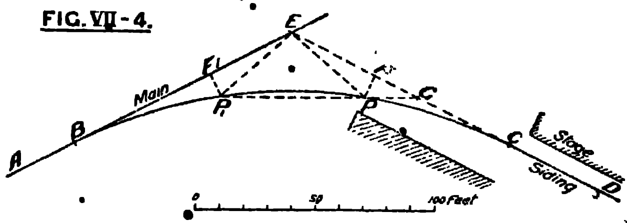
The dotted lines show how the curve may be extended beyond point P, if required. A point F is found, being opposite J, which is the centre of BP. FB then $= FP$. Produce FP to G, and make $PH = BP$, and GH = d.

Problem No. 2 (L. L & P.). Case (1): P. in L.

This case is dealt with under Method B of setting out curves (Chapter VI.)

Problem No. 2 (L. L. & P.). Case (2): P. not in L. (Fig. 4.)

FIG. VII-4.



EXAMPLE.—A curve is required to join the two given straight lines AB and CD. The curve must pass through the fixed point P.

PROCEDURE.—Produce the given lines until they meet in point E.

Find a point P_1, symmetrical with P. This may be done by measuring PE and PF, and making EP_1 and F_1P_1 respectively equal to them.

Draw a line through P_1P to cut line ED at G, and measure P_1G and PG giving say 83′ and 23′.

The point C, from which the curve will spring in the line ED, may then be found, because GC is a tangent from point G and GP_1 is a secant line from the same.

$$\begin{aligned} \text{Therefore, } GC &= \sqrt{P_1G \times PG} \qquad \text{(III., 36.)} \\ &= \sqrt{83 \times 23} \\ &= \sqrt{1909} \\ &= \underline{\underline{43{\cdot}7'}} \end{aligned}$$

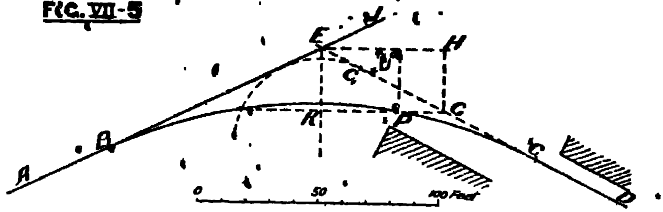
Having fixed point C, fix point B by making $EB = EC$.

The curve may then be set out by Method B, Chapter VI., the Versed sine (V) for BP_1 and PC being $\frac{1}{4}$ FP.

The V for length F_1P may be obtained from measuring a V for this length on the portion of curve already set out, or from the Radius. The Radius $= PC^2 \div 2FP$. (Rule C 2.)

Problem No. 2. Case (2): Alternative method. (Fig. 5.)

FIG. VII-5



Measure $P\,G$ parallel to $E\,H$ which latter is the bisector of angle $J\,E\,D$, and also measure $P\,K$ from P to the bisector of angle $A\,E\,D$.

Suppose $P\,G = 23'$ and $P\,K = 30'$

The tangent line, $GC = \sqrt{GP \times (GP + 2KP)}$ (III., 36.)

$$= \sqrt{23 \times (23 + 2 \times 30)}$$

$$= \underline{\underline{43 \cdot 7'}}$$

To find the Radius of the curve, measure $E\,C$, $E\,H$, and $H\,G$, which suppose $= 100'$, $50 \cdot 4'$, and $25 \cdot 2'$. Then:—

$$R = T \div \text{Tan } D^\circ \text{ *} \qquad \text{(Rule C 35.)}$$

$$\text{That is, } R = EC \div \frac{HG}{EH}$$

$$= 100 \div \frac{25 \cdot 2}{50 \cdot 4}$$

$$= 100 \times \frac{50 \cdot 4}{25 \cdot 2}$$

$$= \underline{\underline{200'}}$$

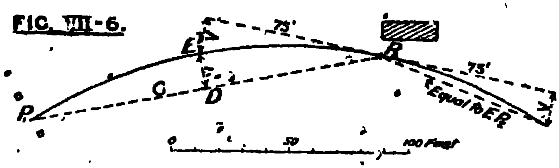
We may notice that $G\,C$ may also be set out towards E, as shown by $G\,C_1$, in which case we should obtain the smaller curve shown dotted, which, though it touches the given lines and passes through the point, is obviously not the curve required. This is one of the many instances of there being more than one solution to a problem.

If the point E is inaccessible, reference should be made to Problem No. 6.

* Dividing by Tan D° is the same thing as multiplying by twice the Centre Line Measure (see Chapter XII.), of the angle of intersection, which in turn is the same as multiplying by the Right Angle Measure of half that angle.

Problem No. 3 (P. P. & R.). (Fig. 6.)

FIG. VII-6.



EXAMPLE.—A curve of Radius = R = 200 feet, is required to pass through the given points P_1 and P_2.

PROCEDURE.—Measure the straight line $P_1\ P_2$.

Suppose this chord (C) = 150 feet.

Calculate the Versed sine (V), for the chord C:—

$$V = R - \sqrt{R^2 - (\tfrac{1}{2}C)^2} \qquad \text{(Rule C 7.)}$$

$$= 200 - \sqrt{200^2 - 75^2}$$

$$= 200 - \sqrt{40000 - 5625}$$

$$= 200 - \sqrt{34375}$$

$$= 200 - 185{\cdot}41'$$

$$= \underline{\underline{14{\cdot}59'}} = \underline{\underline{14'\ 7\tfrac{1}{8}''}}.$$

Bisect the chord $P_1\ P_2$ at D, and set up $D\ E = V = 14'\ 7\frac{1}{8}''$, at right angles. Other points in the curve may be set by "quartering."

Usually it will be near enough to obtain V by Rule C 8, viz.:—

$$V = (\tfrac{1}{2}\ C)^2 \div 2R.$$

The dotted lines show how the curve may be extended by the method shown in Problem (1), Case (2)

Problem No. 4 (B. P. & L.). Case (1): P. in L. (Fig. 3.)

EXAMPLE.—Referring to the diagram of Problem No. 1, a curve is required to spring from the given line $A\ B$ at point B and to pass through the point P.

PROCEDURE.—Produce the given line $A\ B$, to E, and measure the least distance $K\ P$ from this line to point P.

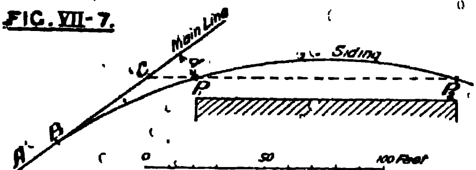
The curve may then be set out by "quartering," the first Versed sine being $\frac{1}{4}$ of $K\ P$ (approx.).

For the radius of the required curve:—

$$R = P\ B^2 \div \text{twice}\ K\ P. \qquad \text{(Rule C 2.)}$$

Problem No. 4 (P. P. L.). Case (2): P_1 not in L. (Fig. 7.)

FIG. VII-7.



EXAMPLE —A curve is required to spring from the given line AB and to pass through the two given points P_1 and P_2.

PROCEDURE.—Through P_2 and P_1 draw a line to intersect the production of the given line at C.

Measure CP_1 and CP_2; suppose they are 20′ and 130′.

For the distance BC, from C to the springing of the curve:—

$$\begin{aligned} BC &= \sqrt{CP_1 \times CP_2} \qquad \text{(III., 36.)} \\ &= \sqrt{20 \times 130} \\ &= \sqrt{2600} \\ &= \underline{\underline{51'}} \end{aligned}$$

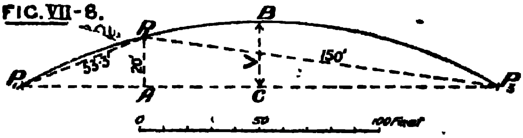
The curve may then be set out as in Case (1) or as in Problem No. 5.

For the Radius, measure d and BP_1, then:—

$$R = BP_1^2 \div 2\text{ d.} \qquad \text{(Rule C 2.)}$$

Problem No. 5 (P. P. P.). (Fig. 8.)

FIG. VII-8.



EXAMPLE.—A curve is required to pass through the three given points P_1, P_2 and P_3.

PROCEDURE—Join the points P_1 and P_3 and measure the square distance, AP_2, from this line to point P_2. AP_2 will be a "Side ordinate". Also measure the chords P_1, P_2 and P_2, P_3.

The radius of the required curve will be:

$$R = \frac{P_1 P_2 \times P_2 P_3}{2 \times P_2 A} \quad \text{(Rule C 3.)}$$
$$= \frac{53 \cdot 3 \times 150}{2 \times 20}$$
$$= \underline{\underline{200'}}$$

The curve may then be set out by "quartering," the first Versed sine (V) being:—

$$V = (\tfrac{1}{2} P_1 P_3)^2 \div 2R \quad \text{(Rule C 8 approx.)}$$
$$= 99 \cdot 03^2 \div 400$$
$$= \underline{\underline{24 \cdot 51'}}$$

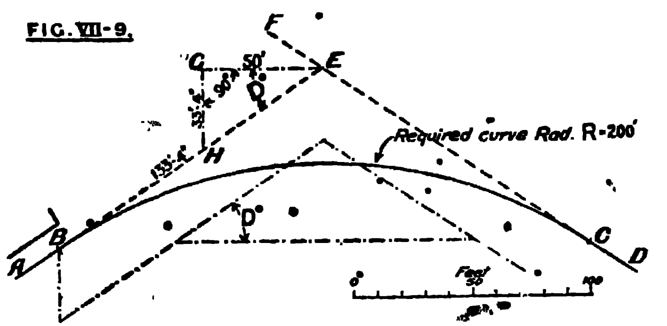
Alternatively the V may be calculated by the exact Rule C 7.

Otherwise we may calculate an approx. V from Rule C 8; then measure $P_1 B$ and find V, thus:—

$$V = P_1 B^2 \div 2R. \quad \text{(Rule C 9.)}$$

Problem No. 6 (L. L. R.). (Fig. 9.)

FIG. VII-9.



Example.—A curve with a radius of 200 feet is required to join the given lines $A B$ and $C D$.

Procedure.—Produce the given lines until one crosses the other. In the diagram, $C D$ is produced to F, crossing the production of $A B$ at E.

Bisect the angle $B E F$ by line $E G$.

Along $E G$ measure any length $G E$ say 50 feet, and measure a square distance $G H$ to the line $B E$. Suppose $G H = 33' \ 4''$.

The length of a tangent line, EB, can then be calculated :—

$$EB = \text{Radius} \times \frac{GH}{GE}^{*} \qquad \text{(Rule C 44.)}$$

$$= 200' \times \frac{33 \cdot 33}{50}$$

$$= \underline{\underline{133'\ 4''}}$$

Make $EC = EB$. B and C will be the ends of the curve, and it may be set out by Method B, Chapter VI.

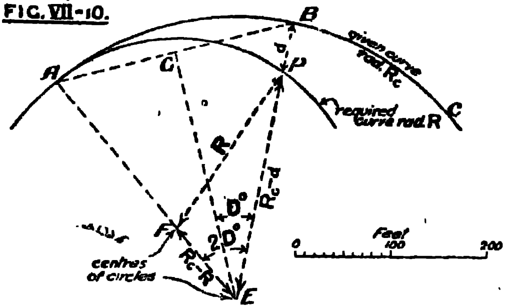
If the given lines cannot be produced until they intersect, then lines parallel to them may be used to find the intersecting angle and the tangent point, as in dotted lines.

Problem No. 7 (P. C. R.). Case (1): P. in C.

At the given point draw a tangent to the given curve. The solution then becomes the same as that of Problem 1, Case 1.

Problem No. 7 (P. C. R.). Case (2) P. on inside of C. (Fig. 10.)

FIG. VII-10.



EXAMPLE.—A curve of Radius $= R = 200'$, is required to spring from the given curve ABC, which has a radius $= R_o = 300'$, and to pass through the fixed point P.

PROCEDURE.—Measure the least distance d, that is PB, between the point and the given curve, and mark point B. Suppose $d = 50'$.

* See footnote to Problem (2) Case (2).

Calculate the distance from B to the springing point A of the required curve:—

$$AB = 2R_c \times \sqrt{\frac{d\,(2R - d)}{4R_c\,(R_c - R - d) + 4dR}}$$

$$= 600 \times \sqrt{\frac{50 \times (400 - 50)}{1200\,(300 - 200 - 50) + (4 \times 50 \times 200)}}$$

$$= 600 \times \sqrt{\frac{17500}{(1200 \times 50) + 40000}}$$

$$= 600 \times \sqrt{\frac{17500}{100000}}$$

$$= 600 \times \sqrt{\cdot 175}$$

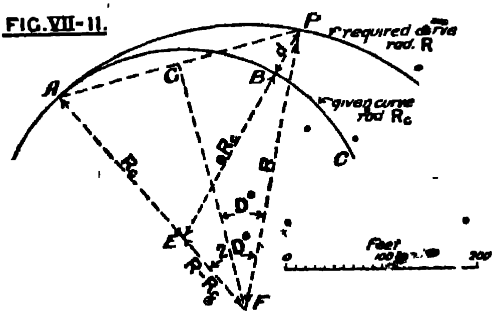
$$= 600 \times \cdot 418$$

$$= \underline{\underline{250 \cdot 8'}}$$

The point A may then be fixed, and the curve set out from a chord AP.

Problem No. 7 (P. C. R.). Case (3): P. on outside of C. (Fig. 11.)

FIG. VII-11.



Example.—A curve of Radius $= R = 300'$, is required to spring from the given curve $A\,B\,C$, which has a radius $= R_c = 200'$, and to pass through the fixed point P.

Procedure.—As in the last example, measure d, which suppose $= 50'$.

In this case calculate the distance from P to the springing point A of the curve:—

$$AP = 2R \times \sqrt{\frac{d\,(2R_c + d)}{4R\,(R - R_c)}}$$

$$= 600 \times \sqrt{\frac{50 \times 450}{1200 \times 100}}$$

$$= 600 \times \sqrt{\frac{22500}{120000}}$$

$$= \underline{\underline{260'}}$$

The curve may then be set out from the chord $A\,P$.

The rules for $A\,B$ and $A\,P$ in Cases 2 and 3, have been obtained by principles beyond those explained in this book.

The procedure is as follows:—

Find the Cosine of the angle 2 D, in terms of the three known sides of the triangle $P\,F\,E$.

From Cosine 2 D obtain Sine D, then in Case 3:—

$$G\,P = R \times \text{Sin D}$$

$$\text{and } A\,P = 2 \times G\,P.$$

Problem No. 8 (P. L. C.). Case (1) P. in C. (Figs. 12 and 13.)

FIG. VII-12.

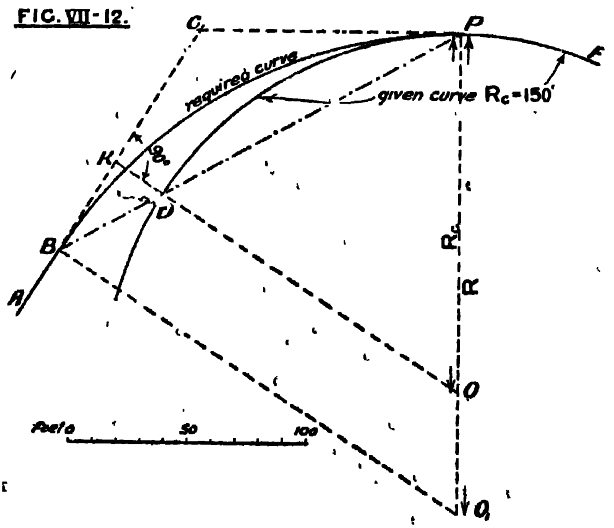
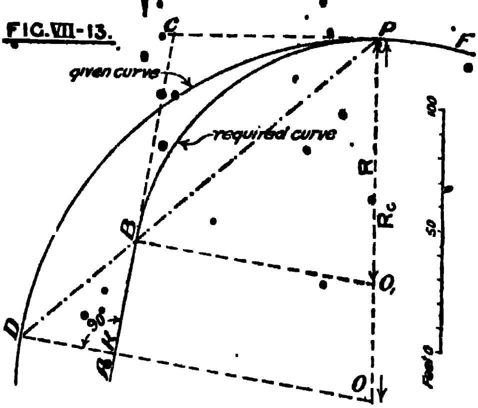


FIG. VII-13.



EXAMPLE.—A curve is required to touch the given curve $D P F$ at point P, and to be tangential to the given line $A B$.

PROCEDURE.—At point P draw a tangent $C P$ to the given curve, and cutting the given line produced at point C.

Measure $C P$ and make $C B$ equal thereto. B will be the commencement of required curve. The curve may then be set out and its radius obtained in accordance with Method B, Chapter VI.

Alternative Method.

Find the points K and D where the given line and curve are at the least distance apart.

Draw a line through P and D, producing it until it cuts the given line at a point B.

B will be the other end of the curve (III. 11a), and because $B O_1$ is parallel to $D O$, we may find the radius, thus:—

$$\frac{R}{R_c} = \frac{PB}{PD} \qquad \text{(VI.—2)}$$

$$\therefore \ R = R_c \times \frac{PB}{PD}$$

The same procedure applies to both cases shown on the two diagrams (Figs. 12 and 13), whichever method is used.

Problem No. 8 (P. L. C.). Case (2): P. in L.

EXAMPLE.—Referring to Figs. 12 and 13, a curve is required to spring from a given point B in the line $A B$, and to touch the given curve $D P F$.

PROCEDURE.—Find the line KD, where the given line and curve are at the least distance apart, and mark the point D.

Draw a line through B and D, producing it until it cuts the given curve at a point P.

P will be the other end of the required curve (III., 11a), and its Radius will be:—

$$R = R_c \times \frac{PB}{PD}$$

Problem No. 8 (P. L. C.). Case (3): P. not in L. or C. (Fig. 14.)

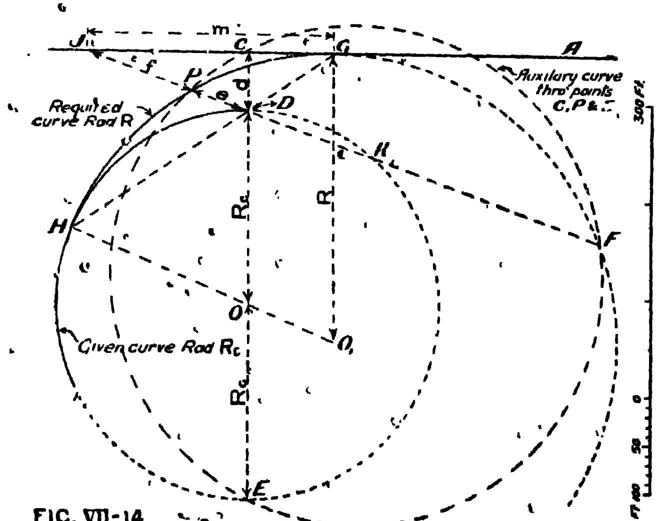


FIG. VII-14.

EXAMPLE.—A curve is required to spring from the given line $J\,C\,G\,A$, to pass through the fixed point P, and to touch the given curve $H\,D\,K$ whose Radius $= R_c$.

PROCEDURE.—Find the line $D\,C$ which represents the least distance between the given line and curve.

Produce the line through D and P to cut the given line at J.

Measure $D\,C = \text{d} = 60'$, $P\,D = \text{e} = 62'$, $\text{J P} = \text{f} = 117'$, and $R_c = 200'$.

Then the distance $m = JG$; (G, being the springing point of the required curve) may be found, thus:—

$$m = \sqrt{f \times \left\{f + e + \left(\frac{d \times 2R_c}{e}\right)\right\}}$$

$$= \sqrt{117 \times (117 + 62 + \frac{60 \times 2 \times 200}{62})}$$

$$= \sqrt{117 \times 566}$$

$$= \sqrt{66222}$$

$$= \underline{\underline{257'}}$$

From J locate point G by the distance m.

The other end H of the required curve, namely where it touches the given curve, may be found by producing the line through G and D until it cuts the given curve at a point H (III., 11a), or otherwise, $DH = \frac{2 \times R_c \times d}{DG}$

To find the Radius (R) of the required curve; because in the triangle $G\,H\,O_1$, $D\,Q$ is parallel to $G\,O_1$,

$$\text{Therefore,} \quad \frac{R}{R_c} = \frac{GH}{DH} \quad \text{(VI.—2.)}$$

$$R = \frac{R_c \times GH}{DH}$$

The curve may then be set out as in Problem 5.

If the point J is inaccessible, $J\,G$ may be calculated by trigonometry, working from the triangle $C\,D\,P$, the sides of which may be measured.

The solution given above is arrived at by imagining $C\,D$ produced through the centre of the circle to E, and an auxiliary circle described through the points $C\,F\,E$; then:—

$$DF \times PD = DC \times DE \qquad \text{(III.—35.)}$$

$$DF = \frac{DC \times DE}{PD}$$

$$= \frac{DC \times 2R_c}{PD}$$

Also:—

$$JG = \sqrt{JP \times JF} \qquad \text{(III.—36.)}$$

but, $JF = JP + PD + DF$

$$\therefore\ JG = \sqrt{JP \times (JP + PD + \frac{DC \times 2R_c}{PD})}$$

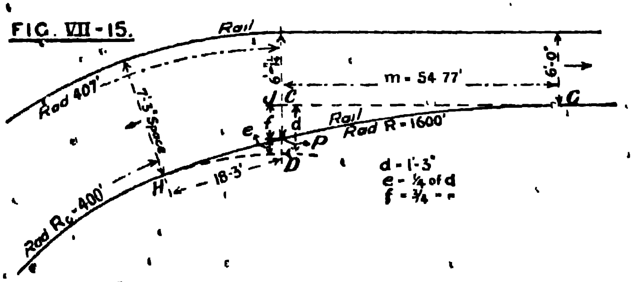
Problem No. 8 (P. L. C.). Case (4) P. on the line which represents the least distance between the given line and curve. (Fig. 15.)

In Case 3 is illustrated what may be termed a general case of Problem 8, but the special case now to be dealt with is more likely to occur in practice.

EXAMPLE.—Fig. 15 shows an example where a curve of 400′ Radius (R_c) springs from a straight, in a double line of railway.

It is desired to widen the space between the roads from 6ft. on the straight, to 7ft. 3in. on the curve, to allow for overthrow of vehicles.

FIG. VII-15.



The same reference letters are used as in the previous case

PROCEDURE.—First decide where P shall lie on DC, thus fixing e and f. Then because $f+e=d$ we have by the previous formula,

$$m = \sqrt{f \times \left\{d + \left(\frac{d \times 2R_c}{e}\right)\right\}}$$

Neglecting the first d,

$$m = \sqrt{\frac{f \times d \times 2R_c}{e}}$$

$$\text{Now } R = \frac{m^2}{2f} \qquad \text{(Rule C 1 approx.)}$$

$$\text{So } R = \frac{f \times d \times 2R_c}{e} \times \frac{1}{2f}$$

$$= \frac{d \times R_c}{e}.$$

In the example we have f as $\frac{3}{4}$ of d, so c is $\frac{1}{4}$ d, and therefore,

$$m = \sqrt{6 \times d \times R_c}$$
$$= \sqrt{6 \times 1{\cdot}25' \times 400'}$$
$$= \underline{\underline{54{\cdot}77'}} \text{ (in the example.)}$$
$$R = R_c \times 4$$
$$= 400 \times 4 = \underline{\underline{1600'}} \text{ (in the example.)}$$

The point of contact with the original curve may be found as in Case 3:—

$$DH = \frac{2R_c \times d}{m} \text{ (approx.)}$$
$$= \frac{2 \times 400 \times 1{\cdot}25}{54{\cdot}77} = \underline{\underline{18{\cdot}3'}} \text{ (in the example.)}$$

To sum up, we find that by commencing at a distance of $\sqrt{6 \times d \times R_c}$ from the tangent point, and laying the inner rail of the 6ft. to a radius of four times that of the original curve we obtain $\frac{3}{4}$ of the increase of space at the original tangent point.

Similarly, it may be shown that by commencing at $\sqrt{2 \times d \times R_c}$ and using a radius of $2 \times R_c$ we obtain $\frac{1}{2}$ of the increase at the tangent point.

It may be noted that a transition curve would form a better, though not so simple a solution.

The clearance between vehicles on the adjacent roads must be tested by travelling outline diagrams of the longest and widest coaches in use, over the plan of the roads as arrived at by any of the above methods.

Problem No. 9 (L. L. L.). (Fig. 16.)

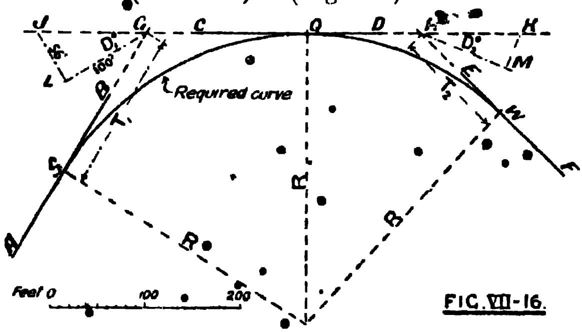


FIG. VII-16.

EXAMPLE.—A curve is required to be tangential to the three given lines AB, CD, and EF.

PROCEDURE.—Produce the two outer lines until they intersect the production of the inner line at G and H.

Measure the tangents of the half angles of intersection (D_1 and D_2), or obtain their Right Angle Measure.-(Chap. XII.)

The Radius of the curve will be :—

$$R = GH \div (\text{Tan } D_1 + \text{Tan } D_2)$$

or $R = GH \times$ the sum of the Right angle Measures of the angles D_1 and D_2.

In the example :—

$$R = 288' \div \left(\frac{56}{100} + \frac{40}{100}\right)$$

$$= 288' \div \frac{96}{100}$$

$$= \underline{\underline{300'}}$$

The distance T_1, from G to the springing point S, will be :—

$$T_1 = R \times \text{Tan } D_1 \qquad \text{(Rule C 44.)}$$

$$\text{or } T_1 = R \div \text{Right Angle Measure of } D_1$$

$$T_1 = 300 \times \frac{56}{100} = \underline{\underline{168'}}$$

Similarly,

$$T_2 = R \times \text{Tan } D_2$$

$$T_2 = 300 \times \frac{40}{100} = \underline{\underline{120'}}$$

When the distances T_1 and T_2 are set along line GH, they should meet at Q, the point where the curve touches the middle line.

Other points in the curve may be set by methods already explained.

The proof of the solution is as follows :—

$$T_1 = R \times \text{Tan } D_1 \qquad \text{(Rule C 44.)}$$

$$T_2 = R \times \text{Tan } D_2 \qquad \text{"}$$

$$\text{By addition } T_1 + T_2 = R \times (\text{Tan } D_1 + \text{Tan } D_2)$$

$$\text{but } T_1 + T_2 = GH \text{ (a known dimension).}$$

$$\therefore R = GH \div (\text{Tan } D_1 + \text{Tan } D_2)$$

Problem No. 10 (L. C. & R.). Case (1): Required curve of greater radius than given curve. (Fig. 17.)

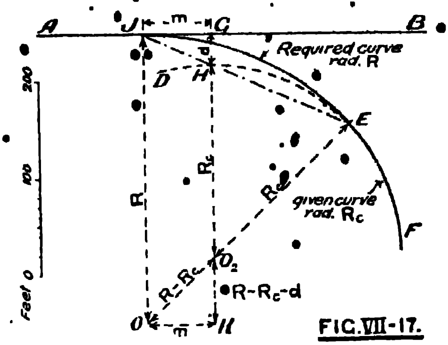


FIG. VII-17.

EXAMPLE.—A curve of radius R = 300′ is required to touch both the given line *AB* and the given curve *DEF*, Radius R_c = 200′

PROCEDURE.—Produce the given curve until its least distance, *GH* = d, from the line, can be measured. Suppose d = 30′.

To find the springing point *J* of the required curve, calculate the distance *JG* = m, using the right-angled triangle $O_1\ O_2\ K$.

$$\begin{aligned} m &= \sqrt{(O_1O_2)^2 - (O_2\ K)^2} \qquad (\text{I., } 47.) \\ &= \sqrt{(R - R_c)^2 - (R - R_c - d)^2} \\ &= \sqrt{d\ (2R - 2R_c - d)} \\ &= \sqrt{30 \times (600 - 400 - 30)} \\ &= \sqrt{30 \times 170} \\ &= \sqrt{5100} \\ &= \underline{\underline{71 \cdot 4'}}. \end{aligned}$$

The other end of the curve may be found by producing the line through *JH* until it cuts the given curve at a point *E* (III., 11a), or alternatively; after measuring or calculating

JH, because in the triangles JO_1H, JO_1 is parallel to HO_2 :—

$$HE \div R_c = JH \div (R - R_c) \quad . \quad \text{(VI.—2.)}$$

$$HE = \frac{R_c \times JH}{R - R_c}$$

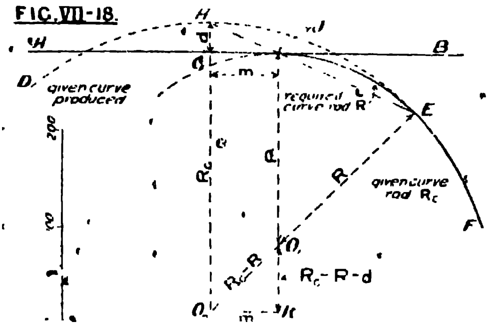
$$= \frac{200 \times 77 \cdot 45}{100}$$

$$= \underline{\underline{154 \cdot 9'}}$$

Knowing the chord JE and the Radius, we have information to set out the curve.

Problem No. 10 (L. C. R.). Case (2) (Fig. 18): Required curve of less radius than given curve.

FIG. VII-18.



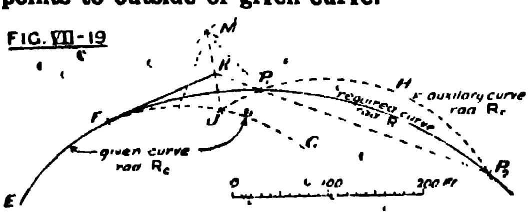
This case may be solved in a similar manner to the above, but:—

$$m = \sqrt{d\,(2\,R_c - 2\,R - d)}$$

$$\text{and } JE = \frac{R \times JH}{R_c - R}$$

Problem No. 11 (P. P. C.). Case (1) (Fig. 19): Given points to outside of given curve.

FIG. VII-19



EXAMPLE.—A curve is required to pass through the points P_1 and P_2, and to touch the given curve EFG, the radius of which is 200 feet.

PROCEDURE.—Set out a curve $P_1 HP_2$ of the same radius as the given curve but to pass through points $P_1 P_2$ (Problem 3).

Produce this curve to cut the given curve at J.

Bisect the angle FJP_1 between the two curves by the line JM. The finely dotted lines show how this may be done.

Produce a line through P_2 P_1 to cut JM at K.

From point K draw a tangent touching the given curve at F

Note that $KF = \sqrt{KP_2 \times KP_1}$ (III—36.)

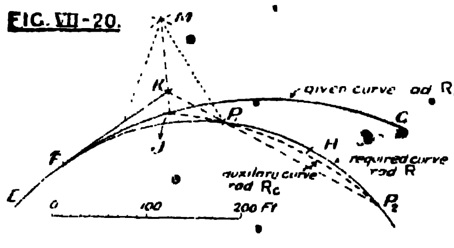
F will be the point where the required curve touches the given curve, and the former may be set out by procedure of Problem No 5 or No. 4.

To obtain the Radius, join FP_2 and measure an ordinate O from line FP_2 to P_1 then :—

$$R = \frac{FP_1 \times P_1P_2}{2 \times O} \qquad \text{(Rule C 3.)}$$

Problem No. 11 (P. P. C.). Case (2): Given points to inside of given curve. (Fig. 20.)

FIG. VII-20.



The procedure is exactly as in the preceding case.

It may be noted that in either of the above cases, another curve may be drawn, passing first through one of the given points, next touching the circle, and then passing through the other given point. Such a curve will not usually be required.

Improvement and Re-alignment of Existing Curves.

With the exception of certain problems included in Chapter X., this subject will not be specially dealt with, as it is considered that if a curve has become irregular or is otherwise faulty, it should usually be re-located on the basis that it must fulfil certain conditions, probably the same as it was originally intended to fulfil. The procedure will thus be the same as in the setting out of a new curve and will be covered by one of the preceding problems.

In the maintenance of track it is important that the Versed sines taken throughout the length of a circular curve, on overlapping chords, should be maintained as nearly equal as possible. In the case of transition curves, permanent marks or "monuments" should be installed to indicate the correct position of the rail and super-elevation.

In many cases, the method of Mr. W. H. Shortt explained in the Proc. Inst. C.E., Vol. 176, 1909, may be employed with advantage in the re-alignment of existing curves. By this method a "chord survey" of the existing work including the obstructions and clearances, is plotted to a distorted scale so that small irregularities may easily be detected. Mr. Shortt's method has been largely used on the L. & S.W. and other Railways, and it has been developed to a fuller extent since the paper was written.

CHAPTER VIII.

REVERSE AND COMPOUND CURVES.

REVERSE CURVE PROBLEMS.

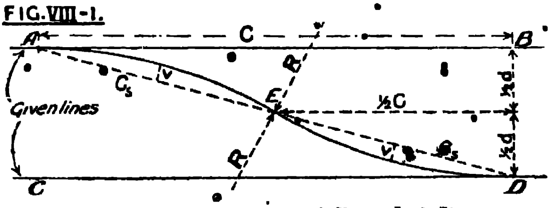
Reverse curves are of frequent occurrence in railway practice; for example, where tracks are diverted to run alongside platforms or over temporary bridges, and in the connecting of two adjacent tracks.

A length of straight should intervene between reverse curves, depending upon the space available, and the nature of the traffic. The minimum length of straight should be long enough to contain all the wheels of the longest fixed wheelbase, though it would be better to contain all the wheels of the longest vehicle. As an ideal, and ~~where~~ considerable speeds obtain, the length should be sufficient to allow for transition curves, with a gradual reversal of the super-elevation due to the circular curves. It is believed that in no previous treatment of reverse curve problems by calculation, has the straight length been taken into account.

When the straight is absent, the curve is named a "Direct Reverse."

Prob. 1. (Fig. 1.) **Direct reverse curves of equal given Radius (R), to join two parallel lines at a given distance (d) apart. To find the further requisite dimensions for setting out.**

FIG. VIII-1.



In Fig. 1 the given lines are $A\,B$ and $C\,D$.

The point of reverse in curvature, E, obviously lies half way between the given lines and half way between A and D, which will be the tangent points.

$A\,E = E\,D =$ the Sub-chord (C_s) for an arc of which the Radius $=$ R, and the Versed sine $= \frac{1}{2}$ d, then:—

$$C_s = \sqrt{2 \times \tfrac{1}{2} d \times R} \qquad \text{(Rule C 17.)}$$

$$= \sqrt{d \times R}$$

Locate the springing points A and D by the skew distance between them, $AD = 2 \times C_3$, and bisect it at E.

In setting out the curves, use the chords AE and ED, taking:— $v = \frac{1}{4}$ of $\frac{1}{2}$ d (approx.)

or use the exact Rule No. C 7.

If required, $C = AB = \sqrt{AD^2 - d^2}$

Prob. 2. (Fig. 1.) **Direct reverse curves as in the last problem, but instead of R, the length C is given. To find R, etc.**

$\frac{1}{2}$ d is a Versed sine for the half chord, $\frac{1}{2}$ C, so:—

$$R = \frac{C^2 + d^2}{4\,d} \qquad \text{(From Rule C 1.)}$$

The curves may then be set out as in the last problem.

Prob. 3. (Fig. 2.) **Direct reverse curves as in Prob. 1, but of unequal Radii (R_1 and R_2). Distance apart of parallel lines = d.**

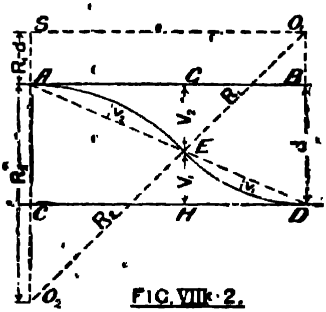


FIG. VIII·2.

The springing points A and D may be located by the skew distance between them AD, or by the length AB.

$$AD = \sqrt{2\,d \times (R_1 + R_2)}$$

$$AB = \sqrt{d \times (2\,R_1 + 2\,R_2 - d)}$$

otherwise $AB = \sqrt{AD^2 - d^2}$

To find the reversing point E; divide AD into two parts, so that one part is to the other as R_1 is to R_2, that is:—

$$\frac{DE}{AD - DE} = \frac{R_1}{R_2}$$

$$\therefore DE = \frac{AD \times R_1}{R_1 + R_2}$$

Along DA mark off DE; then V_1 and V_2 may be measured and the curves set out, using Versed sines, v_1 and $v_2 = \frac{1}{4}$ of V_1 and V_2 respectively.

It may be noted that $V_1 : V_2 = R_1 : R_2$ and because $V_2 = d - V_1$:—

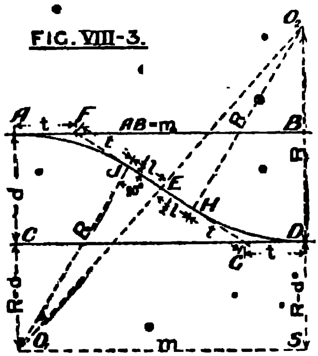
$$\therefore V_1 = \frac{R_1 \times d}{R_1 + R_2}$$

PROOF.—First find $S O_1 = C D$ from the triangle $S O_1 O_2$, then find $A D$ from the triangle $A C D$. Next note that the triangles $A E O_2$ and $D E O_1$ are similar, and so $D E \div E A = R_1 \div R_2$.

Also the triangles $A G E$ and $D H E$ are similar, and so $V_1 \div V_2 = R_1 \div R_2$.

Prob. 4. (Fig. 3.) **Reverse curves of equal given Radius (R), with a given length of straight (l) between them, joining two parallel lines at a given distance apart (d). To find the further requisite dimensions.**

FIG. VIII-3.



To fix point D, assuming that A is given, calculate m, which is the length $A B$.

$$m = \sqrt{(4d \times R) + l^2 - d^2}$$

Next calculate the tangent lengths, t.

$$t = R \times \left(\frac{m - l}{4R - d}\right)$$

From A and D set out $A F$ and $D G$ each $=$ t. Along line $F G$, set $F J$ and $G H$ also each $=$ t. $J H$ should then $=$ l.

The curves may be set out from the chords $A J$ and $D H$. The Versed sine for these chords will be $\frac{1}{4}$ of the distance of J from the line $A B$, or Rule C 10 may be used.

PROOF.—From Rt. A. triangles O_1JE and O_2HE, obtain sides OE from known sides marked R and $\frac{1}{2}$ l. Then from Rt. A. triangle O_1O_2S obtain $O_1S = m$.

The derivation of the Rule for t is too lengthy for inclusion.

Prob. 5. (Fig. 3.) **Reverse curves as in the last problem, but instead of the Radius, the length (m) of the curve, over all, is given. To find R, etc.**

From the last problem:—

$$m^2 = (4d \times R) + l^2 - d^2$$

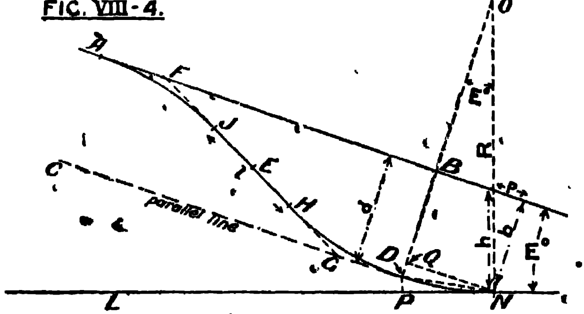
$$\therefore m^2 - l^2 + d^2 = 4d \times R$$

$$\text{that is, } R = \frac{m^2 - l^2 + d^2}{4d}$$

The tangent lengths, t, and the Versed sine, v, may then be obtained as in Problem 4.

Prob. 6. Reverse curves of given radius to join a given point in one line to another line approaching the first line at a given angle. To find the requisite dimensions.

FIG. VIII-4.



In Fig. 4, AB and LN are the given lines approaching each other at an angle E°, which is known or may be measured.

Case 1. (Fig. 4.) **Let N be the given springing point.**

Measure the distances between the lines, h, square to LN and b, square to AB; also measure p.

Note that the angles marked E are equal.

With regard to the angle E calculate the sub-chord ND and the Versed sine $QD = PD$.

$$ND = 2R \times \text{Sine } \tfrac{1}{2} E^\circ$$

$$= 2R \times (\sqrt{h-b} \div \sqrt{2h})^*$$

$$PD = ND^2 \div 2R$$

Otherwise, if PD is required previously to ND,

$$PD = R \times \text{Versed sine } E^\circ$$

$$= R \times (h - b) \div h\ ^*$$

Using ND and PD, fix point D.

Then if a line DC is drawn parallel to AB, any of the types of reverse curve previously treated may be calculated for, commencing at point D. The parallel lines will be AB and CD as in previous problems. Their distance apart, d, will be, b + QD.

It may be noticed that the measurements h, b, and p, are used to obtain the ratios of the angle E. A better method is to obtain them by a line parallel to LN and cutting AB, but the above method is used to simplify the explanation.

Case 2. (Fig. 5.) **Let the given springing point N be at the end of the curve further from the intersection of the given lines.**

Referring to Fig. 5. the explanation for Case 1 will apply word for word.

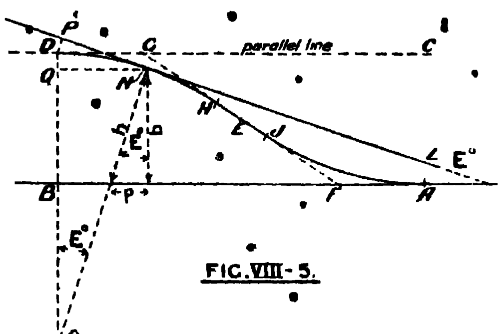


FIG. VIII - 5.

* **These rules are given so that Trigonometry may be dispensed with, if it is preferred to do so.**

Prob. 7. (Fig. 6.) **Direct reverse curves of equal Radius, to join a given point A in a line AQ, to a given point N in a second line LN, approaching the first line at a given angle E. To find the Radius, etc.**

This and the succeeding problem are the most difficult of the series. They frequently arise in practice and are often solved by trial on a plan.

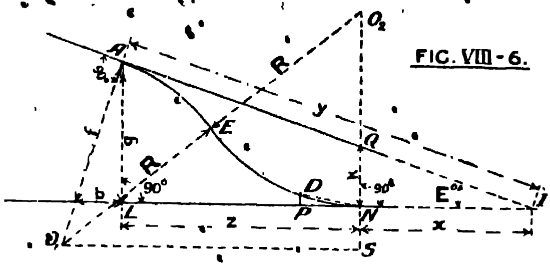


FIG. VIII-6.

First measure the cross widths f, g, and k, and the length $LN, = z$. Then $\frac{z}{g - k} = \text{Cot E}$.

Next calculate the distances x and y, from the given points, to the point I, where the lines would meet if produced.

$$y = f \times \text{Cot E, or } \frac{f \times z}{g - k}$$

$$x = k \times \text{Cot E, or } \frac{k \times z}{g - k}$$

$$R = \frac{\sqrt{1(x^2+y^2) + (x+y)^2 \text{Tan}^2 \tfrac{1}{2}E} - (x+y)}{2 \text{ Tan } \tfrac{1}{2} E}$$

NOTE.—Tan $\frac{1}{2}$ E $= \frac{f - g}{b}$

Then calculate ND and PD, and fix point D as in Problem 6. Bisect DA for point E, and set out the curves as previously shown

PROOF.—The above solution is arrived at by finding the values of the sides of the right-angled triangle $O_1 O_2 S$ in terms of the lengths x and y, the Radius, and the angle E. Equating the squares on these sides to the square on $O_1 O_2$, that is $(2R)^2$, we get a rule for R. The details are too lengthy for inclusion.

Prob. 8. (Fig. 7.) Reverse curves as in the last problem, but with a length of straight (l) between. To find the Radius, etc.

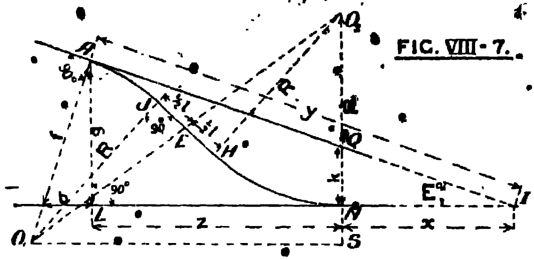


FIG. VIII-7.

The first procedure will be as in the last problem, but the Rule for R will be:—

$$R = \frac{\sqrt{2(x^2+y^2)-l^2+\{(x+y)^2-l^2\}\operatorname{Tan}^2\frac{1}{2}E}-(x+y)}{2\operatorname{Tan}\frac{1}{2}E}$$

The procedure will then be as in Problem 6.

PROOF.—The solution is arrived at on the lines of the last problem, combined with those of Problem 4.

COMPOUND CURVES.

A curve which is formed of two simple circular curves tangential to each other, and of similar flexure, is known as a Compound Curve.

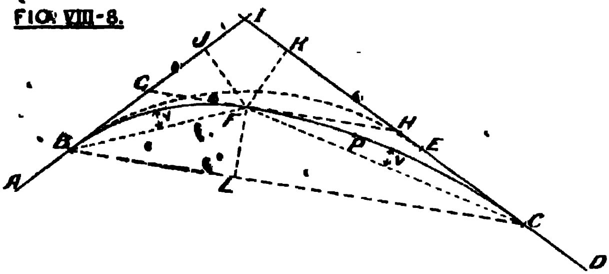
In Chapter VII. it is stated that a simple curve is defined when it has three conditions to fulfil. If four similar conditions to those mentioned are laid down, a number of compound curves may be drawn to fulfil them, and it needs five conditions to be given, to definitely fix the particular compound curve.

In practice it is seldom necessary to deal with purely compound curve problems, because they generally may be resolved into two of the problems in simple curves already dealt with.

Another reason for not devoting much attention to such problems, is the fact that if we have to spring a curve from a point *B* in a line *AB*, Fig. 8, to touch another line *CD*; then the simple curve drawn from *B* to E and connecting the two lines, will be flatter than one of the two

parts of any compound curve springing from B to connect the lines, such as the part $B\,F$ of the curve $B\,F\,C$.

FIG. VIII-8.



Prob. 9. (Fig. 8.) **To join a given point B in a given line $A\,B$, to a given point C in another given line $C\,D$, by a compound curve; the given points being at unequal distances from the intersection, I, of the lines.**

It will be noticed that only four conditions are given, and that a fifth is necessary to fix the curve.

This fifth stipulation may be that the curve must pass through a given point between B and C, such as P, or again that the radius of one part of the curve may be fixed, etc.

Such cases may usually be solved by applying first one and then another of the simple curve problems of Chapter VII.

For instance, in the first case, draw a curve springing from C to pass through P., Problem VII.-1, and then draw a curve springing from B to touch the first curve, Problem VII.-8.

If the second curve does not touch the first until it has passed point P, reverse the operation by first drawing a curve from B to pass through P. One of these operations will usually either satisfy the conditions, or will as nearly do so as is necessary.

If only the four conditions first mentioned are fixed, a suitable compound curve may be put in by either of the two following methods:—

METHOD A.—Join the given points B and C. Bisect angles $I\,B\,C$ and $I\,C\,B$. The intersection F of the bisecting lines will be the tangent point of the two curves. A line $G\,H$ parallel to $B\,C$, will be a common tangent to the two curves.

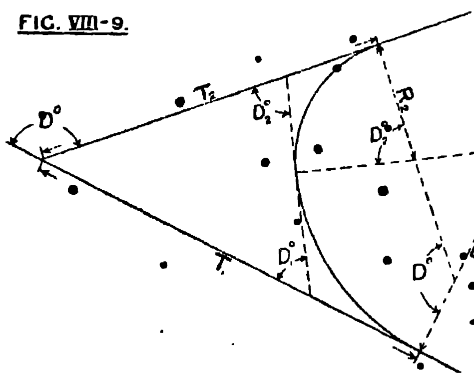
Each curve may then be set out as shown in Chapter VI., Method B.

METHOD B.—Join *B* and *C*, and find point *F* where the three distances *F L*, *F K*, and *F J* are equal, or alternatively it will usually be near enough to bisect *I L* at *F*.

Then with a chord *B F* and a Versed sine, v = ¼ *F L*, set out a curve from *B* to *F*. With the same Versed sine set out a curve from *F* to *C*.

The compound curve set out by either method will be the same, and its parts *B F* and *F C* will have less difference in their radii than those of any other compound curve which can be drawn from *B* to *C*.

FIG. VIII-9.



General Mathematical Formulæ.

Referring to Fig. 9, the following mathematical formulæ may be applied to solve pure compound curve problems.

They are given in an article by A. Llano, in the Engineering News Record (U.S.A.), Vol. 82 (1919):—

(1) $(T_1 + T_2) \text{ Cot } \frac{1}{2} D + (T_1 - T_2) \text{ Cot } \frac{1}{2} D_1 = 2 R_1$

(2) Ditto ditto $+ (T_2 - T_1) \text{ Cot } \frac{1}{2} D_2 = 2 R_2$

(3) $D_1^o + D_2^o = D^o$

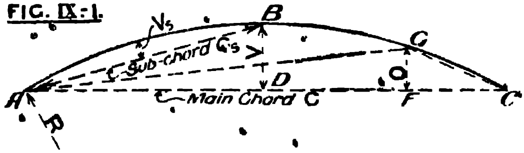
CHAPTER IX.

CURVED TRACK, MISCELLANEOUS DETAILS.

The preceding chapters treat upon the Geometry of curves and their setting out to serve certain purposes. The present chapter will deal with detail matters arising with regard to railway tracks when on the curve.

SHORT PRACTICAL RULES WITH REGARD TO CURVES.

FIG. IX:1.



Rule C 2 given in Chapter VI., namely, that the Radius (R) of a curve equals the square of the Sub-chord (C_s) divided by twice the Versed sine (V), should be carried in the memory on account of its usefulness, not only in lining out but in many technical problems.

It should be memorised in the form,

$$R = \frac{C_s^2}{2 \times V}$$

because in that form it is easily transposed to give C_s or V, when they are the unknown values. If we transfer a value from below the line to the other side of the equation, it goes above the line and *vice versa*, thus we get:—

$$V = \frac{C_s^2}{2 \times R} \quad \text{(Rule C 9.)}$$

and $2 \times V \times R = C_s^2$ or $C_s = \sqrt{2 \times V \times R}$ (Rule C 17.).

Two other simple rules to be memorised are:—

Rule C 26.—Versed sine on the Sub-chord $= \frac{1}{4}$ of Versed sine on Main chord, or $V_s = \frac{1}{4}$ V.

Rule C 12a.—Side ordinate = product of its distances from end of chord ÷ twice the Radius, or $O = \frac{AF \times FC}{2R}$

It may be noted that in many cases it makes no appreciable difference if we take the sub-chord and half the main chord as being equal, so that if C = the main chord, we have:—

$$V = \frac{\left(\frac{C}{2}\right)^2}{2R} = \frac{\frac{C^2}{4}}{2R} = \frac{C^2}{8R} \quad \textbf{Rule IX.—1.}$$

Or in words: Versed sine = Chord squared divided by 8 times the Radius. Transposing we get,

$$C = \sqrt{8 \times V \times R} \qquad \textbf{Rule IX.—2.}$$

$$\text{and } R = C^2 \div 8V. \qquad \textbf{Rule IX.—3.}$$

In the above rules all the measurements are in the same unit. It may be useful to convert the rules to cases where the units vary, for instance:—

If V'' = Versed sine in Inches.
C = a Chord of 1 Chain of 66 feet.
R = Radius in Chains.

$$R = \frac{1^2}{8 \times V} = \frac{1}{8 \times V}. \text{ (All in Chains.)}$$

$$\text{and R (Chs.)} = \frac{1}{8 \times V''} \times 66 \times 12$$

$$\therefore \text{ R (Chs.)} = \frac{99}{V''} \quad \text{(Approx.)} \quad \textbf{Rule IX.—4.}$$

Or in words:—To find Radius in Chains, divide 99 by the Versed sine in inches on a 66ft. chord.

We may also note that Radius in Chains is 99 divided by number of quarter inches in the Versed sine on a 33ft. chord.

EXAMPLE.—Versed sine $7\frac{1}{4}''$ on $66'$ chord.

$$R = 99 \div 7\tfrac{1}{4} = 99 \div 7{\cdot}25 = \underline{\underline{13{\cdot}52}} \text{ Chains.}$$

It is useful to note that the Versed sine for 10 Chains Radius on a 1 Chain chord is practically 10″, and that if the Versed sine is divided by 2 or any other number the Radius must be multiplied by that number.

Tables 14, 15, and 16 will save labour in the calculation of Versed sines and Radii with various chords.

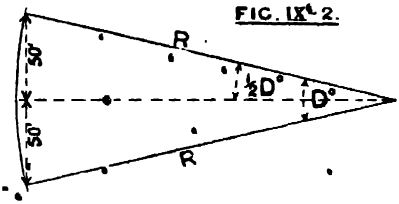
ENGLISH AND AMERICAN MEASUREMENTS OF CURVATURE.

Most engineers, including many of those in U.S.A., agree that in calculations, the Radius of a curve in feet is the most useful method of measurement. The custom of stating the Radius in Chains of 66ft. is so established in this country, however, that it cannot well be altogether discarded.

Conversions of Chains to Feet and *vice versa* are contained in Tables 7 and 8.

The method used in America and most of the Colonies, is to state the curvature in Degrees and Minutes, this being the measurement of the angle at the centre of the circle which is subtended by a 100ft. chord of the curve.

FIG. IX. 2.



To find the Degree of the curve (D°), we have from Fig. IX., 2:—

Sine $\frac{1}{2}$ D° = 50 ÷ R **Rule IX.—5.**

and R = 50 ÷ Sine $\frac{1}{2}$ D° **Rule IX.—6.**

To save labour these rules may be adapted for use by Tables of Reciprocals, thus:

Sine $\frac{1}{2}$ D° = 100 ÷ 2R′

or Sine $\frac{1}{2}$ D° = 100 times the reciprocal of twice the Radius.

Approx. rules are:—

D° = 5737 ÷ R ft. **Rule IX.—7.**

R′ = 5737 ÷ D° **Rule IX.—8.**

Degree of Curve = Number of inches in the Versed sine of a 61ft. 10in. chord. **Rule IX.—9.**

Conversions of Degree of Curve to Radius in Feet and Chains and *vice versa*, are contained in Table 11. These conversions will be found useful in studying American and Indian books on railway work.

BENDING OF RAILS.

Bull-headed rails will generally adapt themselves to curves of over 500 feet radius with slight "Jim Crowing" near the joints. Rails for sharper curves will need "crowing" throughout their length.

Flat bottom rails being less susceptible to bending, will need "crowing" for curves under say 30 chains radius.

The Versed sines for bending rails may be calculated by Rule IX.—1, those for the usual rail lengths and radii being given in Table 16.

SHORT RAILS FOR INSIDE OF CURVES.

In laying rails on a flat curve, the supply of rails of the standard length should be measured with a steel tape, as a slight variation in length will usually be found.* The longer and shorter rails should be reserved for the outer and inner rails respectively.

As the curve becomes sharper it will be necessary to use some of the special short lengths usually provided for use in curves. These lengths are usually 2 to 4 inches shorter than the standard length. With 4 inches decrease, a curve of 424 feet radius will need one 29′ 8″ rail to every 30 feet on the outer rail. With 2 inches decrease, a curve of 1270 feet radius will need one 44′ 10″ rail to each 45 feet on the outer rail.

With curves sharper than these, rails will have to be cut for the inner rail.

In any instance, the procedure may be as follows:—

First obtain Radius and Length of curve on outer rail.

EXAMPLE.—Radius, 12 chains = 792′. Curved length, 620′. Standard rails, 30′ Short rails, 29′ 8″.

The outer rail will need $620' \div 30 = 20$ rails, and a closure of about 20′.

Consulting Table 17, we see that with 30ft. rails and a radius of 798ft., the nearest to that in hand, the decrease for each rail is $2\frac{1}{8}$ins.

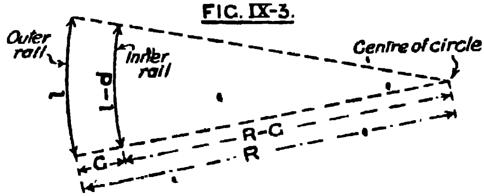
The total decrease will be $20 \times 2\frac{1}{8}'' = 42\frac{1}{2}''$.

* $\frac{1}{16}$in. above or below the standard length is allowed by the British Engineering Standards Association.

As each short rail gives 4ins. decrease, we shall require $42\frac{1}{2} \div 4 =$ say 10 short rails. These should of course be distributed in the curve at as nearly equal distances apart as possible. In this case a short and full rail may be laid alternately.

The Rules for finding the decrease of the inner rail are derived thus:—

FIG. IX-3.



Referring to Fig. 3.

If l = length of outer rail. R = Radius of ditto. G = Gauge. d = decrease of inner rail. (All in feet.)

Then by proportion:—

$$\frac{l-d}{l} = \frac{R-G}{R}$$

$$\therefore \quad Rl - Rd = Rl - Gl$$

$$\therefore \quad Rd = Gl$$

and $\quad d = \frac{G \times l}{R}$ **Rule IX.—10.**

Or in words: To find the decrease, multiply the gauge by the rail length and divide by the Radius. Multiply result by 12 to bring it to inches.

From above, $R = \frac{G \times l}{d}$, upon which rule Table 17 is based.

To find the decrease when the length (l) of outer rail and Versed sine (V′) on it are known, comes easily from Rule IX.—10 and,

$d = \frac{G \times V \times 8}{l}$ (all in feet). **Rule IX.—11.**

From this rule Table 17a is obtained. Table 20 will serve for applying Table 17 to other gauges.

LIMITING RADII OF CURVES FOR VARIOUS PURPOSES.

The amount of curvature of any track, measured either by the Radius or the Degree, is a very important question.

The sharper the curvature, *i.e.*, the less the radius and more the degree, the greater the strain and wear on rolling stock and permanent way, and the more the engine power required.

An American Engineer gives the following figures showing the relative renewals of straight and curved track, based on observations of 114 miles, over a period of 38 years.

Straight.	Curves.				
	Under 1° (87 Chns. Radius)	1° to 2° (87 to 43½ Chs. Rad.)	2° to 3° (43½ to 29 Chs. Rad.)	3° to 4° (29 to 22 Chs. Rad.)	4° to 5° (22 to 17½ Chs. Rad.)
1·00	1·00	1·08	1·31	1·39	1·47

With regard to the additional tractive force required to overcome the resistance on curves, it has been estimated that it takes double the force to draw a train at a speed of 25 miles per hour on a curve of 500ft. radius, which it does to draw it at the same speed on the straight.

The ideal would seem to be to provide the easiest curves, within reason, which the available space will allow, at least for high speed traffic. There are instances, however, where better results may be achieved by increasing the curvature on certain parts of the line, in order to introduce transition curves or to lessen the curvature through points and crossings. The latter, because super-elevation with its steadying effect, is difficult or nearly impossible to obtain, and also because sharp curves through diamonds should be avoided where reasonably possible. (See Chapters XVI. and XX.)

Where points and crossings are concerned, and the speed is moderate or slow, we may naturally take into account the extra cost of lengthy junctions and turnouts

which may be involved by using very flat curves. For instance, it may not be economical to lay an ordinary cross-over road to a radius greater than about 10 chains. This leads to the question of what are suitable curves for various purposes. In points and crossing work permanent way workers are apt to overlook the question of curvature, and first decide upon what angles of crossings and lengths of switches they will use. This is a wrong procedure, and the first question should be, with certain few exceptions, what radius is required?

Curves usually adopted on British Railways of Standard Gauge and with important traffic vary considerably, but the following general notes may be useful:—

Main Line Curves entailing no reduction of express speed.

The minimum radius depends chiefly upon the design and condition of the permanent way and rolling stock, and whether transition curves are used or not. The question of speed in relation to radius is dealt with later in this chapter.

With first-rate track and rolling stock, and speeds of about 50 miles per hour, it is suggested that 20 chains for plain line with transition curves and super-elevated, and 30 chains for points and crossings, be considered as the minimum radii.

Reverse curves (Chapter VIII.) should have a higher limit according to the length of straight or transition between them.

The above limits must only be taken as a general guide subject to the special conditions of each case, and it will be evident that there should be some reduction of speed on curves exceeding these in radius, which must be left to the discretion of the engine drivers, who should be acquainted with Rule No. 148 of the Railway Clearing House Rules.

Curves which call for definite instructions to be issued to reduce speed to certain rates.

These curves are usually listed in the Working Time Tables and in some cases warning boards are erected or distant signals are kept at danger.

It should be noted that the American Railway Engineering Association considers curves under 6° (14½ chains) as speed limiting curves, and the Indian Government limit (5′ 6″ gauge) is 5° (17·4 chains).

Absolute minimum radius of curve upon which main line engines have to run.

Such curves occur when valuable properties or other obstructions have to be avoided. In this case the wheel arrangement of that engine using the line which is least adapted for travelling on curves, will determine the radius.

400 feet is a common limit, but many railways adopt a minimum of 7 chains. In general, Goods Engines are constructed to take sharper curves than Passenger Engines (see later).

Curves for Fork or Connecting Lines.

In these cases, where speed is moderate, and economy in first cost is desired, 12 chains is suggested as a suitable radius as it makes a fairly easy curve, and no check rail is necessary.

The junctions, however, should have easier curves for reasons previously given.

Curves for entrances to Running Loops.

The radius may vary with the type of loop, Fast, Slow, or Goods, but in any case it should allow for the emergency of a fast train taking the facing points into the loop over a reverse curve. In good practice the loop inlets and outlets are made 30 chains radius if possible.

Curves for Junctions and Cross-over Junctions.

The radius for these will depend upon the nature of the traffic and the space, both in length and width, available.

On the question of obtaining the easiest curves for fast running in all directions, reference should be made to the International Railway Congress Report of 1900, where the limiting angle of Vee crossings is taken as 1 in 16, and of Diamonds as 1 in 8.

In studying this question, the Chapters on the various arrangements of Points and Crossings should be consulted.

Curves for Siding Groups.

In Sidings, the most standing room for the least land and material used, is generally the desired object.

The curves, therefore, may approach the minimum radius allowed for the engines. It is advisable to keep the curves fairly uniform in radius, especially in Sorting Sidings,

etc., so that the Shunters may become familiar with the resistance of the curves. For instance, the ruling radius for the first marshalling sidings at Edge Hill, was 7 chains.

Curves for Cross-over Roads and Shunting Connections with the Main Line.

A common radius for the ordinary trailing cross-over road is 10 chains. Where the main lines are on a sharp curve, the turnout which curves in the same direction as the main may have to be reduced in radius to prevent the crossing angle becoming too flat.

MINIMUM RADII OF CURVES UPON WHICH VEHICLES WILL TRAVEL.

This question arises in Yard and Siding Work, where speed need not be considered.

The rules given below allow a less radius than is usually arrived at by calculation. They will, however, be found reasonable in practice.

It may be noted that if the calculations are based on new tyres and rails, a margin of safety arises when either the tyres or rails are worn, or the gauge is slightly widened.

Referring to the diagrams, the following symbols will be used:—

FIG. IX-4.

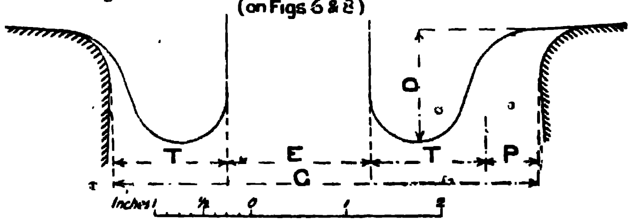


FIG. IX-5.

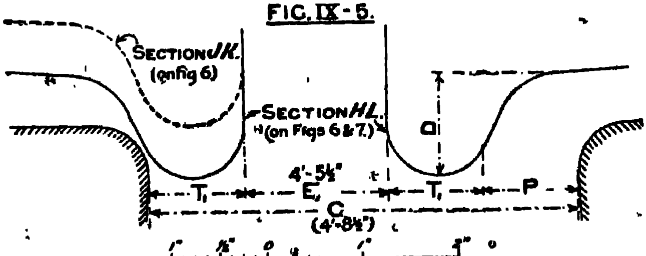


FIG. IX-6.

FIG. IX-7.

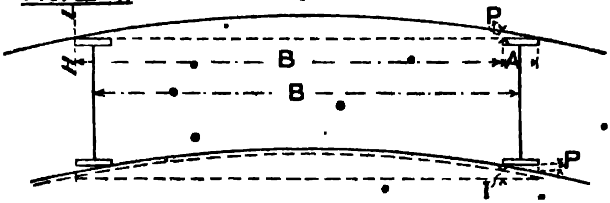


FIG. IX-8.

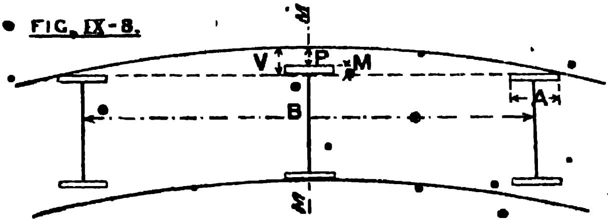
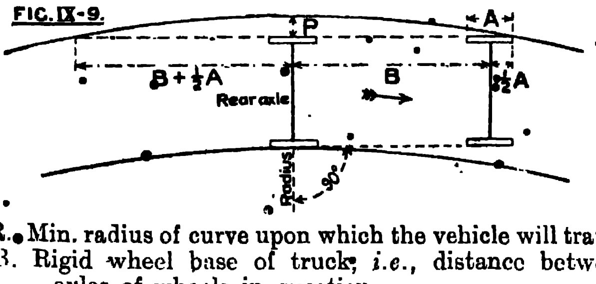


FIG. IX-9.



R. Min. radius of curve upon which the vehicle will travel.

B. Rigid wheel base of truck, *i.e.*, distance between axles of wheels in question.

T. Thickness of wheel flange obtained by taking an outline section of the tyre and placing it against the outline section of the rail as closely as possible and yet maintaining the tyre tread in contact with the top of the rail.

Then T = thickness from back of tyre to edge of rail. (See Fig. 4.)

T_1 = Thickness of wheel flange obtained by placing section of tyre close against rail as in Fig. 5, the tread standing at half the flange depth above the rail, as at *H* in Fig. 6.

E. Clear distance between backs of wheel flanges.

P. Total play between wheel flanges and rails.

$$P = \text{Gauge} - E - 2\,T.$$

D. Depth of wheel flange.

W. Radius of wheel at half depth of flange (Fig. 6).

A. Length of the portion of half flange which is below rail top.

$$A = \sqrt{8 \times \tfrac{1}{2} D \times W} \qquad \text{(From Rule IX.—2.)}$$

To find minimum radius (R) upon which a four-wheel vehicle with rigid wheel base will travel.

The calculations will also apply to bogie vehicles, in which case it is only necessary to consider one of the trucks.

Fig. 7 shows how the flanges will lie with regard to the rails. The flanges alone are shown and these by a sectional plan at half their depth, *i.e.*, as if cut through on the line *H H* (Fig 6). This is because the assumption is made that at half its depth, the front part of the flange begins to touch the rail.

This is where the rules differ from those usually given, in which it is assumed that the flange touches the rail at point *J*, Fig. 6.

Fig. 5 shows in dotted lines the relation of tyre to rail at point *J*, and in full lines the relation at point *H*.

The flange thickness to be taken will be T_1.

From Fig. 7, by Rule C 3, it will be seen that:—

$$R = \frac{B \times A}{2P} \qquad \textbf{Rule IX.—12.}$$

EXAMPLE.—Gauge, 4′ 8½″. Wheel base (B) = 12′. Diameter of wheel on tread, 3′ 3½″, therefore W = 20″. E = 4′ 5½″. Tyres as in Fig. 5, *i.e.*, ½ D = say ½″ and with these tyres T_1 will be 1″.

$$\therefore P = (4'\ 8\tfrac{1}{2}'' - 4'\ 5\tfrac{1}{2}'' - 2'') = 1'' = \tfrac{1}{12}'$$

$$A = \sqrt{8 \times \tfrac{1}{2}'' \times 20''} \qquad \text{(in inches.)}$$

$$= \sqrt{80} = \text{say } 9'' = \tfrac{3}{4}'$$

$$\therefore \text{By Rule IX.—12, } R = \frac{12' \times \frac{3}{4}'}{2 \times \frac{1}{12}'} = \frac{9}{\frac{1}{6}} = 9 \times 6 = \underline{\underline{54' \text{ radius.}}}$$

In practice, 60 feet is the usual limiting radius adopted for four-wheeled wagons of about 12 feet wheel base.

To find the minimum radius (R) upon which a six-wheel vehicle with rigid wheel base will travel.

Fig. 8 shows how the flanges will lie. The flange thickness in this case will be T (Fig. 4).

We get an additional dimension M, which may be called the freedom to move of the middle wheels out of line with the end wheels.

The limiting radius will be seen to be that for a curve whose chord is B + A and Versed sine M + P, and thus from Rule C 2:—

$$R = \left(\frac{B + A}{2}\right)^2 \div 2\,(P + M) \qquad \textbf{Rule IX.—13.}$$

EXAMPLE.—Gauge, 4′ 8½″. Wheel base (B) = 18′. Play of wheels (P) = ⅝″. M = 1¼″. Wheels as in last example, that is A = 9″ = ·75′.

$$R = \left(\frac{18' + \cdot 75'}{2}\right)^2 \div 2\left(\tfrac{5}{8}'' + 1\tfrac{1}{4}''\right)$$
$$= 9\cdot 375^2 \div 3\tfrac{3}{4}''.$$
$$= 87\cdot 89' \div \cdot 312'$$
$$= \underline{\underline{281 \text{ ft.}}}$$

200 feet is usually adopted as the limiting radius for six wheeled wagons, but the gauge is taken as being ¾″ wide thus increasing P by that amount.

To find radius which will avoid rear wheel flange bearing against inner rail.

According to A. M. Wellington, a rigid truck passing round a curve assumes the position shown in Fig. 9, that is the rear axle is on a radius line of the curve, unless the curve is very sharp, when the rear flange will touch the inner rail.

Spiller adds that the rear wheels will slide across the rail when the centrifugal force increases sufficiently.

It may be desired not to use radii below which the rear wheel flange bears against the inner rail, in which case (from Rule C 2),

$$\text{Min. } R = \frac{(B + \tfrac{1}{2}A)^2}{2 \times P} \qquad \textbf{Rule IX.—14.}$$

Locomotives on Curves.

As the wheel arrangements and the devices for suiting engines to take curves are varied, it is difficult to make calculations.

The minimum radius may be arrived at by making a large scale plan of the wheels in their position of greatest allowable movement out of line with each other, and then finding the radius of a curve which will just touch the flanges at the crucial points.

Amongst the devices used to enable engines to travel on curves are:—

Radial Axle Boxes (see Chapter II.).
Bogie Trucks (see Chapter II.).
Pony Trucks (see Chapter II.).
Wheels having thin flanges on the tyres.
Two or more flangeless wheels.

WIDENING OF GAUGE ON CURVES

From the foregoing examples it will be seen that if on curves the gauge is widened, then the play between wheel flange and rail is increased, and thus the passage of the vehicle is made easier.

Further, when we get below the absolute minimum radius, and a truck is "wheelbound," refusing to travel with normal gauge, it will do so if the gauge is widened.

The following methods of calculation may be given for the Increase of Gauge to suit the simpler wheel arrangements. These will apply to sidings, etc., where the traffic consists only of the kind of trucks in question.

For four-wheeled trucks.

Referring to Fig. 7, an increase (I) of gauge may be made, equal to the decrease of flange play (P) due to the curve, and we get from Rule C 13:—

$$I = \frac{B \times A}{2R} \qquad \textbf{Rule IX.—15.}$$

By this rule a usual type of 12ft. wheel base wagons would need:—

Radius.	Increase of Gauge.
200′	$\frac{1}{4}''$
100′	$\frac{1}{2}''$
75′	$\frac{3}{4}''$

For six-wheeled trucks.

In this case an increase of gauge (I) is necessary on very sharp curves to enable the six wheels of the truck to lie between the curved rails.

Referring to Fig. 8, if V equals the Versed sine on a chord of the curve equal to B + A, then :—

$$I = V - (P + M) \qquad \textbf{Rule IX.—16.}$$

With a usual type of six-wheeled wagon it will be found that :—

Radius.	Increase of Gauge.
400′	$\frac{3}{8}''$
200′	$\frac{3}{4}''$

On main lines and sidings where locomotives are in question, the calculations are not straightforward, on account of the variety and complication of wheel arrangements. Amongst the rules adopted from experience on British Lines the following appear to be the most reasonable :—

Curves under 500′ radius	...	Gauge to be 4′ 9″
,, 500′ to 792′ ,,	...	,, ,, 4′ $8\frac{3}{4}''$
Flatter curves...		,, ,, 4′ $8\frac{1}{2}''$

At one time it was customary to make the gauge tight through crossings with the idea of minimising sideways oscillation, but the gauge is now generally made the same as on plain road. It may be noted that where a curve occurs, and the gauge is accordingly widened, the check rail has more effect in keeping the opposite leading wheel flange from striking the crossing nose. The trailing wheels, however, are liable to strike the neck bend of the wing rail, unless the gap at the neck bend has been increased by double bending of the wing rails (see Chapter XII.).

CLEARANCES AND SPACES ON CURVED ROADS. (Fig. 10.)

When a vehicle is on a curved road, its corners overhang the rail to the outside of the curve by an increased amount to that which they overhang when on a straight road. This increase of overhang is known as the "End Throw."

To the inside of the curve the *middle* of the vehicle has an increased overhang, and this increase is known as the "Centre Throw."

The standard minimum clearances to structures on a straight road should, therefore, be increased on curves by the amount of the end and centre throws, on the outside and inside of the curve respectively.

An additional increase to the inside of the curve must be made for the leaning of the vehicle due to super-elevation of the rail. This increase may vary from one-half to twice the super-elevation, according to the height of the structure, the former applying to stages, etc., not higher than half the gauge, and the latter to bridge abutments, etc.

A further allowance of an inch is advisable for the extra oscillation on curves.

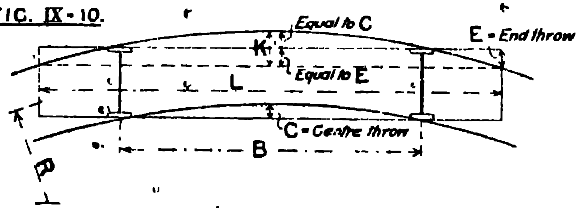
Where unusually wide stock is used, the ordinary 6ft. space will need increasing on curves. This increase should be:—

Centre Throw + End Throw + say, 2in. for Oscillation.

Rule IX.—17.

Care must be taken to have an equal super-elevation on each road.

FIG. IX-10.



The outline of a coach as it would lie when its wheels are on a curved road is shown in Fig. 10. For simplicity, the width of the coach is shown equal to the gauge.

It will be plain that:—

The Centre Throw (C) is the Versed sine on a chord equal to the length (B) of the wheel base, *i.e.*, from Rule IX.—1.

$$C = B^2 \div 8R$$ **Rule IX.—18.**

EXAMPLE.—Wheel base 12ft., Radius of curve 60ft.

$$\text{Centre throw} = 12^2 \div (8 \times 60)$$
$$= 144 \div 480$$
$$= \frac{3}{10}\text{ ft.} = 3\tfrac{5}{8}\text{ ins.}$$

The End Throw (E) is the Versed sine on a chord equal to the length (L) of the vehicle, less the Centre Throw (C), or:—

$$E = K - C$$

$$E = \frac{L^2}{8R} - \frac{B^2}{8R}$$

$$E = \frac{L^2 - B^2}{8R}$$ **Rule IX.—19.**

EXAMPLE.—Wheel base 12ft., Length of Wagon body 22ft., Radius 60ft.

$$E = \frac{22^2 - 12^2}{8 \times 60}$$
$$= \frac{484 - 144}{480}$$
$$= \underline{\underline{\cdot 71'}} = \underline{\underline{8\tfrac{1}{2}''}}$$

Where a bogie vehicle is concerned, B may be taken as the distance between centres of bogies.

In long passenger stock it is common to place the bogies so that the centre and end throws are equal.

Vehicles so arranged are termed "balanced coaches."

In this case:—

$$C = E$$

that is $$\frac{B^2}{8R} = \frac{L^2 - B^2}{8R}$$

or $$2B^2 = L^2$$

that is $\underline{\underline{B = L \div 1{\cdot}414}}$ and $\underline{\underline{L = B \times 1{\cdot}414}}$

Rules IX.—20 and 21.

EXAMPLE.—If a coach body is 60ft. long, what should be the distance between centres of bogies, so that centre and end throws will be equal?

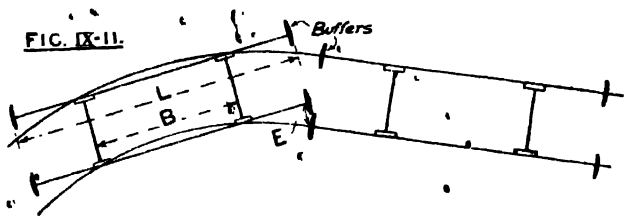
$$B = 60 \div 1{\cdot}414$$
$$= \underline{\underline{42{\cdot}43'}}, \text{ say, } \underline{\underline{42'\ 6''}}$$

Centre and End Throws for many usual wheel bases and coach lengths may be easily obtained by the use of Table 16.

BUFFER LOCKING.

Buffer locking is liable to occur at the springing point of very sharp curves, such as may occur in some town goods yards, etc. The tendency is worse at the reversing point of two sharp curves.

FIG. IX-11.



Inspecting Fig. 11, we see that if the End Throw of a vehicle, taken at the buffer faces, exceeds the width of the face, one face may slip past the other; that is, they will buffer lock. By the application of force in propelling the vehicles, this may happen whilst the buffer faces still have a small overlap or "engagement," say, 2 inches.

To find what Radius (R) of curve springing from a straight, will cause the risk of buffer locking with a given type of vehicle.

EXAMPLE.—Wagon wheel base (B) = 12′; Length over buffers (L) = 24′ 6″; Diameter of buffer faces 1′ 0″.

We will decide that the End Throw (E) must not be more than 2″ less than the diameter of faces, namely, 10″. = ·83′.

$$E = \frac{L^2 - B^2}{8R} \qquad \text{(From Rule IX.—19.)}$$

$$\therefore\ R = \frac{L^2 - B^2}{8E}$$

$$R = \frac{24{\cdot}5^2 - 12^2}{8 \times {\cdot}83}$$

$$= \frac{600 - 144}{6{\cdot}64} = \frac{456}{6{\cdot}64} = \text{say } \underline{\underline{70 \text{ ft.}}}$$

To find what Radius (R) of direct reverse curve will cause the risk of buffer locking with a given type of vehicle.

In this case the wagons will have End Throws in different directions, and thus the End Throws (E) must not exceed half the diameter of face, less half the engagement.

The Radius is proportional to E. And thus the Radius for a reverse curve should not be less than twice that for a simple curve. In the preceding example the safe Radius would be 140 feet.

If buffer locking occurs, the remedies to be considered will be:—

1. To flatten the curve
2. To make the change gradual from straight to curve
3. To increase the width of the buffer faces of the vehicles which buffer lock
4. To prohibit vehicles which have an excessive overhang of buffer faces beyond wheels

It must be noted that the above calculations do not allow for the slightly increased tendency to buffer lock due to play of wheels between the rails.

SUPER-ELEVATION.

The reader should first peruse the references made to this subject at the commencement of Chapter X.

When a car travels upon a curve, a force arises which acts directly outwards from the curve, due to the natural tendency for the movement to be in a straight line.

This force is named Centrifugal Force, and its amount increases as the weight of the car, and as the square of its velocity, and decreases as the radius of the curve increases.

By raising the outer rail of a curve by an amount "E," which is known as the Super-elevation, and sometimes as the "cant," we form an incline down which the car tends to slide, and thus counteract the Centrifugal Force

When "E" is just sufficient to balance the Force caused by a car moving at the highest speed obtaining upon a given radius of curve, it is referred to as the "Theoretical Super-elevation," which will afterwards be denoted by "E" if in feet, and by "e" if in inches.

The rule for E, the derivation of which is explained in books on Mechanics, may be written in various forms, thus, if:—

E = Super-elevation in feet.

e = Super-elevation in inches.

G = Gauge in feet, centre to centre of rails.

g = Acceleration due to gravity = 32·2′ per sec. per sec.

v = Velocity in feet per sec. = V × 1·47.

V = Velocity in miles per hour = v ÷ 1·47.

r = Radius of curve in feet.

R = Radius of curve in chains.

$$\text{Then } E = \frac{G}{g} \times \frac{v^2}{r} \qquad \textbf{Rule IX.—22.}$$

or with 4ft. 11in. gauge—

$$E = \cdot 153 \times \frac{v^2}{r} \qquad \textbf{Rule IX.—23.}$$

Changing E to inches, and v to miles per hour,

$$e = 12 \times \frac{G}{32 \cdot 2} \times \frac{(V \times 1 \cdot 47)^2}{r}$$

$$e = G \times \frac{V^2}{1 \cdot 25r} \qquad \textbf{Rule IX.—24.}$$

which is perhaps the most usual form of the rule. If G is 4ft. 11in., we may write:—

$$e = 3 \cdot 96 \times \frac{V^2}{r} \qquad \textbf{Rule IX.—25.}$$

If the Radius is taken in chains, we get:—

$$e = \cdot 06 \times \frac{V^2}{R} \qquad \textbf{Rule IX.—26.}$$

on which rule many tables have been based.

If the curve is measured in degrees (D°), we get:—

$$e = \cdot 00066 \times D^\circ \times V^2 \qquad \textbf{Rule IX.—27.}$$

which is the usual American rule for the Theoretical e.

The formulæ show that e increases rapidly with the velocity, namely, as the square thereof, and that it decreases directly as the radius increases.

Thus, on the same curve a speed of 60 miles per hour would call for four times the e due to a speed of 30 miles.

Again, with the same speed, a curve of 20 chains radius only calls for half the e due to a curve of 10 chains radius.

Table 18 shows the Theoretical e on the 4ft. 8½in. gauge, calculated by Rule IX.—26, for different Speeds and Curves, both the Radius and Degree of the latter being given.

Practical Super-elevation.

There are further considerations to be taken into account before we arrive at the e to be actually used—that is, what may be termed the "Practical e."

First the Theoretical e may be so great that when a light car is stationary or moving slowly on a curve, it may be in danger of overturning by wind pressure from the outside of the curve.

Spiller shows that a certain van may be blown over by a wind pressure of 20 lbs. per square foot when the e is 4½ins., and suggests limiting e to 4ins.

The present writers consider this maximum rather small, and recommend one of 5ins., remembering that e's of 6ins. and even 7½ins are occasionally used without mishap.

The above considerations place a limit upon the maximum e.

There are other matters which lead to the reduction of e throughout:—

(1) Though e is intended to abolish side pressure from centrifugal force at the maximum and lower speeds, it is generally accepted that the outer leading wheel flange is always in contact with the rail and tending to climb over it. This may be said to be due to the car having to continually change its direction as it passes round the curve. Now at speeds lower than that used to calculate e, super-elevation takes part of the load from the outer wheel, which load tends to keep it down and prevent climbing.

(2) If e is much more than is called for by the greater proportion of the traffic, excessive wear of the table of the inner rail may occur, due to its greater load, and again the joints and fastenings may be unduly stressed, the top of the rail being pushed outwards.

Thus we see that the actual e's should be less than the e's calculated for the highest expected speed; but, so far no writer seems to have laid down a definite working rule to this effect.

The usual advice is similar to that of Cole, who says that "the calculated cant should be approached carefully, super-elevating bit by bit until the engines take the roads comfortably."

In an endeavour to fix upon the actual e to allow in practice, it is suggested that as a rule, with exceptions in special cases, it be taken as $\frac{5}{8}$ of the Theoretical e for the highest expected speed.

This will reduce the 1·25 to 2 in Rule IX.—24, and the 3·96 in Rule IX.—25 to 2·48, or, say, half the gauge (centres of rails), and the rule we suggest for all gauges is:—

$$\text{Practical } e'' = \tfrac{1}{2} G \times \frac{V^2 \text{ (M. per hour)}}{r \text{ (ft.)}} \qquad \textbf{Rule IX.—28.}$$

This reduction is not chosen haphazard, and has the following facts to recommend it:—

The practical e would be the Theoretical e for $\frac{4}{5}$ of the highest speed, and for somewhere near the average speed of passenger traffic.

With such e's it has been found upon one of the Lancashire districts (not electrified) that the inner and outer rails lose weight at about the same rate.

Again, a Theoretical e of 8ins is about the maximum which would arise in extreme cases under working conditions. Now in Shortt's opinion, 3ins. may be taken from such an e without very apparent effect to passengers in a train rounding the curve.

Thus the 5ins. we suggest as a practical maximum is $\frac{5}{8}$ of the theoretical one of 8ins.

It may be noted that many authorities agree that the value of super-elevation has been over-estimated as regards the prevention of overturning, but that it conduces to the comfort of passengers and easy running.

Super-elevation is often omitted of necessity through points and crossings, though in many cases fairly high speeds obtain through them.

The suggested procedure in arriving at the e to allow in practice would, in accordance with the ideas expressed above, be as follows:—

(1) Determine the highest speed usually run upon the curve.

(2) Obtain the curvature by taking the average of several Versed sines on 66ft. chords.

(3) Take the Theoretical e from Table 18 for the speed and curvature.

(4) Obtain ⅔ of e from Table 19, which will give the Practical e to allow.

In sidings where no considerable speed is obtained, the e need naturally only be small, and it is common to limit it to 2 inches.

Obtaining the Super-elevation from the Versed sine of a Chord.

As the Versed sine on a chord of a curve is proportional to the Radius and to the square of the chord; and e is proportional to the Radius and the square of the Velocity, it is possible to calculate the length of a chord, the Versed sine of which will be equal to e

W. H. Cole gives tables applying to many gauges and speeds on this principle.

With 4ft. 8½ins. gauge:—

From Rule IX.—25. Theoretical e = Versed sine on a Chord whose length in feet is 1·625 × Velocity in Miles per hour. **Rule IX.—29.**

From Rule IX.—28, Practical e = Versed sine on a Chord of 1·286 × Velocity in Miles per hour. **Rule IX.—30.**

One American rule is to make e = Versed sine on a chord whose length in feet = distance travelled in one second.

According to this, Rule IX.—25 would become:—

$$e'' = 3{\cdot}24 \times \frac{V^2}{r}$$

Determination of Speed.

Before we can use any of the above rules it is necessary to obtain the Velocity, or "V."

Generally, the rule is to take the speed of the fastest train either by estimating or timing.

It will be apparent that the speed, and therefore the e, will be less on a rising and more on a falling gradient, and in some cases on double lines more e is required on one line than the other.

In single lines the track is generally elevated to suit the descending trains.

To time the speed, measure a fair length of the curve, say, by counting the rails. Then observe the time taken to travel this distance.

One method is to place a man with a stop-watch at each end of the measured length, both watches being in exact agreement, each man to stop his watch as the engine passes him.

EXAMPLE.—Distance 20 rails of $30' = 600'$.

Engine passed first man 10 mins. 37 secs., second man 10 mins. 45 secs.; time 8 secs. Then speed in feet per sec. $= 600 \div 8 = 75$.

To convert Feet per sec. into Miles per hour, divide by 1·47, thus:—75′ per sec. $= 75 \div 1{\cdot}47 = 51$ Miles per hour.

Table 12 shows conversions for various speeds.

Running out of Super-elevation.

The ideal is for e to be gradually attained upon a transition curve, at the springing of which from a straight road, e will be nil, and will increase at a regular gradient to the end of transition and beginning of circular curve, where it will become equal to the e required on the latter.

If there is no transition curve there are difficulties, for if, as is often the case, e is attained on the straight, there is, according to Spiller, more danger at the entrance to the curve than if there was no e at all.

It may be noted, however, that as a curve seldom, if ever, in practice begins suddenly, it should be possible to introduce a little of the e on the straight, and attain its full dimension some distance along the curve, using a gradient of 1in. in 60ft. to 120ft., according to the circumstances. It is most important that e should be maintained constant throughout the main part of the curve.

Maximum Speed allowable on Curves.

Those who have studied this subject will understand that it is difficult to fix upon any rule.

Three rules may, however, be quoted that have been suggested for the 4ft. 8½in. gauge:—

	Maximum Speed in Miles per Hour.	Theoretical e
		Inches.
Wellington...	$\cdot77 \times \sqrt{r}$ ft. or $6\cdot25 \times \sqrt{R}$ Chns.	2·35
Spiller	$\cdot871 \times$ „ „ $7\cdot07 \times$ „	3·00
Shortt	$1\cdot358 \times$ „ „ $11\cdot00 \times$ „	7·30

It will be noted that Shortt's rule allows the highest speed, and it has been pointed out that even the speeds given by this rule are exceeded in everyday working.

Allowable speeds, as usually accepted in British practice, with first-class track, but without transition curves, are shown in Table 18.

It may be noted that if the ideal of all curves being traversed at a speed in proportion to the square root of the radius, obtained in practice, we should require the same theoretical e on every curve, and these are shown opposite the speed rules above.

Super-elevation on Electrified Lines.

The low centre of gravity of electric stock does not alter the theoretical e, and it is usual to allow the same e as on steam-worked lines.

A low centre of gravity reduces the tendency to overturn by centrifugal force, but increases the tendency of the outer wheels to climb, and is responsible for excessive side wear on the outer rail, which cannot be reduced by increasing the e.

Super-elevation on Various Gauges.

The e should be in direct proportion to the gauge, or, to be exact, to the distance between centres of rails; thus, a gauge of 2ft. 4¼in. would require approximately one-half of the e due to a 4ft. 8½in. gauge.

Table 20 shows the figures to multiply the e's on Tables 18 and 19 by, to suit other gauges than 4ft. 8½in.

THE CHECKING OF CURVES.

According to the Board of Trade Rule, curves of 10 chains or less radius must be provided with a check rail to the inside rail of the curve. Where high speeds obtain, and on electrified lines, it is advisable to check curves of greater radius than 10 chains. Each case can only be decided on its circumstances, unless the Ministry of Transport eventually issue a definite rule.

The detailed particulars adopted with regard to checking vary, and a table is given below which may be completed in accordance with the practice the reader is concerned with, as may the particulars of check chairs listed in Chapter III.

Some railways do not elevate the check rail above the running rail. It may be noted, however, that an elevated check acts upon a greater length of the wheel flange than a level check. A serviceable rail is generally used for the check rail, and heavy section running rails are usually checked by old rails of lighter section.

There are difficulties in checking curves throughout points and crossings, especially at the switches. Photo No. 1 shows the check carried as far as possible towards the switch toe in a certain instance.

The purpose of the check rail is to prevent the outer wheel flange from climbing the rail. In order to do this completely, the check should come into contact with the back of inner wheel flange before the outer flange tends to climb.

Thus the acting face of the check should be at a distance from the outer rail, equal to T+E where E = the distance from back to back of flanges and T = thickness of wheel flange taken as in the notes on "Vehicles on Curves" (Fig. IX.-4).

With E = 4′ 5⅜″, and T = 1⅛″, new flanges, T+E would be 4′ 6¾″, and with normal gauge, the Flangeway Clearance (F) between rail and check would be 4′ 8½″ − 4′ 6¾″ = 1¾″.

This is the usual clearance in checking crossings, 2ins. clearance being usually given on plain straight roads when on viaducts, etc.

The dimension of 1¾″ would apply when the curve is nearly straight, but if there is considerable curvature the flange will touch the check earlier, and a wider flangeway will be needed, as may be seen from Figs. IX.-7 and 8.

The ideal flangeway will vary according to the dimensions of the wheel tyres, the wheel base, the curvature, and the elevation of the check rail. It is not practicable, however, to provide numerous patterns of check chairs, and only about three patterns are usually used.

As wheel flanges become worn, or the outer rail becomes side worn, or the road becomes wide to gauge, then if the check is not worn it touches the inner wheel flange before the outer one touches the rail and thus is taking all the side pressure. If the check becomes worn or if it has too wide a flangeway, the inner flange will not touch it until the outer flange tends to climb and to "side-wear" the outer rail

It is open to question whether, and if so, to what extent, the check rail should share side wear with the outer rail. The authors think that it should share it about equally and have attained this result by inserting packings in the check chairs, which packings are removed as the channel becomes too wide by wear, the thickness of the keys being changed as required. Additional wear may also be obtained out of the check rail by reversing it.

Curve Checking adopted on..........Railway.

Check Rails to be provided:—

On Passenger Running Lines :—

With speeds over.........miles per hour to curves of......... radius and under.

With speeds under........miles per hour to curves of......... radius and under.

On Electrified Running lines to curves of......radius & under.

On Goods „ „ „ „ ...... „ „

On Sidings used by passenger trains „ „ ...... „ „

On Goods sidings „ „ ...... „ „

On Viaducts and Bridges......ft. in length and over.

Over Paved Level Crossings. (Usually to both rails.)

Check Rail on Running Lines to extend......ft. beyond each end of curve.

For list of Check Chairs, see Chapter III.

CHAPTER X.

TRANSITION CURVES.

The papers of Shortt and Spiller in the Proc.Inst.C.E., Vol. 176, 1909, with the discussion thereon show much of the theory and practice of leading railway engineers with regard to transition curves and super-elevation. These papers have been largely quoted by various writers. The discussion exemplifies the difference of opinion and the extensive theorising which may take place upon these subjects, obliging us to decide upon and explain a certain line of practice without devoting space to the theory and proofs of the rules in every case.

A transition, sometimes called easement, curve may be defined as a curve whose degree of curvature gradually increases so as to make an easy change from a straight line to a circular curve, or from one circular curve to another of different radius.

Alternatively, a transition curve may be defined as a curve of constantly changing radius so as to allow the super-elevation due to a circular curve to be attained at a regular gradient; the curve being designed so that the elevation at any point is correct for the curvature at that point.

According to the idea of Shortt, the latter definition hardly applies, because he states that a curve with no super-elevation is more in need of easing than one with it.

We are in agreement with this, and advocate in practice that similar transitions be used for the same radius, whether the curve is super-elevated or not.

A curve which provides for the conditions of the second definition is the Cubic Parabola, the transition curve generally adopted in this country, and to which the following notes apply. Its use is by no means new, as it was propounded by Froude about 1860. Mention may be made of other curves as follows:—

10 Chord Spiral.	**Adopted by American Railway Eng. Association.**
Holbrook ,,	U.S.A.
Crandall ,,	,,
Talbot ,,	,,
Searles ,,	,,
Lemniscate of Bernoulli.	**A curve often used on Tramways, but which has been advocated for Rlys.**

It must be noted that the easing of the sharp curves used on tramways, presents a different problem to that of railway curves.

Prob. I. Transition Curve between a Straight Line and a Circular Curve.

If a circular curve, afterwards referred to as an arc, springs from, that is touches, a straight line, and it is desired to place a transition curve between them, then either the arc, or the line, or both, must be moved so that they do not touch, and there is a certain minimum gap between them.

This gap is called the Shift (S) and it is one of the three main dimensions with regard to a transition curve.

A second dimension is the length (L) of the transition curve from its springing point to its termination where it joins the arc.

A third dimension is the Radius (R) of the arc.

R being known, either L or S may be chosen. If L is decided upon, S may be found thus:—

$$S = L^2 \div 24\,R \qquad \textbf{Rule X—1.}$$

Or, if S is known, and L is required:—

$$L = \sqrt{24 \times S \times R} \qquad \textbf{Rule X.—2.}$$

L, however, is usually the correct dimension to choose, and there are two methods of making a suitable choice.

1ST METHOD.—By taking the super-elevation to be given to the arc and making L sufficient to run it out at an easy gradient.

One inch in 66 feet or 1 in 792 is a commonly used "run-out." Thus, if the elevation is 5ins., L would be made $5 \times 66 = 330$ft. It may be noted that Froude used a gradient of 1 in 300.

2ND METHOD.—By making L bear a definite relation to the Radius, thus Shortt advocates that:—

$$\left.\begin{array}{lcl} \text{L in chains} & = & \sqrt{\text{R in chains}} \\ \text{or L in feet} & = & 8\cdot12 \times \sqrt{\text{R in feet}} \end{array}\right\} \textbf{Rule X.—3.}$$

Thus for a curve of 25 chains Radius, L would be made $\sqrt{25} = 5$ Chains; and for a curve of 1600ft. Radius, L would be $8\cdot12 \times \sqrt{1600} = 8\cdot12 \times 40 = 325$ ft.

In the second method, the less the Radius, the less the transition length; whilst by the first, the reverse will generally be the case.

The second method is recommended chiefly because the very fact of a small radius existing, usually means less length available for transition.

Now if a rule is adopted such that L shall bear a certain relation to R, such as L chains $= \sqrt{R \text{ chains}}$, that is $R = L^2$, then S will be a constant, because with all dimensions in chains:—

$$S = \frac{L^2}{24R}$$

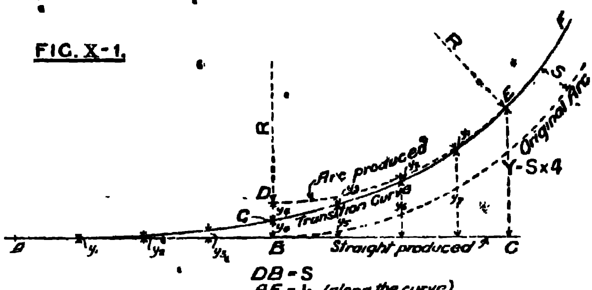
$$\text{or} \quad S = \frac{L^2}{24 \times L^2}$$

$$\text{i.e.} \quad S = \frac{1}{24} \text{ Chain} = \underline{\underline{2' \ 9''}}$$

This is the Shift worked to by Shortt on high speed curves, but with sharp curves or in confined places, we may have to be satisfied with much less shift and shorter transition curves.

From Table 21 may quickly be calculated the transition lengths (L) for various Shifts.

FIG. X-1.



Referring to Fig. 1, having calculated S from R and L, or in some cases L from S and R, the original arc shown in dotted lines must be moved to DEF, parallel to its first position, or alternatively the line ABC may be moved, until there is a gap DB between the arc and the line equal to S.

This gap will be opposite the original tangent point B.

A transition curve has then to be run from the line ABC to the arc DEF.

The offset from the line ABC to G, the centre of transition, is $\frac{1}{2}$ S, and it will be found that the offset $CE =$ Y, to the end of transition, is $S \times 4$.

To obtain intermediate points, other offsets (y) from the line ABC may be set.

Now all offsets are proportional to the cube of their distances from the springing point A, measured along the curve.

This principle is used in obtaining other offsets.

The basis of the proportion may be the end offset. Y = S × 4.

Then for any offset y at a distance x from A, we have:

$$y : x^3 = 4\,S : L^3$$

$$\therefore y = 4S \times \frac{x^3}{L^3} \qquad \textbf{Rule X.—4.}$$

The number of offsets required depends upon the length of curve, etc. Table 22 enables offsets to be readily calculated when L is divided into 8, 10, or 12 parts.

If only 4 offsets are needed, use the Table for 8 offsets omitting the 3rd, 5th, and 7th. For 5 or 6 offsets use Tables for 10 or 12 offsets, omitting the odd numbered offsets.

It must be understood that if the transition curve is divided into a certain number of equal parts, then whatever its length may be, the list of offsets is always the same for a certain shift.

Thus if L is divided into eight equal parts and the shift is 2′ 9″ or 132″, the end offset will be 2′ 9″ × 4 = 11′ 0″, and the other offsets will be:—

		ins.				ft.	ins.
1st offset	=	132	×	·002	=		0¼
2nd ,,	=	,,	×	·0156	=		2
3rd ,,	=	,,	×	·053	=		7
4th ,,	=	,,	×	·125	=	1	4½
5th ,,	=	,,	×	·244	=	2	8¼
6th ,,	=	,,	×	·422	=	4	7¾
7th ,,	=	,,	×	·670	=	7	4½
8th ,,	=	,,	×	1·000	=	11	0

A further fact which may be of use, is that offsets to the transition curve taken from the part of the arc produced, $E\,D$, beginning at E, are practically equal to those from the line $A\,B\,C$ beginning at A.

Prob. 2. Transition Curve between Reverse Curves.

In this, as in the former case, there must be a gap or Shift (S) before a transition curve can be attained.

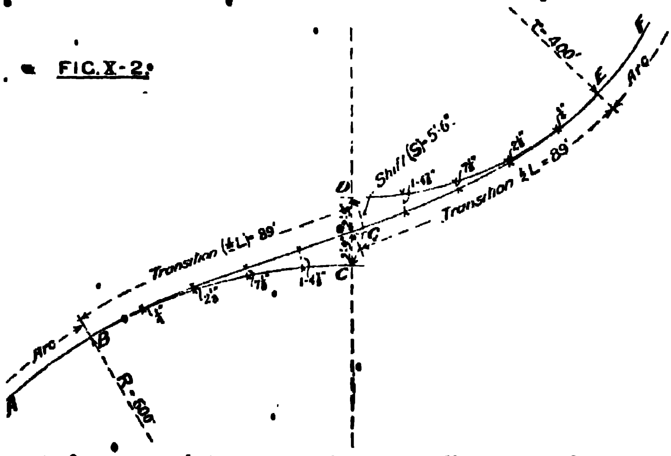
The shift in this case is the least distance between the two arcs. Let R and r indicate the radii of the arcs, R being the greater radius. One ideal for a transition curve would be one whose total length $L = \sqrt{R}$ chains $+ \sqrt{r}$ chains, and to arrange each of the two parts as if it sprang from a straight line, this line being a common tangent to the two parts of the reverse transition curve.

This ideal can very seldom be attained because the shift between the arcs would be roughly eight times the 2′ 9″ shift due to each curve from a straight, or 22′ 0″.

Another method is to make the two parts of the transition curve duplicates of each other, except that the one for the arc of less radius is made longer than the other.

Froude's method,† an explanation of which will now be given, will be found easy to use and to give a satisfactory curve when the shift is pre-determined, as will nearly always be the case in practice.

FIG. X-2.



Referring to Fig. 2 which shows a dimensioned example, as follows:—

Shifted arc $A\,B\,C = 600'$ Radius (R).

,, ,, $D\,E\,F = 400'$,, (r).

Shift $C\,D = 5'\ 6''$ (This is where the arcs are nearest to one another).

† See Manual of Civil Engineering, Rankine.

The rule for L, with all dimensions in the same unit, will be:—

$$L = \sqrt{24 \times S \times \frac{R \times r}{R + r}} \ddagger \qquad \textbf{Rule X.—5.}$$

In the example $L = \sqrt{24 \times 5{\cdot}5 \times \frac{600 \times 400}{600 + 400}}$

$$= \sqrt{24 \times 5{\cdot}5 \times \frac{240000}{1000}}$$

$$= \sqrt{24 \times 5{\cdot}5 \times 240}$$

$$= \sqrt{31680}$$

$$= \underline{\underline{178'}}$$

If L is the given dimension, and S is required:—

$$S = L^2 \div \left(24 \times \frac{R \times r}{R + r}\right) \ddagger \qquad \textbf{Rule X.—6.}$$

The middle point of the transition curve will be at *G*, the centre of the shift *C D*. Its ends may be marked off by *G B* and *G E* each equal to ½ L or 89′.

The offsets to each half of the transition curve may be set from each arc produced, beginning at the end *B* for curve *B G*, and at *E* for *E G*.

The offsets will increase proportionally to the cube of their distances from *B* and *E*. Table 22 may be used to calculate the offsets required; thus suppose the last offset *G C*, which is half the shift, equals 2ft. 9ins. or 33ins. and 5 points are required in each curve, then from Table 22, taking even offsets only,

	ins.		ins.	ft.	ins.
1st offset	= 33 ×	·008 =	·26 =		$0\frac{1}{4}$
2nd ,,	= ,, ×	·064 =	2·11 =		$2\frac{1}{8}$
3rd ,,	= ,, ×	·216 =	7·128 =		$7\frac{1}{8}$
4th ,,	= ,, ×	·512 =	16·896 =	1	$4\frac{7}{8}$
5th ,,	= ,, ×	1·000 =	33·0 =	2	9

In Fig. 2 all dimensions are shown for curves of 400ft. and 600ft. radius with 5ft. 6ins. shift.

‡ These are the expressions for "Equivalent Radius" see Chap. XIII.

Prob. 3. Transition Curve between Curves of Similar Flexure.

Shortt shows that it is not necessary to use a transition curve unless the difference in radii of the curves exceeds 15 per cent. His rule for the transition length is:—

$$L = \left(1 - \frac{r}{R}\right) \times \sqrt{r} \qquad \textbf{Rule X.—7.}$$

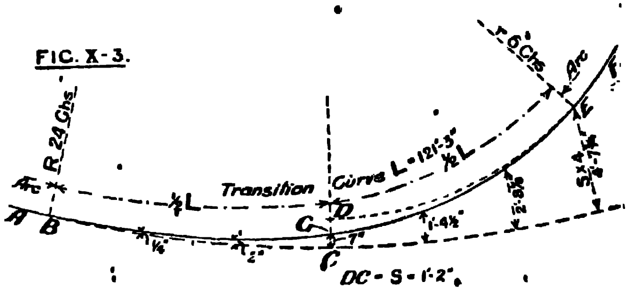
where r is the lesser radius, and all lengths are in chains

The formulæ connecting L, S, and R, each in the same unit will be:—

$$L = \sqrt{24 \times S \times \frac{R \times r}{R - r}} \ddagger \qquad \textbf{Rule X.—8.}$$

$$S = L^2 \div \left(24 \times \frac{R \times r}{R - r}\right) \ddagger \qquad \textbf{Rule X.—9.}$$

FIG. X-3.



The procedure will be similar to that explained above for reverse curves, but with reference to Fig. 3, which shows all dimensions for curves of 6 and 24 chains radius and a pre-determined shift of 1ft. 2ins.

The offsets may all be set from the flatter arc, the end offset being S × 4.

It will be found that unless the radii are very different, Rule X.–7 will give a very short transition, and Rule X.–9 a very small shift, for example: with curves of 20 and 10 chains radius, L will only be 1·58 chains, and S will only be 4⅛ins.

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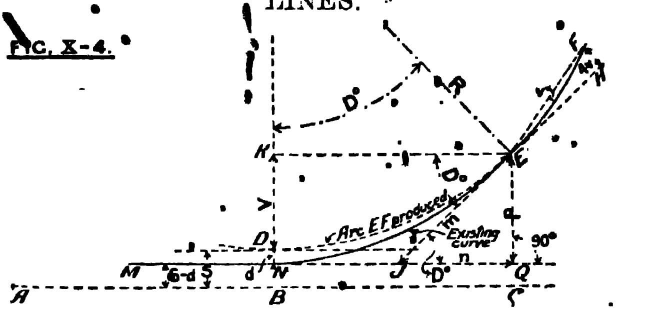
‡ These are the expressions for "Equivalent Radius" see Chap. XIII.

INSERTION OF TRANSITION CURVES IN EXISTING LINES.

FIG. X-4.



Prob. 4. Under an existing alignment, an arc NEF, more or less regular, joins a straight MNQ, Fig. 4. To insert a Transition Curve.

1st.—Obtain Radius of arc (R) from a Versed sine (v) on a chord EF, which is estimated to be just clear of the transition curve.

2nd.—At E draw a tangent EH to the arc and produce it to cut MQ at J. Draw EQ square to MQ. Measure EJ = m, JQ = n, and EQ = p.

3rd.—Find point D where arc EF when produced will touch line MNQ, or if not that line, a line parallel to it, thus:—

$$QD = EK = \mathrm{R} \times \frac{\mathrm{p}}{\mathrm{m}} * \quad \text{(From Rule C 40.)} \quad \textbf{Rule X.—10.}$$

$$KD = \mathrm{V} = \mathrm{R} - \left(\mathrm{R} \times \frac{\mathrm{n}}{\mathrm{m}}\right) \dagger \quad \text{(From Rule C 43.)} \quad \textbf{Rule X.—11.}$$

$$DN = \mathrm{d} = \mathrm{p} - \mathrm{V}$$

If the arc produced touches the line MQ, d will be nil and the arc and line must be moved apart by the required shift (S).

If, owing to some easement having already being made, d is sufficient to be made the shift, a transition may be put in without alteration of the main parts of the arc or line, its centre being at the middle of point d.

If V is greater than p, the arc produced cuts the line and they must be moved apart by d + S.

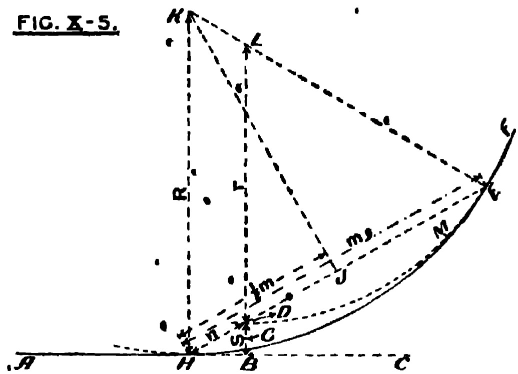
If d is smaller than the necessary shift, the line and arc must be moved apart by S − d. This is the case shown on the diagram, where the line MNQ has been shifted to ABC.

* Sin D° † Cos D°

After the movement, the transition curve may be lined out as described in Prob. 1, the same reference letters applying, but point E will need altering to conform to the calculated length.

Prob. 5. To insert a Transition Curve between a straight AHC and an arc HEF of radius R, which touch at H, when neither the straight nor the main part of the arc may be shifted. (Fig. 5.)

FIG. X-5.



This is a very common case in practice.

In order to obtain a shift it will be necessary to reduce the arc to one of smaller radius (r) as it approaches the straight, and to run the transition curve to the new arc.

Case 1.—When the arc cannot be altered beyond a certain point E.

1st.—Choose the shift (S).

2nd.—Draw a line EH and on it find point D which is at a distance S from AC. Draw DB square to AC and bisect it at G, which point will be the middle of the transition curve.

3rd.—Measure $EH = \mathrm{m}$, and $DH = \mathrm{n}$, and calculate r.

$$\mathrm{r} = \frac{\mathrm{R} \times (\mathrm{m} - \mathrm{n})}{\mathrm{m}}$$ **Rule X.—12.**

This is deduced from the similar triangles KHE and LDE.

4th.—The transition curve may be set out as previously explained, its length (L) will be :—

$$\mathrm{L} = \sqrt{24 \times \mathrm{S} \times \mathrm{r}}$$ (by Rule X.—2.)

In Fig. 5 it is assumed that the transition curve will join the new arc at M. The distance $M\,E$ should not be too small, about an engine length being suitable. If point M falls beyond point E, the shift must be reduced.

Case 2.—When it is decided to reduce the arc to a certain radius (r), Fig. 5.

In order that a transition curve shall not be necessary between the two arcs the reduction in radius should not exceed 15 per cent.

1st.—Choose the shift (S).

2nd.—The tangent point E of the two arcs, and the point D, where the new arc will be nearest to the line $A\,C$, are required, so calculate m and n, thus:

$$m = \sqrt{\frac{2 \times S \times R^2}{R - r}} \qquad \textbf{Rule X.—13.}$$

$$n = \frac{2 \times S \times R}{m} \qquad \textbf{Rule X.—14.}$$

3rd.—From point H fix E by distance m, and on line $H\,E$ fix point D by distance n.

Draw $D\,B$ which will equal S, and bisect it at G which will be the middle point of the transition curve.

4th.—Length of transition curve (L):—

$$L = \sqrt{24 \times S \times r}$$

The above rules for m and n arise thus:—

From similar triangles $H\,D\,B$ and $K\,H\,J$.

$$\frac{n}{S} = \frac{R}{\frac{1}{2}m} \quad \therefore n = \frac{2\,S\,R}{m}$$

and from triangles $K\,H\,E$ and $L\,D\,E$,

$$\frac{R}{m} = \frac{r}{m - n} \quad \therefore n = \frac{m\,(R - r)}{R}$$

$$\text{So} \qquad \frac{2\,S\,R}{m} = \frac{m\,(R - r)}{R}$$

$$\therefore m = \sqrt{\frac{2\,S\,R^2}{R - r}}$$

CHAPTER XI.

THE SWITCH.

The terms usually adopted with regard to the parts of the switch and its appurtenances, are shown on Figs. XI.-2 and 3.

The switches in use for railway track are as a rule of the straight type, that is, when placed against a straight stock or back rail, the running edge of the switch is a straight line.

It will readily be seen that when such a switch is placed close against a back rail, which is bent to conform to a curved main line, the switch will take up the same curvature as the main line, because whatever curvature there is in the back rail, is transmitted to the switch.

There are exceptions to the rule, the shorter switches of some railways having their running edges curved to some fixed radius, not necessarily that of a turnout curve. This kind of switch is generally adopted on tramways.

A type of switch which can hardly be called an exception to the rule, is the "springing" switch (Fig. 3). In this, the switch is straight from the toe to a point where the wheel flange clearance is obtained; here there is no joint. but a "virtual heel" about which the switch pivots. The remainder of the switch is held firmly in chairs and forms part of the curve of the diverging road.

For purposes of calculation, the length (S) of this switch must be taken as the distance from the toe to the virtual heel and not to the heel joint.

The Switch Heel Divergence.

The heel divergence of the switch and the gauge of the railway are the basic dimensions for nearly all point and crossing calculations. The former is thus a very important dimension.

The heel divergence which will afterwards be referred to as "H," is the distance between gauge lines at the switch heel, and so is equal to the width of the rail head plus the flangeway.

On British main lines the flangeway varies from $1\frac{3}{4}$ins. to 2ins. Rail heads vary from $2\frac{9}{16}$ins. for 80 lb. section, to $2\frac{3}{4}$ins. for 90 lb. section and over. Thus the heel divergences may vary from $4\frac{1}{4}$ins. to $4\frac{3}{4}$ins.

It would be an advantage if H could be fixed as a standard dimension. In practically all the examples and tables H will be taken as $4\frac{1}{2}$ins. This, with rail heads $2\frac{3}{4}$ins. wide, will give $1\frac{3}{4}$ins. flangeway. It must be noted that $4\frac{1}{2}$ins. is $\frac{3}{8}$ft., and this gives simple values for obtaining the switch angle, which is the angle the switch makes with the stock rail in the main line.

The Switch Angle.

One way of measuring the switch angle is by its rate of divergence. This is the ratio of H to the switch length (S)

To find M, when the Angle of Divergence is "1 in M." divide S by H, or:—

$$M = S \div H \qquad \textbf{Rule XI.—1.}$$

Example. $S = 12'$ and $H = 4\frac{1}{2}''$

$$\text{Then } M = 144'' \div 4\frac{1}{2}''$$
$$= \underline{\underline{32}}$$

The practical mind will see that with these flat angles it is immaterial whether they are measured by Right Angle or Centre Line Measure,* and in this volume we shall change from one to the other as suits the purpose. The term "Cot A" will be used for the Right Angle Measure, whilst the symbol "M" will be used for the Centre Line Measure.

Cot A and M will be taken at the same value, which in the example given above is 32.

For some purposes it is necessary to know the measure of the switch angle in degrees, minutes, and seconds, this measurement being referred to as "A."

To find the angle A: Divide H by S (in the same units), which will give the Tangent of the angle; then consulting Trigonometrical Tables, the angle A will be found opposite to a tangent of this value.

It may be noted, however, that A is practically proportional to the switch length; for instance, with $4\frac{1}{2}$ins. H, we may obtain A by dividing 21° 28′ 52″ by S in feet. The error with S = 6 feet is 12 seconds, and with S = 10 feet is 1 second.

*See Chapter XII.

It is not necessary to find the angle A to obtain its own ratios, for instance:—

$$\text{Cosec A } (\text{Cot A}) = S \div H \quad \text{Cot A} =$$

$$\text{and } \cos A = \frac{S}{\sqrt{(S^2 + H^2)}}$$

The angles of switches with various divergences, and also their Cotangents and Cosines will be found in Table 28.

The following table will form a guide to various heel divergences in use, but the reader should carefully ascertain the dimension in the particular switches he is concerned with.

TABLE XI.—1.

	Gauge.	Heel divergence (H).	Cot A, or rate of divergence (M).	G—H or W.
	ft. ins.	Ins.		ft. ins.
British Main Lines	4 8½	4½ 4¾	S × 2⅔ S × 2·526	4 4 4 3¾
Irish Ditto	5 3	4½	S × 2⅔	4 10½
Light and temporary railways using rails about 2″ wide	4 8½ ,,	4 3½	S × 3 S × 3·714	4 4½ 4 5
Light railways of narrow gauge	narrow	3½ 3	S × 3·714 S × 4	
India	5 6	4¼*	S × 2·824	5 1¾
Do.	3 3⅜ (metre)	3 11/16*	S × 3·254	2 11 1/16
Do.	2 6	2¾*	S × 4·364	2 3¼
Canada	4 8½	5½ & 5¾		
U.S.A.	4 8½	6	S × 2	4 2½

* W. H. Cole.

× 2·82353.

The Curved Switch.

Though the straight switch has survived as the type more acceptable in general practice, the curved switch possesses at least two advantages which should not be overlooked.

The first is, that with the same length of switch, an easier deflecting angle from the main is obtained, and at the same time a slightly flatter turnout curve for the same length of lead.

The second is, that a turnout to the outside of a curved main, may be laid to a better line.

If we consider such a turnout in a main of fairly sharp curve, which must be maintained throughout, we see that an originally straight switch is bent to conform to the main curve; then at the heel occurs a reverse to follow the turnout curve.

Now a switch which is curved originally, will on being placed close against a stock rail with the same amount of curvature in the opposite direction, become straight.

A comparison between the two sketches (Fig. 1) will show that a better line is formed for the turnout in the latter case.

On those railways where straight switches are the standard, the bend in the switch with contra turnouts is sometimes avoided by cutting 3ft. or so from the end next to the heel and also by packing out the heel as far as the fastenings will allow. If such devices are adopted, the necessary allowances must be made in calculations.

The disadvantages of the curved switch are of a constructional nature, grinding in addition to planing being required to form the curve, also the switch becomes rather thin at the middle.

In Chapter XV. it is shown that an equivalent length of straight switch can be found, which will have the same angle as that of a given curved switch, thus making the remaining calculations the same in both cases.

Length of Switches.

The question of what the switch length (S) should be in relation to either the Radius of the turnout curve (R),* the Crossing Angle Number (N), or the length of the Lead (L),† will now be dealt with.

* If the main is curved, R must be taken as the Equivalent Radius of the turnout, see Chapter XIII.

† For definitions of these terms see Chapter XIII.

S is a matter of choice, but it may be chosen with certain considerations in mind.

It is here recommended that S should be sufficient to ensure that a number of wheels attached to the same frame, will suffer no more deflection from a straight line, when passing over the switch angle than they do when traversing the turnout curve.

This condition will be fulfilled if the switch length is not less than * $\sqrt{R} \times \sqrt{H}$, or with $4\frac{1}{2}''$ H, * $\sqrt{R'} \times \cdot 612$.

When S is longer than half the length over the wheels, the length obtained by this formula more than attains the object, so from this and several other considerations, the following rules are recommended for the minimum length of switch (S) in feet, with the usual H's of $4\frac{1}{2}$ins. or thereabouts.

$S = N \times 1\frac{3}{4}$	**Rule XI.—2.**
$S = {}^{*}\sqrt{R'} \times \cdot 581$	„ **XI.—3.**
$S = L' \div 4 \cdot 08$	„ **XI.—4.**

These relations may be looked upon as ideal, and subject to modification in certain cases, for example, it is usual not to allow switches less than about 15ft. long in the main line, because with usual distances apart of wheels, a following wheel comes on to such a switch before a leading wheel gets past the heel chair, and thus "rocking" is minimised.

Again, with very long turnouts from the inside of a curved main, it is often necessary to be satisfied if S is not less than $N \times 1\frac{1}{2}$, whilst with short turnouts to the outside of a curve, it may be advisable to make $S = N \times 2$ or more.

In a turnout the dimensions N, $\sqrt{R}$,* and L bear definite relations to one another, as explained in Chapters XIII. and XXIV., and on this basis the three rules are devised to give the same result for S whichever dimension is used in finding it.

Table 29 shows the maximum R, N, and L which should be used with given lengths of switches, in accordance with the rules above.

*** If the main is curved, R must be taken as the Equivalent Radius of the turnout, see Chapter XIII.**

In Tables 46 and 47, will be found the relations between S and $\sqrt{R}$, and between S and L, corresponding to other given relations between S and N, for instance:—

$$\text{When } S = N \times 1\tfrac{1}{2}$$
$$\text{then } S = \sqrt{R} \times \cdot 493$$
$$\text{and } S = L \div 4\cdot 62$$

SWITCHES, PRACTICAL DETAIL DIMENSIONS.

The following tables of dimensions with which the reader should be familiar, are intended to be completed in accordance with the practice of the railway upon which he is engaged.

The reference letters thus (a), show where the dimensions are marked on Figs. XI.–2 and 3.

Fig. 2 shows an ordinary type of switch pivotting at the heel joint, and Fig. 3 shows a springing switch pivotting at the place marked "Virtual Heel."

TABLE XI.—2.

SWITCH LENGTHS FOR VARIOUS PURPOSES.

(Refer also to Table 29.)

..............................R'ly.	Switch length.
Minimum to be used in main line ...	....
Usual for crossover roads and trailing connections in main lines	....
Main line junctions, with curves entailing no reduction of speed for fast traffic	....
Usual for gathering lines in siding groups	....
For Catch points	....

FIG. XI-1.

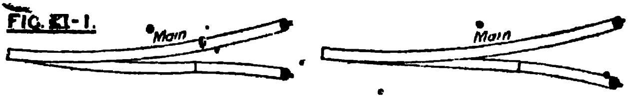


FIG. XI-2.

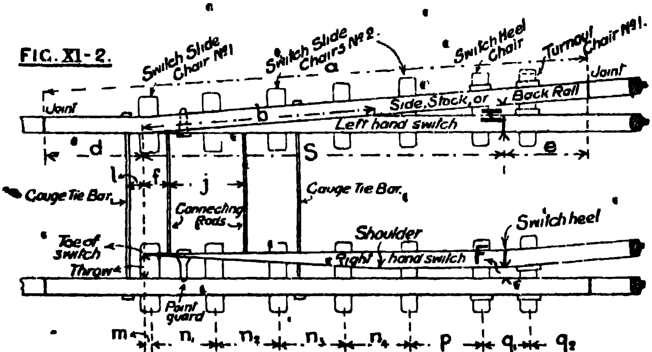


FIG. XI-3.

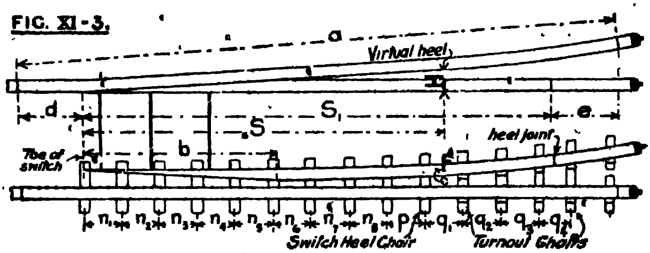


TABLE XI.—3.

SWITCHES, DETAIL DIMENSIONS.

Refer to Figs. XI.—2 & 3.	Standard Lengths of Switches (S. & S_1)						
Flangeway clearance at heel of ordinary switches (F) ...							
Heel divergence of ordinary switches (H)..................							
Length of stock* rails (a) ...							
Length of cutting or planing (b)							
Toe of stock rail to toe of switch (d)							
Heel of switch to heel of stock rail (e)							
Toe of switch to point driving rod (f)							
Toe of switch to 1st stretcher rod (g)							
1st to 2nd stretcher rod (j)...							
2nd to 3rd ditto (k)...							
Gauge tie bar to toe of switch (l)							
Toe to "virtual heel" of springing switches (S)							
Toe to centre of 1st slide chair (m)							

* Also named side rails or back rails.

TABLE XI.—4.

SWITCHES DETAIL DIMENSIONS—*continued.*

Refer to Figs. XI.—2 and 3, and Figs. III.—13 and 14.	Switch lengths						
Distances apart of chairs † (centre to centre):—							
Slide chairs .... n_1							
*n_2							
n_3							
n_4							
n_5							
n_6							
n_7							
n_8							
Last Slide and Heel chair. p							
Heel chair and Turnout chair ... . q_1							
Turnout chairs........... *q_2							
" " .... *q_3							
" " .... *q_4							
Divergences (H) of chairs:—							
At Heel chair							
" " " springing switch							
" Turnout chair No. 1 ...							
" " " " 2 ...							
" " " " 3 ...							
" " " " 4 ...							

* These distances may vary according to the Radius of the turnout curve.
† For particulars of switch chairs see Chapter III.

Throw of toe of switch at:—

Facing points } ....

Trailing points } ....

Catch points ... .

Points in sidings ....

Distance between facing and trailing points, with toes of standard stock rails butting:—............ .

NOTE.—This is an objectionable arrangement, forming a bad road from one turnout to the other

Distance between facing and trailing points to allow for locking bar of standard length (......), but with no cut rails:—...............

Other distances with regard to switches will be found in Chapter II (Signalling arrangements), and a list of Switch Chairs is included in Chapter III.

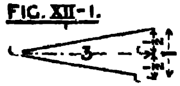
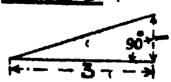
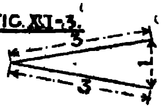
CHAPTER XII.

THE CROSSING.

The terms usually adopted with regard to the parts of the Acute or Vee Crossing and its appurtenances, are shown on Figs. XII.–8 and 9.

Methods of measuring the Angle.

Three of the practical methods of measuring the angle of a crossing, which are, or have been used in railway work, are explained by Figs. 1, 2, and 3, a 1 in 3 crossing being shown, as measured by each method.

	Remarks.
Centre Line Measure (C.L.M.) FIG. XII-1. 	Adopted on many British railways. Is the standard in U.S.A. Used by authors—Donaldson, 1871. W. P. Hales, 1889.
Right Angle Measure (R.A.M.) FIG. XII-2. 	Is the standard in India. Used by authors—W. H. Cole. L. H. Young (Proc. Inst. C.E.)
Isosceles Measure (Isos. M.) FIG. XII-3. 	Is the British Engineering Standards Association standard for Tramway Track (1919). Used by authors—F. W. Atkinson. J. Whitelaw.

In addition, there are the scientific measures of an angle

1. In Degrees, Minutes, and Seconds, which it is necessary to use in developing true mathematical formulæ. The principles of this measure are explained in Chapter V.

2. Circular or Radian Measure, which is useful in arriving at formulæ to give turnout leads when measured along the curve, as developed by E. H. Young, Inst.C.E. Proc., 1901-2. The principles of this measure are given in Table 1.

There is room for much discussion upon the relative merits of the three practical ways of measuring the angle.

A point in favour of R.A.M. is, that it is the method of measuring angles used in nearly all other branches of engineering.

Isosceles measure is possibly the easiest by which an angle can be set out.

C.L.M. has certain disadvantages, one being that it does not appear to be used in any other kind of work, another that it is not easy to set out the angle against a fixed line.

It will be seen, however, on going deeper into the subject, that it possesses a strong advantage, namely, that the conversion of strictly accurate mathematical formulæ into practical rules is fairly simple, and when these rules are further simplified for quickly calculating approximate dimensions, more accuracy is obtainable with C.L.M. than with other measures.

Though the method of measurement makes very little difference in flat angles, the difference is more marked in angles of small number, and it need hardly be said that it is most important that a uniform method of measurement should be standardised and thoroughly understood by all concerned.

The C.L.M. has been adopted on many British railways, but so far has not been standardised thereon, though the question has been brought forward by the Permanent Way Institution.

For reasons which will appear from the foregoing remarks, C.L.M. will be adopted in this volume; In Table 30, however, will be found the values of C.L.M. angles of a given Number, when measured by R.A. and Isosceles Measures as well as by Degrees and Minutes.

Conversion of Angle Measures.

The following table will show how one measure may be converted to another, many of the formulæ, however, being only of mathematical interest.

In each case N is the Number of the angle by the given measure, N_1 being the required number by another measure, and B the angle in degrees.

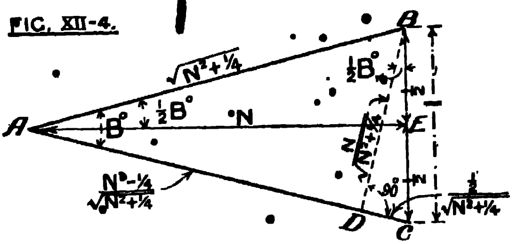
TABLE XII.—1.

To convert.	into	Rule	Rule No. XII.
Degs. & Mins. (B)	C. L. M.	$N_1 = \frac{1}{2}$ Cot of $\frac{1}{2}$ Angle B.	1
Do. Do.	R.A.M.	N_1 = Cot of Angle B.	2
Do. Do.	Isos. M.	$N_1 = \frac{1}{2}$ Cosec of $\frac{1}{2}$ Angle B.	3
C. L. M.	Degs. & Mins.	B = Twice the Angle whose Cot is 2 × N or otherwise Cot $\frac{1}{2}B = 2N$.	4
Do.	R. A. M.	$N_1 = N - \dfrac{1}{4N}$	5
Do.	Isos. M.	$N_1 = \dfrac{\sqrt{(2N)^2 + 1}}{2}$ or $N_1 = \sqrt{N^2 + \frac{1}{4}}$	6
R. A. M.	Degs. & Mins.	B = The Angle whose Cot is N.	7
Do.	C. L. M	$N_1 = \dfrac{N + \sqrt{N^2 + 1}}{2}$	8
Do.	Isos. M.	$N_1 = \frac{1}{2}\sqrt{(N + \sqrt{N^2 + 1})^2 + 1}$	9
Isosceles M	Degs. & Mins.	B = Twice the Angle whose Cosec is 2 × N or otherwise Cosec $\frac{1}{2}B = 2N$.	10
Do.	C. L. M.	$N_1 = \sqrt{4N^2 - 1} \div 2$	11
Do.	R. A. M.	$N_1 = \dfrac{N^2 - \frac{1}{2}}{\sqrt{N^2 - \frac{1}{4}}}$	12

Trigonometrical Ratios of Crossing Angles.

The number (N) of a C.L.M. angle bears fairly simple relations to the Trigonometrical Ratios of the angle (B).

FIG. XII-4.



In Fig. 4 the angle B is 1 in N by C.L.M.

$$AB = AC = \sqrt{N^2+(\tfrac{1}{2})^2} = \sqrt{N^2+\tfrac{1}{4}} \qquad \text{(Euclid I., 47).}$$

The triangles BCD and ABE are similar,

$$\text{Therefore } BD = \frac{BC \times AE}{AB} = \frac{N}{\sqrt{N^2+\frac{1}{4}}}$$

$$\text{And } DC = \frac{BC \times BE}{AB} = \frac{\frac{1}{2}}{\sqrt{N^2+\frac{1}{4}}}$$

$$\text{And } AD = AC - DC$$

$$= \sqrt{N^2+\tfrac{1}{4}} - \frac{\frac{1}{2}}{\sqrt{N^2+\frac{1}{4}}}$$

$$= \frac{N^2-\frac{1}{4}}{\sqrt{N^2+\frac{1}{4}}}$$

Then, taking the sides of the right-angled triangle ABD, we get :—

$$\text{Cot B} = \frac{N^2-\frac{1}{4}}{\sqrt{N^2+\frac{1}{4}}} \div \frac{N}{\sqrt{N^2+\frac{1}{4}}} = \frac{N^2-\frac{1}{4}}{N} \text{ or } N - \frac{1}{4N}$$

Rule XII.—13.

$$\text{Cosec B} = \sqrt{N^2+\tfrac{1}{4}} \div \frac{N}{\sqrt{N^2+\frac{1}{4}}} = \frac{N^2+\frac{1}{4}}{N} \text{ or } N + \frac{1}{4N}$$

Rule XII.—14.

$$\text{Cos B} = \frac{N^2-\frac{1}{4}}{\sqrt{N^2+\frac{1}{4}}} \div \sqrt{N^2+\tfrac{1}{4}} = \frac{N^2-\frac{1}{4}}{N^2+\frac{1}{4}} \text{ or } 1 - \frac{2}{(2N)^2+1}$$

Rule XII.—15.

The Tan, Sin, and Sec, if required, are reciprocals of the Cot, Cosec, and Cos respectively.

The above formulæ are useful in compiling a table of the Trigonometrical Ratios of C.M. Angles similar to Table 30, and also in converting trigonometrical formulæ into rules which do not involve the use of trigonometry.

The formulæ may be expressed in words, thus:—

For the Cot; deduct from N, the reciprocal* of 4 times N. **Rule XII.—13.**

(This rule also gives the R.A.M. of the angle).

For the Cosec; add to N, the reciprocal of 4 times N. **Rule XII.—14.**

For the Cosine; to the square of twice N add 1, multiply the reciprocal of the result by 2, and deduct the product from 1. **Rule XII.—15.**

By R.A.M. the ratios are:—

$$\text{Cot} = N \qquad \textbf{Rule XII.—16.}$$

$$\text{Cosec} = \sqrt{N^2 + 1} \qquad \textbf{Rule XII.—17.}$$

$$\text{Cos} = \frac{N}{\sqrt{N^2 + 1}} \qquad \textbf{Rule XII.—18.}$$

By Isosceles Measure, the ratios are:—

$$\text{Cot} = \frac{N^2 - \frac{1}{2}}{\sqrt{N^2 - \frac{1}{4}}} \qquad \textbf{Rule XII.—19.}$$

$$\text{Cosec} = \frac{N^2}{\sqrt{N^2 - \frac{1}{4}}} \qquad \textbf{Rule XII.—20.}$$

$$\text{Cos} = 1 - \frac{1}{2N^2} \qquad \textbf{Rule XII.—21.}$$

Sums and Differences of Angles.

In double turnout, and some other problems, it is necessary to find the sum or difference of two angles.

One way of obtaining an exact result is to convert the angles to degrees and then add or deduct them in this measure, finally reconverting the result into C.L. or other measure in use.

---

* The reciprocal is the result of dividing the number into 1; see Chapter IV.

It is possible, however, to obtain the sum of two angles directly from their numbers:—

Let S = the Number of the Sum of two angles whose Numbers are N and M; and let D = the Number of their Difference.

Then exact rules by C.L.M. are:—

$$S = \frac{(N \times M) - \frac{1}{4}}{M + N} \qquad \textbf{Rule XII.—22.}$$

$$D = \frac{(N \times M) + \frac{1}{4}}{M - N} \qquad \textbf{Rule XII.—23.}$$

and for approximate rules we may neglect the $\frac{1}{4}$, and say:—

S = Product of numbers ÷ their sum. **Rule XII.—24.**

and D = ,, ,, ÷ ,, difference. **Rule XII.—25.**

EXAMPLES:—

1. Required the sum of the angles 1 in 3 and 1 in 5.—

By exact rule, $S = \frac{(3 \times 5) - \frac{1}{4}}{5 + 3} = \frac{14\frac{3}{4}}{8} = 1 \text{ in } 1\frac{27}{32}$

By approx. ,, $S = \frac{3 \times 5}{5 + 3} = \frac{15}{8} = 1 \text{ in } 1\frac{7}{8} \text{ or } 1\frac{28}{32}$

2. Required the difference of the angles 1 in 4 and 1 in 6:—

By exact rule, $D = \frac{(4 \times 6) + \frac{1}{4}}{6 - 4} = \frac{24\frac{1}{4}}{2} = 1 \text{ in } 12\frac{1}{8}$

By approx ,, $D = \frac{4 \times 6}{6 - 4} = \frac{24}{2} = 1 \text{ in } 12$

3. What difference is made by increasing the number of an angle by 1.

In this case $M - N = 1$

$$\therefore \text{Approx. } D = \frac{N \times M}{1} = N \times M$$

Thus the difference between an angle of 1 in 8 and 1 in 9 is:— 1 in $(8 \times 9) = 1$ in 72.

On the same principle, the difference for $\frac{1}{4}$ is $4 \times N \times M$, for example, the difference between 1 in 8 and 1 in $8\frac{1}{4}$ is:—

$$4 \times 8 \times 8\tfrac{1}{4} = 1 \text{ in } 264$$

and the difference in the spread at the end of 12ft. "legs" will be $12' \div 264 = \cdot 55''$.

The approximate rules will serve with very little error for Isos.M. as well as C.L.M., but it is better to use exact rules for R.A.M., which are:—

$$S = \frac{(N \times M) - 1}{M + N}$$ **Rule XII.—26.**

$$D = \frac{(N \times M) + 1}{M - N}$$ **Rule XII.—27.**

Setting out of Crossing Angles.

The setting out of an angle of 1 in N by C.L.M. needs no explanation unless the position of one "leg" of an angle is fixed in a given line.

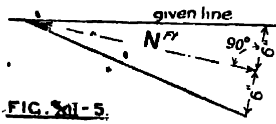


FIG. XII-5.

One method in this case, is to place a one foot rule at right angles to a tape at N feet thereon, the tape ring being held at the crossing point and one end of the rule touching the given line. The other end will then give the line of the crossing leg, as indicated in Fig 5.

Generally, however, both in drawing and setting out, a better process is to convert the angle to Isos M. (*i.e.*, use Table 30). Then with the tape ring held at the crossing point, sweep an arc with a radius—the Isos.M or $\sqrt{N^2 + \frac{1}{4}}$ (Rule XII. 6), and mark off a one foot chord.

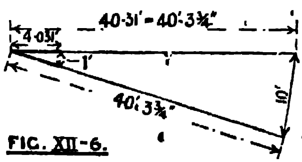


FIG. XII-6.

Greater accuracy will be obtained by first multiplying the Isos.M. and the one foot by some number. Fig. 6 shows the line of a 1 in 4 crossing produced by this method, the multiplier being 10.

The measurement of the Angles of existing Crossings.

When taking particulars for relaying, etc., care must be taken not to measure the spread of a crossing where it is affected by the legs having been bent to a curve.

When using an ordinary pocket rule it is recommended that the distance between the points where the gauge lines are 1in. and 7ins. apart should be measured. This distance in feet multiplied by 2 will give the number.

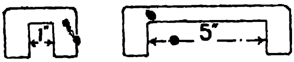


FIG. XII-7.

A simple way of measuring angles is to use two thin metal templates as shown in Fig. 7.

Place these where they fit on the crossing, and the distance between them in inches, divided by 4, will give the angle.

Crossings on the Curve.

The crossing angle as obtained by calculation is the angle between two straight lines, and where curves extend through the crossing legs, these straight lines are the tangents to the curves at the fine point of the crossing.

It will be evident that when the crossing is put into the road, its legs will need bending to conform to the curve, but that the length of the crossing which is held by the splice, cannot be bent.

To help in conforming the legs to the curve the use of a slightly wider angle than the calculated angle is recommended when one crossing leg has to bend away from the other, and a narrower angle in the reverse case

EXAMPLE.—If one leg of a crossing which is calculated at 1 in 7 has to be bent outwards to a 7 chains curve, we should use a 1 in $6\frac{3}{4}$, which at 4ft. from the point would have $\frac{1}{4}''$ more spread, and thus nearly give the offset due to the curve at 4ft., after which point the leg may be curved as required.

Each case will need special consideration bearing in mind that wide angles mean short splices which may not appreciably interfere with the curve, whilst a long splice may mean a flat curve entailing very little alteration to the spread.

DETAIL DIMENSIONS OF VEE CROSSINGS.

The notes under this heading mainly consist of a list of the particulars with which the reader should be familiar in the practice of the railway he is engaged upon, spaces being left to receive the figures which apply to the said practice.

Dimensions and notes are given in some cases as a guide to the usual practice.

The reference letters thus (a) indicate the dimensions as marked on Figs. XII. 8 and 9, which show typical crossings of wide and narrow angles respectively.

FIG. XII-8.

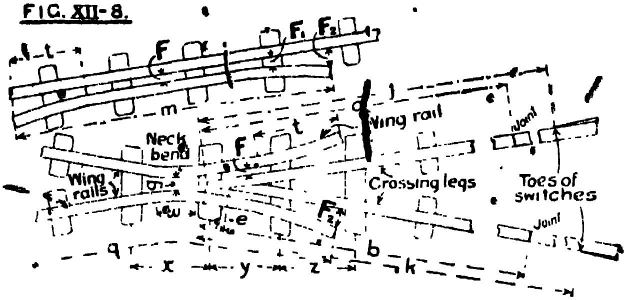
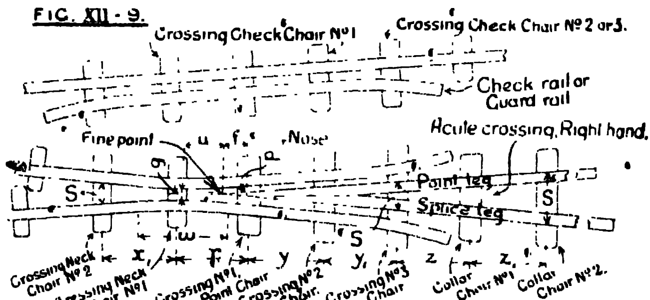


FIG. XII-9.



Flattest Angle of Vee Crossing.

	Trailing.	Facing.	Exceptional.	Very exceptional.
Usual	14	12	16	20
............Rly.	......	......	......	......

Angles of Crossings available from Stock.

.............Rly..........................

Width of Crossing Nose (p).

Usual $\frac{1}{2}''$ and $\frac{3}{4}''$.

...............Rly. ....

Nose of Crossing to Fine Point, or "Fine Point Distance" (f).

This equals width of nose multiplied by Number of crossing (N). See Table 38. **Rule XII.—28.**

Length of Rails out of which Crossing Legs are made.

............Rly.	Point rail (a).	Splice rail (b).
	....	....
	....	....

It may be noted that long crossing legs tend to smooth running, but may take up valuable space in siding groups, etc.

Again, when the heels of the legs are opposite, we may obtain better timbering, but the change from crossing timbers to sleepers cannot be made as early as when the heels are not opposite.

A point worthy of consideration is that it may be advisable for certain crossings to be made with legs of a suitable length to suit the timbering in an ordinary crossover road with a 6ft. space.

Splice Distance, *i.e.*, Crossing Nose to Splice End (e).

..............Rly. 1 in...... 1 in...... 1 in...... 1 in......

....

Distance from Crossing Nose to Toe of a following set of Switches with no cutting of Rails.

............................Rly.

Switches.	Along point rail (j).	Along splice rail (k).			
		1 in	1 in	1 in	1 in
		....	....	....	....

Suitable lengths to shorten legs by, when switches following immediately after a crossing must be nearer to crossing nose than a full leg will allow.

..........................Rly.

Cut from point rail.			Cut from splice rail.		
1 in	1 in	1 in	1 in	1 in	1 in

NOTE.—As it becomes necessary to situate the toe of a switch very near to the crossing nose, the crossing leg must be specially made so as to form a stock rail for the switch.

Length of Check or Guard Rails (m).

.................Rly. ....

Position of Check Rails.

Usually end of check rail is opposite to end of wing, which usually makes centre of check about opposite neck of wing.

Length of Wing Rails (q).

.................Rly. ....

When wings are bent on site their length is made to suit the particular case in hand.

Position of Wing Rail.

The free end is just clear of the collar chair making the length from crossing nose to free end of wing.

.............Rly.	1 in......	1 in......	1 in......	1 in......
	....	....	....	....

Flangeway clearance between Check Rail or Parallel Wing Rail* and Running Rail.

.........Rly.	Normal (F).	At End Check Chairs (F_1).
	....	....

At Check Rail End (F_2).

....

In the case of modern main line track, the following four dimensions cannot well be inserted in this table. The bends are formed by compound curves, to decrease the effect of the knocking of wheel flanges against them. Such bends are usually formed in the workshops. A paper by J. T. Lee in the "Per. Way Inst. Journal" for 1910, Vol. III., describes how "double bending" may be done by the platelayer.

* This refers to the type of wing in which the portion held in chairs is parallel to the running rail.

Length of Check and "Parallel"* Wing Rail Bends (t).

..............Rly.

Centre of Neck Bend of Wing to Fine Point of Crossing (u).

With ordinary single circular bends this equals:—

Wing rail clearance (F) multiplied by crossing number (N) or:—

$u = F \times N$. **Rule XII.—29.**

EXAMPLE:— $N = 8$ and $F = 1\frac{3}{4}''$

Then $u = 1\frac{3}{4}'' \times 8 = 1'\ 2''$.

Gap between Wing Rails at Neck (g).

..............Rly. ....

Length of Wing Rail Neck Bend (w).

With ordinary single circular bends this equals:—

g minus F, multiplied by 4 times No. of crossing, or:—

$w = (g - F) \times 4 \times N$ **Rule XII.—30.**

EXAMPLE:— $N = 8$, $F = 1\frac{3}{4}''$, and $g = 2\frac{1}{2}''$.

Then $w = (2\frac{1}{2}'' - 1\frac{3}{4}'') \times 4 \times 8$

$= 2'\ 0''$.

Position of Crossing Blocks.

..............Rly.	Neck block.	Vee block.
	....	....

TABLE XII. 2.

CROSSING CHAIRS. SPREAD, POSITION, ETC.

(Refer to Chapter III. and Figs. III.-6 to 10.)

Spread (S) = Width between gauge lines of rails at the centre of the chair in question.

Distance of a chair (centre) from fine point of crossing will be S multiplied by No. of crossing.†

Distance (y and z Fig. XII.-8 and 9), between two chairs will be the Difference of their Spreads multiplied by No. of crossing.

The Table will serve as a list of chairs available.

* This refers to the type of wing in which the portion held in chairs is parallel to the running rail.

† This will not apply to a neck chair when it is situated on the neck bend of the wing rails.

Chairs for Angles of :— (......lbs. section).	SPREADS (S).								
	Neck Chairs.			Crossing chairs.			Collar chairs.		
	3	2	1	1	2	3	1	2	3

Minimum Lengths of Checking.

Where one crossing is near to another or near to switches, as in double turnouts, scissors cross-overs, and slip roads, it may be necessary to determine minimum distances of ends of check and wing rails from nose of crossing, or in other words, the minimum length of checking that is necessary for safety. Such cases will of course only occur where the usual checking cannot be obtained.

It is difficult to lay down rules, but in the case where the crossings are not flatter than say 1 in 8, the following remarks may be of service.

Since the purpose of the check rail is to keep the wheel flanges moving in a straight line over the crossing gap, we must in the case of a wheel approaching the crossing in a facing direction, at least get the portion of the wheel flange which is below rail level, that is the length A on Fig. XII.-10, into the straight portion of the check rail, just before the opposite flange reaches the bend of the wing.

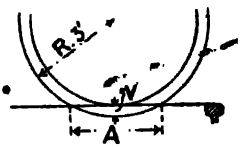


FIG. XII-10.

To fix upon the length A, we will take a wheel 3ft. radius (R) as being near to the largest in ordinary use; with a very far-worn tread causing the flange to be $1\frac{1}{2}$ins. deep (V) then:—

$$A = \sqrt{8 \times V \times R} \qquad \text{(Rule IX.—2.)}$$

$$= \sqrt{8 \times 1\tfrac{1}{2} \times 36} \qquad \text{(all in inches.)}$$

$$= \sqrt{432}$$

$$= 21'' = \underline{\underline{1'\ 9''}}$$

The straight portion of the check should at least continue to guide the wheel flange until the length A is clear of the crossing nose. The straight portion of the check thus becomes:—

$$A + \tfrac{1}{2}w + u + f + A$$

which with a 1 in 8 crossing and $g = 2\frac{1}{4}''$, $F = 1\frac{3}{4}''$, and $p = \frac{3}{4}''$ becomes:—

$$1'\ 9'' + 1'\ 0'' + 1'\ 2'' + 6'' + 1'\ 9''$$

$$= \underline{\underline{6'\ 2''}}$$

To this we must add the length of two bent ends, which we will make shorter than usual, say 1ft. 5ins. each, and thus obtain the minimum length of check rail as 9ft., 5ft. 4ins. of which is in front of fine nose, and 3ft. 8ins. behind it.

The wing should naturally extend as far as the check, and its length behind $\frac{3}{4}$in. nose according to the above is 3ft. 2ins. for all angles, which shows that the above must be taken with certain reservations.

Further notes applying to this matter are included in the chapter on scissors cross-overs, and double turnouts.

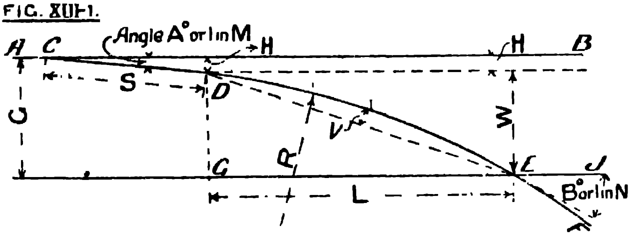
CHAPTER XIII.

SWITCHES AND CROSSINGS, THEIR CURVES AND DISTANCES. GENERAL RULES

It is now proposed to describe a series of leading rules which may be applied in the solution of points and crossing problems. These rules are collected in Tables 25, 26, and 27, and will afterwards be referred to as the "General Rules."

Their mathematical proofs will be found in Chapter XIV. and examples of their use are given in the treatment of the various track arrangements.

FIG. XIII-1.



The arrangement to which the General Rules apply is where a "turnout" is concerned and may be described as follows:—

A straight main rail *A B* (Fig. XIII.–1) is given, against which a straight switch line *C D* of given length (S) makes a given angle (A°).

A curve *D E F* springs from the switch and is tangential thereto at its heel *D*. This curve crosses a line *G E J* parallel to the line *A B*: in a turnout this is the other rail forming the main track.

At the first perusal of this chapter and the next, the reader is advised to have in mind the idea of the simple turnout, with its crossing of the other rail of the track alone; then to go over the matter again, extending his view to the case of the turnout curve when it crosses some other track parallel to the line *A B*. He will then see that the rules are applicable in many varied cases. It will only be the cross width from the switch heel to the rail on which the crossing lies, which will differ in the various cases.

Amongst the problems which the rules given will serve to solve, are those of the double line junction and the cross-over road where one of the lead curves continues through both crossings. Later it will be shown that the solutions of certain double turnout and scissors cross-over problems follow from the same leading rules.

The following symbols will be used to represent the dimensions concerned:—

A° = The switch angle in degrees and minutes.

S = The length of the switch.

H = The divergence at switch heel.

M = The number of the switch angle by centre line measure. M = S ÷ H.

*G = Gauge of the track.

W = Width from switch heel to a rail on which a crossing in question lies; in the case of a turnout W = G – H.

B° = Angle of crossing in degrees and minutes, and is the angle the main line makes with a tangent to the turnout curve at the crossing point *E*.

N = Number of the crossing angle by centre line measure.

L = Lead or distance from the switch heel to the crossing line point, measured along the main line when the rail is straight.

V = Versed sine of the lead curve on a chord extending from switch heel to crossing point.

R = Radius of the outer rail in the turnout curve.

Whatever the problem given, we must always know beforehand:—

(i.) The Gauge.

(ii.) The Switch Heel Divergence.

(iii.) The Switch Length.

The choice of a suitable length of switch may however, be left to the calculator, in which case the rules given in Chapter XI. should be used.

When dealing with anything beyond the simple turnout, the width W must also be known.

In addition to the above known dimensions, there must be given one of the following further dimensions, before a

* In this volume the inside edges of the rails are used in all calculations, for reasons given in Chapter XXV. The General Rules will apply when the outside edges of the rails are used, if the symbol "G" is taken as representing the gauge plus the width of two rail heads, or with usual standards, 5 feet 2 inches.

problem can be set for solution, namely, either:—

The Crossing Angle,
The Radius, or
The Lead.

One of these being given, the problem will be to find the other two.

The Rules collected in Tables Nos. 25, 26, and 27 can be applied to solve any problem which may thus be given.

Rules are also given to find W when, as in some special problems, this is the required dimension.

The process employed in making up the Tables has been as follows:—

(1) The Rules Nos. 1a to 8a (Table 25) are deduced from first principles by Geometrical Theories, supplemented by Trigonometry and Algebra.

The Formulæ obtained are accurate and easily used by those possessing an elementary knowledge of Trigonometry, including the use of Tables of Cosines, Cotangents, etc. The Tables of Switch and Crossing Angles (28 and 30) will prove of great assistance in the calculations.

The formulæ are naturally the same as those arrived at by other writers when working upon purely mathematical lines.

(2) The second procedure is to convert the first set of formulæ into rules enabling the Crossing and Switch Angles to be taken as they are measured in practice; that is, not in Degrees, but by their Rates of Divergence, which for reasons explained in Chapter XII. are taken as the Centre Line Measures of the Angles.

Rules 1b to 7b (Table 26) are thus obtained. These may be employed by using ordinary arithmetic.

(3) In the last-mentioned Rules, notice is now taken of terms which may be neglected without unduly affecting accuracy. Then, after simplification, Rules 1c, 1d, 2c, 2d, 3c, 5c, and 7c (Table 27), are obtained.

Much labour in calculation is saved by the use of these Rules, especially if Tables of Squares and Reciprocals are employed, and they will give results sufficiently accurate for many purposes.

(4) It will be noticed that Rules numbered 4a, 4b, 4c, 6a, 6b, and 6c are not given. Such rules would be complex, and are not necessary, because after finding the Crossing Angle by Rule 3 or 5, the Lead or the Radius, as the case may be, can be found by Rule 1 or 2.

In case it is needed, however, to find L or R without previously finding N, Approximate Rules 4d and 6d are given, which are arrived at by a different principle to the other rules. This principle may be named the "Offset principle." Rules 8c to 11d are also based on this principle. Rule 4e is derived from Rules 1c and 7c. Rule 6e arises from Rule 4e.

(5) The General Rules arrived at may next be adapted to the standard dimensions obtaining on any particular railway. In this work, this is not carried out except in the operations of the next process.

(6) When the particular standards namely, Gauge, Switch Heel Divergence, and Spaces between Tracks are fixed, a system may be developed which may be named the "*System of Factors.*"

The details of this system will be dealt with in Chapter XXIV; for the present, it may be stated that it consists in finding simple numbers by which we may multiply or divide a known dimension in a turnout or other arrangement, to give the unknown dimensions.

Explanations of Tables 25, 26, and 27.

As an example, suppose we are given the Radius of a Turnout, and are asked to find the Number of the Crossing, and the Lead.

Looking down the first column of each of the three Tables for the dimension given, namely, the Radius, opposite to it are seen the Rules Nos. 3a, 3b, and 3c, by any one of which can be found the Angle of the Crossing.

If conversant with Trigonometrical Tables and having them at hand, we may use Rule No. 3a (Table 25.). If not, we may use Rule No. 3b (Table 26), noticing that the result is obtained directly in Centre Line Measure.

If it is wished to reduce the labour of calculation to a minimum, we may use Rule No. 3c (Table 27), the results not being quite accurate, but near enough for usual purposes.

To find the Lead after having found the Crossing Angle, Rule No. 1a, 1b, or 1c may be employed, according to which type of rule we are using.

An approximate Rule, No. 4d, is included, which will give the Lead without previously finding the Crossing Angle. It is, however, usually better to first find the Crossing Angle, even if it is not required.

The above will serve to explain the use of the Tables as far as Rules 7a, 7b, and 7c. These Rules are to serve in the solution of such a problem as the following:—

A junction line of 30 chains Radius is to spring from a double straight main line; what must the Space between the mains be, in order that the diamond crossing may not exceed 1 in 8?

In this problem R and N are given, and W is required.

From either Rule 7a, 7b, or 7c, W may be found, and the Space can be arrived at by deducting G – H.

Rules 8c, 9c, and 10c may be used at the conclusion of the other calculations, to find the Versed sine (V) of a turnout curve on a chord stretched between switch heel and crossing point.

This V may be used in the method of "quartering" to set out the curve at as many points as is wished, which method may be considered preferable to any other, such as offsets from the main rail.

Application of the Rules to the case where the Main Line is Curved.

So far in the explanation the Main Line has been assumed to be straight; the case where the Main Line is curved will now be considered.

It is possible to evolve exact mathematical rules to solve all the problems in this case.

The earliest work in which the authors have met with a mathematical treatment of the turnout from a curved main, based upon a switch which would be straight if placed against a straight main rail, is that of W. Prior Hales, published in 1889.

The problems have also been treated by Preston and Barnard, Proc. Inst. C.E., 1901, supplemented by Young, Proc. Inst. C.E., 1902; the latter applying the rules so that the Leads may be measured along the curved rails instead of along an imaginary line at right angles to the line across the heels of the switches.

Whilst these investigations are interesting, and exact in treatment, the working rules are by no means simple.

By making certain approximations, there is, however, a way of simplifying the whole question which will now be explained.

The Principle of Equivalent Radius.

It is an important fact, and a very fortunate one, that all Turnouts which have the same Angle of Crossing and the same Switch, will have practically the same Length of Lead, whether the main line is straight or curved to any radius. The Lead referred to is measured in all cases along a tangent to the main line at Heel of Switch, to a point opposite the Crossing.

Further, if the main is curved, there will be a definite relation between the Radius of the Main, the Radius of the Turnout from the Curve, and the Radius of the Turnout if it had been from a Straight Main.

This last is known shortly as the Equivalent Radius, or more fully as the Equivalent Radius for the particular Crossing, and its Lead when on a Straight Main.

The approximate rules as to Equivalent Radii were recognised by Donaldson and other early writers, also by the authors referred to above, and it is believed that the convenient term, "Equivalent Radius," is due to F. W. Atkinson.

To show the relations between the three Radii mentioned, let

R_m = Rad of Main.

R_t = Rad. of Turnout.

R_e = Equiv. Radius from the Straight.

Then:—

Rules XIII.—1 to 9.

Turnouts of Contra Flexure.	Turnouts of Similar Flexure where Turnout Radius is less than Main Line Radius.	Turnouts of Similar Flexure where Turnout Radius is greater than Main Line Radius.
$R_e = \frac{R_m \times R_t}{R_m + R_t}$ (1)	$R_e = \frac{R_m \times R_t}{R_m - R_t}$ (4)	$R_e = \frac{R_m \times R_t}{R_t - R_m}$ (7)
$R_t = \frac{R_m \times R_e}{R_m - R_e}$ (2)	$R_t = \frac{R_m \times R_e}{R_m + R_e}$ (5)	$R_t = \frac{R_m \times R_e}{R_e - R_m}$ (8)
$R_m = \frac{R_t \times R_e}{R_t - R_e}$ (3)	$R_m = \frac{R_t \times R_e}{R_e - R_t}$ (6)	$R_m = \frac{R_t \times R_e}{R_t + R_e}$ (9)

These very simple rules mean that the product of two known Radii divided by their sum or difference, as the case may be, enable the required radius in any problem to be found.

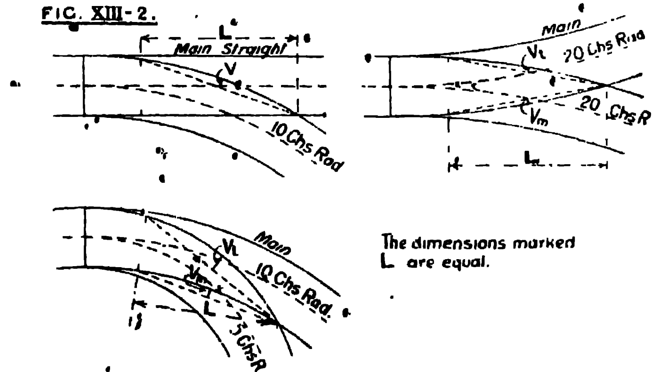
Examples will be given in the general problems treated later and also in Chapter XXIV. The radii should all be taken as that of the rail on which the crossing lies.

It is interesting to notice that if, without altering its length, we transform a turnout from the straight into one of contra flexure, with radii of main and turnout equal, we shall double the turnout radius.

Again, if we transform the turnout into one of similar flexure, curving the main to the same radius as the turnout was from the straight, we shall halve the radius of the turnout.

For example, all the turnouts in Fig. XIII.–2 will have the same crossing, lead, and equivalent radius.

FIG. XIII-2.



The last remarks are sufficient to show the foundation for the following approximate, but serviceable, rules for the :—

Versed Sines in Turnouts from the Curve.

Rule XIII.–10. The V Sine on the Lead Curve for a Turnout of Contra-flexure. = V.S. on same Lead Curve from the Straight, less V.S. of Main Line on same length of chord.

Rule XIII.–11. The V Sine on the Lead Curve for a Turnout of Similar flexure. = V.S. on same Lead Curve from the Straight, plus V.S. of Main Line on same length of chord.

Or referring to Fig. XIII.-2:—

For Turnout of Contra-flexure, $V_t = V - V_m$ **Rule XIII.—10.**

For Turnout of Similar-flexure, $V_t = V + V_m$ **Rule XIII.—11.**

To adapt the Equiv. Radius Formulæ for use with Tables of Reciprocals, divide the numerator and denominator by the numerator, thus:—

$$R_e = \frac{R_m \times R_t}{R_m + R_t}$$

$$R_e = \frac{1}{\frac{1}{R_t} + \frac{1}{R_m}}$$

This in words means:—

To find R_e, add the Reciprocals of the known Radii, and take the Reciprocal of the sum.

Lead measured along the Curve.

Rules have been developed by some writers for obtaining the distance from switch heel to fine point of crossing measured along the curved rails of a Turnout.

It does not appear to be necessary to deal in detail with this question, as occasion for the use of the rules seldom arises in practice. The reader who is interested, may refer to the paper by E. H. Young, in the Proc. Inst. C.E., Vol. 150, 1902, and to the articles in the "Railway Engineer," commencing October, 1920, by C. H. Lobban.

The following approximate rules will give within ¼in. the increase in Length of Lead measured on the curved turnout rail, over the Lead measured on the straight main rail.

Gauge.	Increase.	**(Rule XIII.—12.)**
5ft. 6ins.	21ins. ÷	No. of Crossing.
5ft. 3ins.	20ins. ÷	,, ,,
4ft. 8½ins.	18ins. ÷	,, ,,
3ft. 6ins.	13½ins. ÷	,, ,,
Metre	12½ins. ÷	,, ,,

CHAPTER XIV.

SWITCHES AND CROSSINGS. PROOFS OF THE GENERAL RULES.

A complete list of the rules is contained in Tables 25, 26, and 27. A description of the general problem and of the general application of the rules in practice is given in Chapter XIII., where a list of the symbols used appears.*

In the following proofs the General Curve Rules of Chapter VI. will be brought into service, and in this respect the similarity of the portion $ODKE$ of Fig XIV.-1 to Figs. VI.-1 and 2. should be noticed

Trigonometrical Rules (a).

Rule No. 1a.

To prove that $L = W \times \text{Cot}\left(\frac{A + B}{2}\right)$

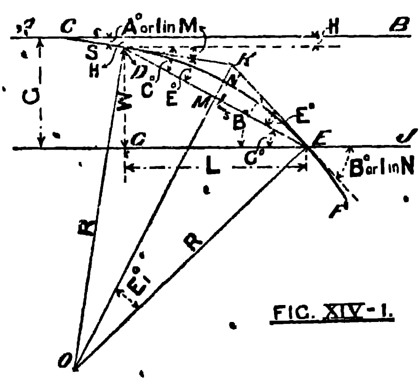


FIG. XIV-1.

In Fig. 1:—

$$\text{Angles } C + E = \text{Angle } B$$
$$\text{,, } C - E = \text{,, } A$$

Adding, $\quad 2C = A + B$

* In this Chapter an additional symbol L_S is used to denote the Lead measured on the skew.

Dividing by 2, $C = \frac{A+B}{2}$ (I.)

Subtracting, $2E = B - A$

Dividing by 2, $E = \frac{B-A}{2}$ (II.)

Now, $\frac{L}{W} = \text{Cot } C$

or, $L = W \times \text{Cot } C$

∴ from I., $L = W \times \text{Cot } \frac{A+B}{2}$ **(Rule 1a.)**

Transposing, $\text{Cot } \frac{A+B}{2} = \frac{L}{W}$ **(Rule 5a.)**

Rule No. 2a.

Angle $E_1 = E = \frac{B-A}{2}$ From II.

$$\text{Sine } E_1 = \frac{ME}{OE}$$

$$\therefore OE = \frac{ME}{\text{Sine } E}$$

that is, from II., $R = \frac{\frac{1}{2} \times L_s}{\text{Sine } \frac{B-A}{2}}$ (III.)

Again,

$$\text{Sine } \frac{A+B}{2} = \text{Sine } C = \frac{W}{L_s}$$

$$\therefore L_s = \frac{W}{\text{Sine } \frac{A+B}{2}}$$

Substituting this in III.,

$$R = \frac{\frac{1}{2} \times \frac{W}{\text{Sine } \frac{A+B}{2}}}{\text{Sine } \frac{B-A}{2}}$$

$$R = \frac{W}{2 \times \text{Sine } \frac{B+A}{2} \times \text{Sine } \frac{B-A}{2}}$$

∴ By Trig., "Products into differences" $R = \dfrac{W}{\text{Cos A} - \text{Cos B}}$ **(Rule 2a.)**

Transposing, $\text{Cos B} = \text{Cos A} - \dfrac{W}{R}$ **(Rule 3a.)**

Also „ $W = R \times (\text{Cos A} - \text{Cos B})$ **(Rule 7a.)**

Rule No. 8a.

$$\text{Angle } E_1 = \frac{B - A}{2}$$

$$\text{Cos } E_1 = \frac{MO}{OE}$$

$$\therefore \quad MO = OE \times \text{Cos } E_1$$

that is, $$MO = R \times \text{Cos} \frac{B - A}{2}$$

but $$MO = R - V$$

$$\therefore \quad R - V = R \times \text{Cos} \frac{B - A}{2}$$

$$\therefore \quad V = R - \left(R \times \text{Cos} \frac{B - A}{2}\right)$$

$$V = R \times \left(1 - \text{Cos} \frac{B - A}{2}\right)$$

that is, $$V = R \times \text{Vers} \frac{B - A}{2}$$ **(Rule 8a.)**

Centre Line Measure Rules (b, c, and d).

In the chapter upon the Crossing it is shown that when the angles A° and B° are 1 in M and 1 in N by C.L. Measure:—

$\text{Cot} \dfrac{A}{2} = 2M$ $\quad\quad$ $\text{Cot} \dfrac{B}{2} = 2N$ $\quad\quad$ (Rule XII.—4).

$\text{Cos A} = \dfrac{4M^2 - 1}{4M^2 + 1}$ $\quad\quad$ $\text{Cos B} = \dfrac{4N^2 - 1}{4N^2 + 1}$ $\quad\quad$ („ „ 15).

Rule No. 1b.

By Rule 1a, $$L = W \times \text{Cot} \frac{A + B}{2}$$

Then by the rules of Trig. for compound angles, $$L = W \times \frac{\left(\text{Cot} \dfrac{A}{2} \times \text{Cot} \dfrac{B}{2}\right) - 1}{\text{Cot} \dfrac{A}{2} + \text{Cot} \dfrac{B}{2}}$$

Substituting the values above for Cot $\frac{A}{2}$ and Cot $\frac{B}{2}$ $\Big\}$ $L = W \times \frac{(2M \times 2N) - 1}{2M + 2N}$

$$L = W \times \frac{4MN - 1}{2(M+N)}$$

Rearranging, $$L = 2W \times \frac{MN - \frac{1}{4}}{M + N} \quad \textbf{(Rule 1b.)}$$

By neglecting the $\frac{1}{4}$ we get **Rule 1c**, and then by dividing top and bottom by 2 M N we get **Rule 1d.**

Rule No. 2b.

By Rule 2a, $$R = \frac{W}{\text{Cos } A - \text{Cos } B}$$

Substituting the values above for Cos A & Cos B $\Big\}$ $R = \dfrac{W}{\dfrac{4M^2 - 1}{4M^2 + 1} - \dfrac{4N^2 - 1}{4N^2 + 1}}$

$$R = \frac{W}{\dfrac{16M^2N^2 - 4N^2 + 4M^2 - 1 - 16N^2M^2 + 4M^2 - 4N^2 + 1}{16M^2N^2 + 4M^2 + 4N^2 + 1}}$$

$$R = W \times \frac{16M^2N^2 + 4M^2 + 4N^2 + 1}{8M^2 - 8N^2}$$

$$R = W \times \frac{4M^2N^2 + M^2 + N^2 + \frac{1}{4}}{2(M^2 - N^2)} \quad \textbf{(Rule 2b.)}$$

By neglecting the $+ M^2 + N^2 + \frac{1}{4}$ we get **Rule 2c,** and then by dividing top and bottom by $2\,M^2\,N^2$ we get **Rule 2d.**

Rule No. 3b.

By Rule 2b, $R = \dfrac{W \times (4M^2N^2 + N^2 + M^2 + \frac{1}{4})}{2M^2 - 2N^2}$

Cross multiplying,

$$2RM^2 - 2RN^2 = 4WM^2N^2 + WN^2 + WM^2 + \frac{W}{4}$$

Transposing, $$2RM^2 - WM^2 - \frac{W}{4} = 2RN^2 + 4WM^2N^2 + WN^2$$

$$M^2(2R - W) - \frac{W}{4} = N^2(2R + 4WM^2 + W)$$

$$\frac{M^2(2R - W) - \dfrac{W}{4}}{2R + 4WM^2 + W} = N^2$$

$$N = \sqrt{\frac{M^2(2R - W) - \frac{W}{4}}{2R + 4WM^2 + W}} \quad \textbf{Rule 3b.}$$

By neglecting the $\frac{V}{4}$ and the $+W$, we may place the M^2 outside of the root sign and cancel, which will give **Rule 3c.**

Rule No. 5b.

By Rule 1b, $$L = W \times \frac{2MN - \frac{1}{2}}{M + N}$$

$$\therefore \quad \frac{2MN - \frac{1}{2}}{M + N} = \frac{L}{W}$$

Cross Multiplying, $$2MNW - \frac{W}{2} = LM + LN$$

Transposing, $$2MNW - LN = LM + \frac{W}{2}$$

$$N = \frac{LM + \frac{W}{2}}{2MW - L} \qquad \textbf{Rule 5b}$$

Rules Nos. 7b and 7c.

These are arrived at by transposing in Rules 2b and 2c.

Rule No. 4e.

Take Rule 3c and instead of W substitute its value by Rule 7c. By cancelling, Rule 4e is obtained.

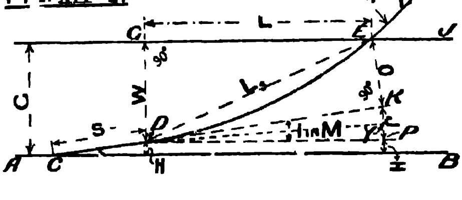
Rule No. 6e.

This is arrived at by transposing Rule 4e.

The remaining rules are based upon what may be termed the "Offset Principle" in which certain approximations are made at the outset. These do not seriously affect the accuracy of the results.

It may be noted that not only the rules which follow, but approximate rules to cover practically all the problems may be evolved upon this principle without the use of Trigonometry.

FIG. XIV-2.



Rule No. 4d.

Referring to Fig. 2 we see that DE, which will be named the "Skew Lead" or L_s, is a chord for the arc DE of the turnout curve whose radius is R.

Also when the switch line CD is prolonged to K, O is the offset for the same arc, therefore:—

$$L_s^2 = 2 \times O \times R \quad \text{By Curve Rule 17.}$$

$$\text{But} \quad L_s^2 = L^2 + W^2 \quad \text{Euclid I. 47.}$$

$$\therefore \quad L^2 + W^2 = 2 \times O \times R$$

$$\text{or} \quad L^2 = (2 \times O \times R) - W^2 \qquad \text{(I.)}$$

Now draw DP parallel to CB. We then have KDP as the switch angle (1 in M) with DY as its centre line. Then because EKP is nearly parallel to W,

$$O = W - J \quad \text{(approx.)}$$

$$\text{And because,} \quad DY = L, \quad (\;\text{,,}\;)$$

$$\text{and} \quad J = \frac{L}{M} \quad (\;\text{,,}\;)$$

$$\text{Therefore,} \quad O = W - \frac{L}{M}$$

Substituting this in (I.),

$$L^2 = \left\{2 \times \left(W - \frac{L}{M}\right) \times R\right\} - W^2$$

$$\text{Simplifying,} \quad L^2 = 2WR - \frac{2RL}{M} - W^2$$

$$\text{Transposing,} \quad L^2 + \frac{2R}{M}L = 2WR - W^2 \qquad \text{(II.)}$$

This is a quadratic equation, so, add to both sides the square of half the co-efficient of the unknown, *i.e.*:—

$$\text{Add, } \left(\frac{R}{M}\right)^2, \quad L^2 + \frac{2R}{M}L + \left(\frac{R}{M}\right)^2 = 2WR - W^2 + \frac{R^2}{M^2}$$

$$\text{Take } \sqrt{} \quad L + \frac{R}{M} = \sqrt{2WR - W^2 + \frac{R^2}{M^2}}$$

$$\text{Transpose,} \quad L = \sqrt{W(2R - W) + \frac{R^2}{M^2}} - \frac{R}{M}$$

Rule 4d.

Rule No. 6d.

From (II.) above,

$$L^2+W^2 = 2WR - \frac{2R}{M}L$$

$$= R(2W - \frac{2L}{M})$$

$$\frac{L^2+W^2}{2W-\frac{2L}{M}} = R$$

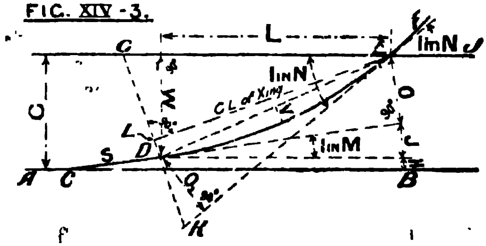
or,
$$R = \frac{M(L^2+W^2)}{2(WM-L)}$$ **Rule 6d.**

Rule No. 11c.

This is obtained by solving the quadratic equation II. above, for W.

Rules Nos. 9c and 10c.

FIG. XIV-3.



Referring to Fig. 3 draw EL the centre line of the crossing and GK at right angles thereto, then,

$$GK = \frac{EL}{N}$$

but, $EL = L$ approx. so,

$$GK = \frac{L}{N}$$

Again,

$$DK = GK - GD$$

but $DK = O_1$ and $GD = W$ (approx.)

So,
$$O_1 = \frac{L}{N} - W$$

but $V = \frac{1}{4}O$ or $\frac{1}{4}O_1$

So, $$V = \frac{\frac{L}{N} - W}{4} \qquad \textbf{Rule 9c.}$$

Under Rule 4d above, it was shown that

$$O = W - \frac{L}{M} \text{ (approx.)}$$

So, $$V = \frac{W - \frac{L}{M}}{4} \qquad \textbf{Rule 10c.}$$

In spite of the approximations, Rules 9c and 10c will give the V in an ordinary turnout with no more than $\frac{1}{16}$in. error.

Rule No. 8c.

Multiplying Rule 9c by 4N, $\quad 4VN = L - WN$

" " 10c " 4M, $\quad 4VM = WM - L$

Adding $\quad 4V(M+N) = W(M-N)$

$$4V = \frac{W(M-N)}{M+N}$$

$$V = \frac{W}{4} \times \frac{M-N}{M+N}.$$

Rule 8c.

This rule is not intended for use in practice, but is given to show that V is not dependent upon the Radius or the Lead.

Rule No. 11d.

In Fig. 2, $\quad W = O + J \quad$ (approx.)

and, $\quad O = \frac{L^2}{2R} \quad$ (")

" $\quad J = \frac{L}{M}$

$$\therefore \quad W = \frac{L^2}{2R} + \frac{L}{M} \qquad \textbf{Rule 11d.}$$

CHAPTER XV.

THE TURNOUT

Types of Turnout.

The various types of turnout in use will first be described, the remarks applying to the case where the main is straight.

Type (1), Fig. XIII.-1.—Having a straight switch, and a crossing either straight or curved and with a true circular curve between tangential to the switch at its heel, and to the crossing at its point

This is the usual method in British practice, and it is the case to which practically all the Rules and Tables for points and crossings given in this volume apply.

Type (1a), Fig. XI. 3 —Similar to Type (1), but where only the portion of the switch between its toe and a point where the wheel flange clearance is obtained, is straight: the remainder of the switch, towards the heel, forming part of the turnout curve. This point may be regarded as the virtual heel of the switch.

The General Rules and Tables may be used with this type, the switch length being taken as the distance from the toe to the virtual heel, at which point the divergence "*H*" must be measured.

Type (1b), Fig. XV.-3.—Similar to Type (1), but with the straight line of the switch continued to a point beyond its heel.

This type will occur when it is necessary to use a certain angle of crossing with a lead which is longer than its correct lead for a turnout of Type (1).

A compound curve would do away with the straight behind the heel, but would introduce a curve of less radius than the simple curve.

Type (1c), Fig. XV.-4.—Similar to Type (1), but with a straight length in front of the crossing point, the turnout curve being tangential to this straight.

This is the usual case in American practice, where the crossings are of the "built-up" type, and therefore the portion of the wing rails in front of the crossing, and the legs of the crossing, cannot well be bent to a curve. It has the advantage of giving wheels a straight run over the crossing gap.

This type will occur when it is necessary to use a certain angle of crossing with a lead which is shorter than its correct lead for a turnout of Type (1). The curve, however, will be of less radius than if the crossing had its full lead.

In the "System of Factors" are included factors applying to the turnout of Type (1c) when D is 6ins or 9ins. (Table 18)

Type (1d).—Similar to Type (1), but where the curve changes in radius before reaching the crossing point.

This can hardly be looked upon as a general type. It may occur through some necessity of the general outline of the work. It is a case in which drawing or setting out is the only practical means of solution.

Type (2).—Having a straight switch and a crossing either straight or curved, with a curve between which is originally calculated to be tangential to the main line, and afterwards adjusted to be tangential to the switch. The existence of this type is due to a method of calculation which should be discarded. Its supporters argue that the calculations are simpler, and that it enables a branch line junction to be laid in exactly to the centre line pegs of the new branch railway, which are set out tangential to the main line.

With regard to the first contention, there may be certain cases in which the assumption of curves tangential to the main line will solve problems with sufficient accuracy and more easily than by the correct method.

Generally, however, the calculations based upon the curve being tangential to the switch are little more difficult than the others, and they have the advantage of being in keeping with practice.

Many of the existing tables of points and crossings are based on the "tangential to main line" method. In order that the reader may examine such tables, and also note the

difference in the dimensions by the two methods, the following table is given:—

TABLE XV.-1.

Gauge 4' 8½" H = 4½"								
Crossing No.	Switch.	By tangent to main method.			By tangent to switch method.			
		Lead.		Radius.	Lead.		Radius.	
		ft.	ins.	ft.	ft.	ins.	ft.	
4	6	26	11½	153	27	7	150	
4	9	,,		,,	29	8	145	
8	12	54	0½	606	55	5	594	
8	15	,,		,,	57	9	580	
12	18	81	1	1357	83	2	1335	
12	21	,,		,,	85	7	1311	

Under the System of Factors are included in Tables 46 and 47, factors which form a ready means of showing the discrepancies between the two methods of calculation. Table 46 also shows that the "tangent to main" method is only correct when the switch length in feet, is 1⅓ times the crossing number.

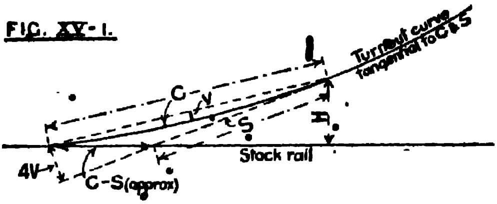
With regard to the second contention mentioned above, the cases of branch line junctions are few in number, compared with the points and crossings for other purposes. Even then, with the Tangent to Switch method a curve may be followed which is parallel to the curve pegged out, and only at a slight distance, usually about 2¼ins. from it.

Type (3).—Having a switch curved to a fixed radius and with the lead curve tangential to the switch at its heel.

Very few of the main British railways adopt this kind of switch, and then only for shorter switches.

The calculations may be resolved into the same as those for a straight switch, thus:—

FIG. XV-1.



Referring to Fig. XV.–1, where C is the length of curved switch, V its Versed sine, H its heel divergence, and S the length of equivalent straight switch.

By the principle of similar triangles (Euclid VI.–2):—

$$\frac{S}{H} = \frac{C - S}{4V}$$

$$4VS = HC - HS$$

$$4VS + HS = HC$$

$$S(4V + H) = HC$$

$$S = \frac{H \times C}{4V + H} \quad \textbf{Rule XV.—1.}$$

It will be noticed that the approximations made will not affect the accuracy of the results in all practical cases, and that the equivalent straight switch will be a tangent to any turnout curve which is tangential to the curved switch.

EXAMPLE.—What length of straight switch must be taken in using the General Rules or Table 31, when the switch is a curved one 16ft. long with ½in. Versed sine and 4½in. divergence?

With all dimensions in inches; by Rule XV.–1:—

$$S = \frac{4\frac{1}{2} \times 192}{(4 \times \frac{1}{2}) + 4\frac{1}{2}} = \frac{864}{6\frac{1}{2}} = \frac{1728}{13} = 133'' = \underline{\underline{11'\ 1''}}$$

Therefore in Table 31, an average between the figures given for 10ft. and 12ft. switches may be taken.

Remarks upon the advantages of curved switches will be found in the chapter upon the Switch.

Type (4).—A straight switch with a transition curve tangential thereto, and terminating either before, at, or beyond the crossing, where it joins a circular curve.

Though there are advantages in this type for fast running junctions, there seems no occasion for its general adoption. Its introduction may in some cases decrease the radius of the circular curve following it. Again, there are sufficient difficulties in obtaining accuracy with plain circular curves through points and crossings, without introducing further complications unless the advantage gained is important.

PRACTICAL DETAILS OF TURNOUTS.

Fig. XV.–2 shows a usual English method of timbering; crossing timbers about 14ins. by 7ins. being used throughout.

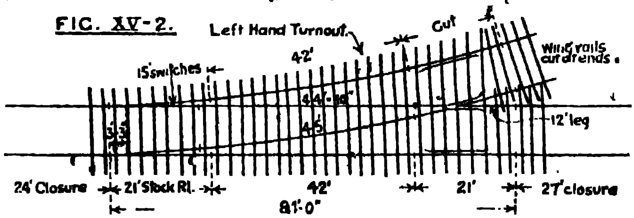
On some of the Scotch and Irish railways, sleepers are used for the whole or part of the turnout.

The rail joints should be "grouped" so that four joints lie in one timber space. There is a better opportunity of doing this with a minimum of rail cutting when the wing rails are bent and cut on the site than when they are delivered ready made.

The joints are naturally "squared" at the crossing heels.

Model turnouts may be designed with most Company's materials, the lead lengths being arranged to avoid cutting of rails, especially in the main line. Fig. XV.–2 shows an example of a turnout in L. & Y. Ry. materials, which may be inserted in a main line laid with 45ft. rails without cutting. It is assumed that the position of the points may be altered slightly to make them fall at a multiple of 3ft. plus 3ins., from an existing joint, and yet not give closure rails which would be too short. A series of articles by C. H Lobban, commencing October, 1920, in "The Railway Engineer," deal with this subject.

FIG. XV-2.



The "hand" of a turnout is decided by standing at the toe of the switches and facing the crossing. If the turnout road diverges to the left as in an ordinary trailing crossover road, it is named a "Left-hand Turnout."

CALCULATIONS FOR THE TURNOUT.

The following examples will show how the General C.L.M. Rules (Approx.), explained in Chapter XIII and collected in Table 27, may be applied in solving the various problems.

The examples apply to a Gauge (G) of 4′ 8½″ = 4·708′, and to a Switch Heel Divergence (H) of 4½″ = ⅜′ or ·375′, and therefore to a Width (W) of 4′ 4″, which = 4⅓′ or 4·333′.

TURNOUTS FROM STRAIGHT MAINS.

Prob. 1. Given the Number (N) of the Crossing Angle.: to find the remaining dimensions.

EXAMPLE.—Crossing Number (N). 1 in 8.

1st.—Choose the Switch length (S).

$$S = N \times 1\tfrac{3}{4} \qquad \text{(See Rule XI.-2 and Table 29.)}$$
$$= 8 \times 1\tfrac{3}{4}$$
$$= \underline{\underline{14'}}$$

Assuming that 15′ is the nearest stock length, we will decide upon:—

$$S = \underline{\underline{15'}}.$$

2nd.— $$M = S \div H \qquad \text{(Rule XI.-1.)}$$
$$= 15 \div \tfrac{3}{8}$$
$$= 15 \times \tfrac{8}{3}$$
$$= \underline{\underline{40}}$$

3rd.—To find the Lead (L).

$$L = 2 \times W \times \frac{M \times N}{M + N} \qquad \text{(Rule 1c.)}$$
$$= 2 \times 4\tfrac{1}{3} \times \frac{40 \times 8}{40 + 8}$$
$$= 8\tfrac{2}{3} \times \frac{320}{48}$$
$$= \frac{\overset{13}{\cancel{26}}}{3} \times \frac{40}{\underset{3}{\cancel{6}}}$$
$$= \frac{520}{9}$$
$$= \underline{\underline{57{\cdot}78\text{ft.}}} = \underline{\underline{57\text{ft. } 9\tfrac{1}{3}\text{ins.}}}$$

Note.—The increase in Lead if measured along the outer rail of the turnout curve will be:—

$$18'' \div N \qquad \text{(Rule XIII.-12.)}$$
$$= 18'' \div 8$$
$$= \underline{\underline{2\tfrac{1}{4}''}}.$$

4th.—To find the Radius (R).

$$R = \frac{L \times N \times M}{M - N} \qquad \text{(Rule 6e.)}$$
$$= \frac{57\cdot78 \times \cancel{8}^{1} \times \cancel{40}^{10}}{\cancel{32}_{4}}$$
$$= \underline{\underline{578'}}.$$

The Radius obtained by the more accurate Rules Nos. 2a and 2b will be 580′.

5th.—To find the Versed sine (V).

$$V = \frac{(L \div N) - W}{4} \qquad \text{(Rule 9c.)}$$
$$= \frac{(57'\ 9'' \div 8) - 4'\ 4''}{4}$$
$$= \frac{7'\ 2\tfrac{5}{8}'' - 4'\ 4''}{4}$$
$$= \frac{2'\ 10\tfrac{5}{8}''}{4}$$
$$= \underline{\underline{8\tfrac{5}{8}''}} \text{ (say)}$$

Prob. 2. Given the Radius (R) of the Turnout: to find the remaining dimensions.

EXAMPLE.—Radius 10 chains = 660′.

1st.—Choose the Switch length (S).

$$S = \sqrt{R} \times \cdot581 \qquad \text{(See Rule XI.-3 and Table 29.)}$$
$$= 25\cdot69 \times \cdot581$$
$$= \underline{\underline{14\cdot92'}}$$

So we may decide upon :—

$$S = \underline{\underline{15'}}.$$

2nd.— $M = \underline{\underline{40}}$ as Problem 1.

3rd.—To find the Crossing Number (N).

$$N = M \times \sqrt{\frac{R - \frac{1}{2}W}{R + (2 \times W \times M^2)}} \quad \text{(Rule 3c.)}$$

$$= 40 \times \sqrt{\frac{660 - 2{\cdot}17}{660 + (2 \times 1{\cdot}333 \times 1600)}}$$

$$= 40 \times \sqrt{\frac{657{\cdot}83}{660 + 13866{\cdot}67}}$$

$$= 40 \times \sqrt{\frac{657{\cdot}83}{14526{\cdot}67}}$$

$$= 40 \times \sqrt{\cdot 04528}$$

$$= 40 \times \cdot 2128$$

$$= \underline{\underline{8{\cdot}512}}$$

4th.—To find the Lead (L).

Now that N is known, apply Rule 1c as in the 3rd step of Problem 1.

L will be 60·81′ or 60′ 10″.

Prob. 3. Given the Lead (L) : to find the remaining dimensions.

EXAMPLE.—Lead (L) = 40′.

1st.—Choose the Switch length (S).

$S = L \div 4{\cdot}08$ (See Rule XI.-4 and Table 29.)

$= 40 \div 4{\cdot}08$

$= \underline{\underline{9{\cdot}80'}}$

In this case a 10′ switch would not be unsuitable, but a longer switch than this is to be preferred, so we will decide upon :— $S = \underline{\underline{12'}}$.

2nd.—$M = 12' \div \frac{3'}{8} = \underline{\underline{32}}$. (Rule XI.-1.)

3rd.—To find the Crossing Number (N).

$$N = \frac{L \times M}{(2 \times W \times M) - L} \quad \text{(Rule 5c.)}$$

$$= \frac{40 \times 32}{(2 \times 4 \cdot 333 \times 32) - 40}$$

$$= \frac{1280}{277 \cdot 33 - 40}$$

$$= \frac{1280}{237 \cdot 33}$$

$$N = \underline{\underline{5 \cdot 39}} \quad \text{say } 5\tfrac{1}{4}$$

4th.—To find the Radius (R).

Now that N is known, apply Rule 6c as in the 4th step of Problem 1.

R will be $\underline{\underline{259'}}$.

TURNOUTS FROM CURVED MAINS.

In these Problems, the principle of Equivalent Radius explained in Chapter XIII may be applied. As regards Turnouts of Contra-flexure, reference should be made to the remarks upon the Curved Switch in Chapter XI.

Prob. 4. Given the Number (N) of the Crossing, and the Radius (R_m) of a Curved Main; Turnout of similar flexure; to find the remaining dimensions, including Radius (R_t) of Turnout.

EXAMPLE.—N, 1 in 8. R_m 2302′.

The 1st to 4th steps will be as in Problem 1, the 4th step will give the Equivalent Radius (R_e), namely 578′, then:—

5th.—

$$R_t = \frac{R_m \times R_e}{R_m + R_e} \quad \text{(Rule XIII.—5.)}$$

$$= \frac{2302 \times 578}{2302 + 578}$$

$$= \frac{1330556}{2880}$$

$$= \underline{\underline{462'}} = \underline{\underline{7 \text{ Chs.}}}$$

6th.—Equivalent Versed sine (V_e) can be found as in Problem 1.

$$V_e = \underline{\underline{8\tfrac{3}{8}''}}$$

7th.—For Versed sine on Main (V_m):—

$$\begin{aligned} V_m &= L^2 \div 8\,R_m \quad \text{(from Rule IX.—1.)} \\ &= 57 \cdot 8^2 \div (8 \times 2302) \\ &= 3340 \cdot 84 \div 18416 \\ &= \underline{\underline{\cdot 182'}} = \underline{\underline{2\tfrac{1}{4}''}} \end{aligned}$$

8th.—For Versed sine on Turnout (V_t).

$$\begin{aligned} V_t &= V_e + V_m \qquad \text{(Rule XIII.—11.)} \\ &= 8\tfrac{3}{8}'' + 2\tfrac{1}{4}'' \\ &= \underline{\underline{10\tfrac{5}{8}''}} \end{aligned}$$

Prob. 5. As Prob. 4, but with Turnout of Contra Flexure.

EXAMPLE.—N, 1 in 8. R_m 2302'.

The procedure until the Equivalent Radius (R_e) of 578' is found will be as in Problem 1, then:—

5th.

$$\begin{aligned} R_t &= \frac{R_m \times R_e}{R_m - R_e} \qquad \text{(Rule XIII.—2.)} \\ &= \frac{2302 \times 578}{2302 - 578} \\ &= \frac{1330556}{1724} \\ &= \underline{\underline{772'}} \end{aligned}$$

6th.—As in Problem 4:—

$$V_e = \underline{\underline{8\tfrac{3}{8}''}}$$

7th.—As in Problem 4:—

$$V_m = \underline{\underline{2\tfrac{1}{4}''}}$$

8th.—For Versed sine on Turnout (V_t).

$$\begin{aligned} V_t &= V_e - V_m \qquad \text{(Rule XIII.—10.)} \\ &= 8\tfrac{3}{8}'' - 2\tfrac{1}{4}'' \\ &= \underline{\underline{6\tfrac{1}{8}''}} \end{aligned}$$

Prob. 6. Given the minimum allowable Radius of Turnout (R_t) and maximum allowable Number of Crossing (N): to find the sharpest Main Line Curve into which a Turnout of similar flexure may be laid.

EXAMPLE.—R_t, 7 chains = 462′. N, 1 in 14.

1st.—Choose switch length, say:—

$$S = 21'$$

$$M = 21 \times \tfrac{8}{3} = 56$$

2nd. $$R_e = W \times \frac{(4 \times M^2 \times N^2) + M^2 + N^2}{2 \times (M^2 - N^2)} \qquad \text{(Rule 2b.)}$$

$$= 4\tfrac{1}{3} \times \frac{(4 \times 56^2 \times 14^2) + 56^2 + 14^2}{2 \times (56^2 - 14^2)}$$

$$= 4\tfrac{1}{3} \times \frac{(4 \times 3136 \times 196) + 3136 + 196}{2 \times (3136 - 196)}$$

$$= 4\tfrac{1}{3} \times \frac{2458624 + 3136 + 196}{2 \times 2940}$$

$$= 4\tfrac{1}{3} \times \frac{2461956}{5880}$$

$$= 4\tfrac{1}{3} \times 418 \cdot 7$$

$$= \underline{1814'} = 27 \text{ chs. } 32 \text{ ft.}$$

This would be the Radius of the turnout from a straight main.

3rd.—To find Radius of Main when R_t is 462′.

$$R_m = \frac{R_e \times R_t}{R_e - R_t} \qquad \text{(Rule XIII.—6.)}$$

$$= \frac{1814 \times 462}{1814 - 462}$$

$$= \underline{\underline{620'}} \text{ or } 9 \text{ chs., } 26 \text{ ft.}$$

Prob. 7. Given a Turnout of Contra Flexure, and with the maximum Number of Crossing: to find the Radii of Main and Turnout when they are equal. This gives the easiest possible curves in both directions.

EXAMPLE.—N = 1 in 14. Switch 21′.

1st.—Find R_e as in Problem 6,

$$R_e = \underline{\underline{1814'}}$$

2nd. $R_e = \frac{R_m \times R_t}{R_m + R_t}$, (Rule XIII.—1.)

But $R_m = R_t =$ say R, so,

$$R_e = \frac{R^2}{2R} = \frac{R}{2}$$

that is, $\underline{\underline{R = 2 R_e}}$

$\therefore R = 2 \times 1814'$

$= \underline{\underline{3628'}} = 54$ chains, 64 feet.

Prob. 8. Given the Radii of Main (R_m) and of Turnout (R_t). Similar Flexure.

EXAMPLE.—$R_m = 23\frac{1}{3}$ chains $= 1540'$. $R_t = 7$ chains $= 462'$.

1st.—Find Equivalent Radius (R_e).

$$R_e = \frac{R_m \times R_t}{R_m - R_t} \qquad \text{(Rule XIII. —4.)}$$

$$= \frac{23\frac{1}{3} \times 7}{23\frac{1}{3} - 7} \quad \text{(in chains)}$$

$$= \frac{163\frac{1}{3}}{16\frac{1}{3}}$$

$= \underline{\underline{10 \text{ chains}}} = 660'$

To find N and L, proceed as in Problem 2, working with the above Radius (10 chains).

To find the Versed sine proceed as in Problem 4.

Prob. 9. Given the Radii of Main (R_m), and of Turnout (R_t). Contra Flexure.

EXAMPLE.—$R_m = 30$ chains. $R_t = 15$ chains.

1st. $$R_e = \frac{R_m \times R_t}{R_m + R_t} \qquad \text{(Rule XIII.—1.)}$$

$$= \frac{30 \times 15}{30 + 15}$$

$= \underline{\underline{10 \text{ chains.}}}$

N and L may then be found as in Problem 2, and V as in Problem 5.

Prob. 9a. Required the Lead and Crossing when the Main and Turnout are both of the minimum allowable Radius. This will give the least Lead and Crossing Number which should be used.

This of course refers to a turnout of contra-flexure.

EXAMPLE.—$R_m = R_t$ = 7 chains. Switch 12′, *i.e.*, M = 32.

1st. $$R_e = \frac{7 \times 7}{7 + 7} = \underline{\underline{3\tfrac{1}{2} \text{ chains}}} = \underline{\underline{231'}}$$

Working as in Problem 2 :—

$$N = \underline{\underline{5{\cdot}06}}$$

$$L = \underline{\underline{37'\ 11''}}$$

Prob. 10. Given the Lead (L) and the Radius (R_m) of the Main.

EXAMPLE.—Lead (L) = 40′. R_m = 878′.

The 1st to 4th steps will be as in Problem 3, the 4th step will give the Equivalent Radius (R_e) = 259′, then :—

If the Turnout is of Similar flexure :—

$$R_t = \frac{R_m \times R_e}{R_m + R_e} \qquad \text{(Rule XIII.—5.)}$$

$$= \frac{878 \times 259}{1137}$$

$$= \underline{\underline{200 \text{ ft.}}}$$

If the Turnout is of Contra-flexure :—

$$R_t = \frac{R_m \times R_e}{R_m - R_e} \qquad \text{(Rule XIII.—2.)}$$

$$= \frac{878 \times 259}{878 - 259}$$

$$= \underline{\underline{367 \text{ ft.}}}$$

CALCULATIONS FOR TURNOUTS OF SPECIAL TYPES.

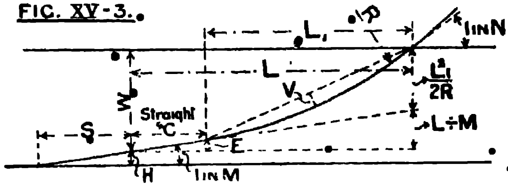
In the following problems, the special rules Nos. XV.–2 to 7, for turnouts of Types 1b and 1c, have been obtained on an offset principle. These rules are not exact but are near enough for the purpose.

The General Rules 4e and 6e, which are independent of W, and also Rules 9c and 10c, apply to Turnout Types 1b and 1c, as well as to the ordinary type.

Turnout Type 1b. (Fig. XV—3.)

In this type the Lead (L) is longer than is normal for the Crossing Number (N). As previously mentioned, it is best to continue the straight line of the switch a certain distance (C) beyond the heel, the remaining portion (L_1) of the Lead being curved.

FIG. XV-3.



Prob. 11. Turnout Type 1b. Given the Crossing Number (N), and the full Lead (L): to find the Length of Curve (L_1), the Radius (R), and the Vers sine (V).

EXAMPLE.—Crossing (N) 8, Lead (L) 64', Switch 15' (M = 40).

NOTE.—The Normal Lead and Radius would be 57' 9" and 580' (from Table 31).

$$L_1 = \frac{2 \times N\{(W \times M) - L\}}{M - N} \quad \textbf{(Rule XV.—2.)}$$

$$= \frac{2 \times 8 \times \{(4\tfrac{1}{3} \times 40) - 64\}}{40 - 8}$$

$$= \frac{16 \times (173\tfrac{1}{3} - 64)}{32}$$

$$= \frac{16 \times 109\tfrac{1}{3}}{32}$$

$$= \underline{\underline{54\tfrac{2}{3}'}}$$

$$C = L - L_1$$

$$= 64 - 54\tfrac{2}{3} = \underline{\underline{9\tfrac{1}{3}'}}$$

$$R = \frac{L_1 \times N \times M}{M - N} \qquad \text{(Rule 6e.)}$$

$$= \frac{54{\cdot}67 \times 8 \times 40}{32}$$

$$= \underline{\underline{547'}}$$

$$V = \frac{W - (L \div M)}{4} \qquad \text{(Rule 10c.)}$$

$$= \frac{4'\ 4'' - (64' \div 40)}{4}$$

$$= \frac{4'\ 4'' - 1'\ 7\tfrac{1}{4}''}{4}$$

$$= \frac{2'\ 8\tfrac{3}{4}''}{4}$$

$$= \underline{\underline{8\tfrac{1}{4}''}}$$

Prob. 12. Turnout Type 1b, with a given Crossing Number (N): to find to what length the Lead may be lengthened without decreasing the Radius below a certain limit.

This means, given N and R, to find L.

EXAMPLE.—$N = 8$. $M = 40$. Radius limit 433′.

1st.
$$L_1 = \frac{R \times (M - N)}{N \times M} \qquad \text{(Rule 4e.)}$$

$$= \frac{433 \times 32}{8 \times 40}$$

$$= \underline{\underline{43\tfrac{1}{3}'}}$$

2nd.
$$E = W + \frac{L_1^2}{2R} - \frac{L_1}{N} \qquad \textbf{(Rule XV.—3.)}$$

$$= 4 \cdot 33 + \frac{43 \cdot 3^2}{2 \times 433} - \frac{43 \cdot 33}{8}$$

$$= 4 \cdot 33 + 2 \cdot 16 - 5 \cdot 42$$

$$= \underline{\underline{1 \cdot 07'}}$$

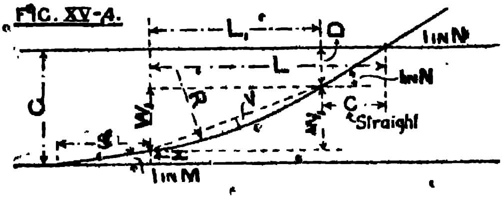
3rd.
$$L = L_1 + (E \times M) \qquad \textbf{(Rule XV.—4.)}$$

$$= 43 \cdot 33 + 1 \cdot 07 \times 40$$

$$= \underline{\underline{86 \cdot 13'}}$$

Turnout Type 1c. (Fig. XV.—4.)

FIG. XV-4.



In this type the Lead (L) is shorter than is normal for the Crossing Number (N), being used. The straight length C will be in front of the crossing, the curved portion (L_1) of the Lead extending from the straight to the switch heel.

Prob. 13. Turnout Type 1c. Given the Lead (L), and Crossing (N): to find L_1, R, and the Vers sine (V) on L_1.

EXAMPLE.—Lead (L) 52′, Crossing (N) 8, Switch 15′ (M = 40).

1st.— $$L_1 = \frac{2 \times M\{L - (N \times W)\}}{M - N} \qquad \textbf{Rule XV.—5.}$$

$$= \frac{2 \times 40 \times \{52 - (8 \times 4\tfrac{1}{3})\}}{40 - 8}$$

$$= \frac{80 \times (52 - 34\tfrac{2}{3})}{32}$$

$$= \frac{80 \times 17\tfrac{1}{3}}{32}$$

$$= \underline{\underline{43\tfrac{1}{3}'}}$$

2nd.— $$R = \frac{L_1 \times N \times M}{M - N} \qquad \text{(Rule 6e.)}$$

$$= \frac{43{\cdot}33 \times 8 \times 40}{32}$$

$$= \underline{\underline{433'}}$$

3rd.— $$V = \frac{(L \div N) - W}{4} \qquad \text{(Rule 9c.)}$$

$$= \frac{(52' \div 8) - 4'\ 4''}{4}$$

$$= \frac{6'\ 6'' - 4'\ 4''}{4}$$

$$= 2'\ 2'' \div 4$$

$$= \underline{\underline{6\tfrac{1}{2}''}}$$

Prob. 14. Turnout Type 1c. Given the Crossing Number (N), the length of straight (C), or, alternatively, the gap (D) between Main and Turnout, where straight joins curve: to find Lead (L) and Radius (R).

1st.— If C is given,

$$D = C \div N$$

or if D is given,

$$C = D \times N$$

2nd.— $$W_1 = G - H - D$$

3rd.— $$L_1 = 2 \times W_1 \times \frac{M \times N}{M + N} \qquad \text{(Rule 1c.)}$$

4th.— $$L = L_1 + C$$

5th.— $$R = \frac{L_1 \times N \times M}{M - N} \qquad \text{(Rule 6c.)}$$

Prob. 15. Turnout Type 1c. Given the Radius and the Distance (D): to find N and L.

EXAMPLE.—Radius = 433'. D = 1' 1", M = 40.

1st.—$W_1 = G - H - D = 4'\ 8\tfrac{1}{2}'' - 4\tfrac{1}{2}'' - 1'\ 1'' = \underline{\underline{3'\ 3''}}$

2nd.—$$N = M \times \sqrt{\frac{R - \frac{1}{2} W_1}{R + 2\, W_1\, M^2}} \qquad \text{(Rule 3c.)}$$

$$= 40 \times \sqrt{\frac{433 - 1{\cdot}6}{433 + (2 \times 3\tfrac{1}{4} \times 1600)}}$$

$$= 40 \times \sqrt{\frac{431{\cdot}4}{433 + 10400}}$$

$$= 40 \times \frac{\sqrt{431{\cdot}4}}{\sqrt{10833}}$$

$$= 40 \times \frac{20{\cdot}77}{104{\cdot}1}$$

$$= \frac{830{\cdot}80}{104{\cdot}1}$$

$$= 7{\cdot}98 \text{ say } \underline{\underline{8}}$$

3rd.—$L_1 = \dfrac{R \times (M - N)}{N \times M}$ (Rule 4e.)

$$= \frac{433 \times \overset{4}{\cancel{32}}}{\underset{1}{\cancel{8}} \times \underset{10}{\cancel{40}}}$$

$$= \underline{\underline{43\tfrac{1}{3}\text{ ft.}}}$$

4th.— $C = D \times N$
$= 1'\ 1'' \times 8$
$= \underline{\underline{8'\ 8''}}$.

5th.— $L = L_1 + C$
$= 43'\ 4'' + 8'\ 8''$
$= \underline{\underline{52}}$.

Prob. 16. Turnout Type 1c, with a given Crossing Number (N): to find to what length the Lead may be shortened without decreasing the Radius below a certain limit.

This means, given N and R, to find L.

EXAMPLE.—N = 8, M = 40, Radius limit 433′. (The Normal Lead and Radius would be 57′ 9″ and 580′, from Table 31).

1st.— $L_1 = \dfrac{R \times (M - N)}{N \times M}$ (Rule 4e.)

$$= \frac{433 \times 32}{8 \times 40}$$

$$= \underline{\underline{43\tfrac{1}{3}'}}$$

2nd.— $D = W - \dfrac{L_1^2}{2R} - \dfrac{L_1}{M}$ **(Rule XV.—6.)**

$$= 4{\cdot}33 - \frac{43{\cdot}33^2}{2 \times 433} - \frac{43{\cdot}33}{40}$$

$= 4{\cdot}33 - 2{\cdot}17 - 1{\cdot}08$
$= 1{\cdot}08 = \underline{\underline{1'\ 1''}}$

3rd.— $L = L_1 + (D \times N)$ **(Rule XV.—7.)**
$= 43{\cdot}33 + (1.08 \times 8)$
$= 43{\cdot}33 + 8{\cdot}64$
$= \underline{\underline{51{\cdot}97'}}$.

It will be noticed that the Radius decreases more rapidly by shortening the normal Lead than by lengthening it.

CHAPTER XVI.

THE DOUBLE LINE JUNCTION.

A new double line junction or the improvement of an old one, is a work of such importance as to warrant, in almost every case, a carefully prepared large scale drawing.

In the design, the following points must have attention:—

1. The lines bearing the faster traffic should have the easier curves.
2. The curves through the points and crossings should be eased as far as possible, even at the expense of rather sharper curves in the plain line beyond. This is because super-elevation, with its steadying effect, also checking if necessary, can be better obtained on the latter.
3. Both tracks through the diamond crossing should be as nearly straight as is practicable.

The Radii of the curves will be limited by the Board of Trade Rule that the diamonds must not be flatter than 1 in 8; unless by special permission,* switch diamonds are used, in which case the flattest desirable angle of Vee crossing may become the limit.

By widening the spaces between tracks, dividing the divergence of the roads, etc., easier curves may be obtained without flattening the diamond. This is explained in the Report of the Railway Congress of 1905; whilst high speed junctions with transition curves are illustrated in J. W. Spiller's paper in the Inst.C.E. Proc., Vol. 176.

PRACTICAL DETAILS OF JUNCTIONS.

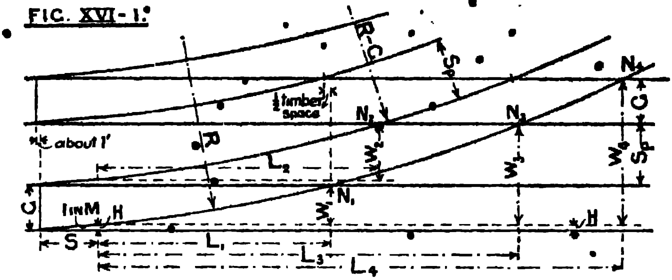
The diagrams of the timbering of the Turnout, the Crossover Road, and the Diamond, will jointly show the usual timbering of a Junction. (Also see "Inner Road of Junction," below.) A good example by E. Treacher may be seen in the Per. Way Inst. Journal of August, 1916.

* Special permission may become unnecessary under the future regulations of the Ministry of Transport.

CALCULATIONS (FIG. XVI.-1).

The case of a simple junction with all tracks parallel, is the only one which lends itself conveniently to calculation. The principles in this case will naturally apply whenever a track crosses, with a continuous curve, one or more tracks parallel to the one it springs from.

FIG. XVI-1.



Prob. 1. Given the Radius of a branch line from a double straight main line to find the angles of the crossings and their distances or "leads."

EXAMPLE.—Radius of branch on centre line of six foot $=12$ chains $=792'$ Spaces (Sp) between tracks, 6' clear or 6' 5½" gauge lines.

A suitable length of Switch must first be decided upon, thus:—

$$S=\sqrt{R}\times\cdot 581 \qquad \text{(Rule XI.-3.)}$$
$$=28\cdot 14'\times\cdot 581$$
$$=\underline{\underline{16\cdot 35'}}$$

16' 0" or 18' 0" would be suitable, but we will assume that 15' 0" is the only standard available, so

$$S=\underline{\underline{15'}}$$

Alternative methods of procedure.

Various correct methods of procedure from this point are:—

1. In accordance with F. W. Atkinson's "Junction Diagram" (Railway Engineer) in which an imaginary line parallel to the main is used. To this line the turnout curve is tangential at a point near to the switch toe.

From this tangent point, "theoretical leads" are obtained by the rule for chords, when the Versed sines are known.

2. By the same first principle, but using Trigonometrical Tables in solving the right-angled triangles, whose sides are the Radius (R), the theoretical Lead (L), and R - Y; Y being the distance from the parallel line to the rail on which the crossing lies.

This method has been used in preparing Table 33.

3. By the General Trigonometrical Rules (Table 25) of the present work.

3a. By the General C.L.M. Rules (Tables 26 and 27) of the same, which, though not the least laborious method, may be employed without the use of tables, and which will now be explained in detail.

1st.—With 792′ Radius on centre line of six foot:—

$$\text{Radius of outer rail (R)} = 792' + \tfrac{1}{2}\,Sp + G$$
$$= 792' + 3'\ 2\tfrac{3}{4}'' + 4'\ 8\tfrac{1}{2}''$$
$$= \underline{\underline{800'}} \text{ say.}$$

$$\text{Radius of next rail (R-G)} = \underline{\underline{795{\cdot}3'}}$$

2nd.—With 15′ switch and 4½″ heel divergence,

$$M = S \div \sqrt{H}$$
$$M = 15 \times \frac{8}{3} \qquad \text{(Rule XI.-1.)}$$
$$= \underline{\underline{40.}}$$
$$\text{And } M^2 = \underline{\underline{1600.}}$$

3rd.—The widths from switch heel line to rails on which the crossings lie, are:—

$$W_1 = G - H$$
$$= \underline{\underline{4'\ 4''}}$$
$$W_2 = Sp - H$$
$$= 6'\ 5\tfrac{1}{2}'' - 4\tfrac{1}{2}''$$
$$= \underline{\underline{6'\ 1''}} = 6{\cdot}083'$$
$$W_3 = W_1 + Sp$$
$$= 4'\ 4'' + 6'\ 5\tfrac{1}{2}''$$
$$= \underline{\underline{10'\ 9\tfrac{1}{2}''}} = 10{\cdot}79'$$
$$W_4 = W_3 + G$$
$$= 10'\ 9\tfrac{1}{2}'' + 4'\ 8\tfrac{1}{2}''$$
$$= \underline{\underline{15'\ 6''}} = 15{\cdot}5'$$

4th.—For number of 1st crossing (N_1):—

$$N_1 = M \times \sqrt{\frac{R - \frac{1}{2}W}{R + (2 \times W \times M^2)}} \quad \text{(Rule 3c.)}$$

$$= 40 \times \sqrt{\frac{800 - 2\cdot16}{800 + (2 \times 4\cdot333 \times 1600)}}$$

$$= 40 \times \sqrt{\frac{797\cdot84}{800 + 13867}}$$

$$= 40 \times \sqrt{\frac{797\cdot84}{14667}}$$

$$= 40 \times \sqrt{\cdot05439}$$

$$= 40 \times \cdot2332$$

$$\underline{\underline{9\cdot328}}$$

5th.—For Lead (L_1) of 1st crossing:—

$$L_1 = W_1 \times \frac{2 \times M \times N_1}{M + N_1} \quad \text{(Rule 1c.)}$$

$$= 4\tfrac{1}{3} \times \frac{2 \times 40 \times 9\cdot328}{40 + 9\cdot328}$$

$$= 4\tfrac{1}{3} \times \frac{746\cdot24}{49\cdot328}$$

$$= 4\tfrac{1}{3} \times 15\cdot128$$

$$= \underline{\underline{65\cdot55}} \text{ say, } \underline{\underline{65'\ 6\tfrac{1}{2}''}}$$

6th.—For Number of 2nd crossing (N_2), the Radius to be taken will be $R - G = 795\cdot3'$, thus:—

$$N_2 = M \times \sqrt{\frac{R - G - \frac{1}{2}W_2}{R - G + (2 \times W_2 \times M^2) + \frac{1}{2}W_2}}\,* \quad \text{(Rule 3c.)}$$

$$= 40 \times \sqrt{\frac{795\cdot3 - 3\cdot04}{795\cdot3 + (2 \times 6\cdot083 \times 1600) + 3\cdot04}}$$

$$= 40 \times \sqrt{\frac{792\cdot26}{795\cdot3 + 19465\cdot6 + 3\cdot04}}$$

$$= 40 \times \sqrt{\frac{792\cdot26}{20264}}$$

$$= 40 \times \sqrt{\cdot0391}$$

$$= 40 \times \cdot1977$$

$$= \underline{\underline{7\cdot908}}$$

*** This $\frac{1}{2}$W is neglected in Table 27 but should be included as W becomes greater.**

$$7th.—L_2 = 6\tfrac{1}{12} \times \frac{2 \times 40 \times 7{\cdot}908}{40 + 7{\cdot}908} \qquad \text{(Rule 1c.)}$$

$$= 6\tfrac{1}{12} \times \frac{632{\cdot}64}{47{\cdot}908}$$

$$= 6\tfrac{1}{12} \times 13{\cdot}205$$

$$= 80{\cdot}33' = \underline{\underline{80'\ 4''}}.$$

N_3, L_3, N_4 and L_4 may be found as shewn for N_1 and L_1, using the respective W's of 10′ 9½″ and 15′ 6″, they will be :—

$$N_3 = 5{\cdot}998$$
$$L_3 = 112{\cdot}59' = 112'\ 7''$$
$$N_4 = 5{\cdot}015$$
$$L_4 = 138{\cdot}14' = 138'\ 2''$$

The particulars may be then put into the form of a table, thus:—

Radius of Branch on Centre Line.	Switch.	Crossing numbers by calculation.				Numbers of Crossings to be ordered.				Leads from switch heel.			
		1	2	3	4	1	2	3	4	1	2	3	4
										ft. in.	ft. in.	ft. in.	ft. in.
12 Chs. (792′)	15′	9·338	7·908	5·998	5·015	9½	8	6	5	65 6½	80 4	112 7	138 2

Inner Road of Junction.

A further set of switches and a 1 in 9½ crossing will be required for the other road of a double branch. The radius of this road will be Sp + G = 11′ 2″, less than the outer road, but this will not appreciably alter the angle of the crossing or the lead.

It is suggested that the switches and crossing be situated with regard to those of the outer road, as shown in Fig. 1. This is to aid in interlacing the timbers, and at the same time to keep a through channel for the point rods.

Main Line Curved.

In this case the Equivalent Radius from a straight main must be obtained, as explained in Chapter XIII. This Equivalent Radius must be used as the "R," in calculations exactly similar to those given above. The leads, however, should be measured along a tangent to the main line at the switch heel.

Prob. 2. Given the length overall of a double line junction: to find all the required particulars which will give regular curves of main and branch lines throughout.

In relaying, and at the same time improving, a junction, it may be desirable not to alter the positions of the switch and the last crossing. In such a case, the length overall will be the given dimension.

EXAMPLE.—Length from switch heel to last crossing (L_4) 166′. Switch 18′. (Sp) between tracks 6′ $5\frac{1}{2}''$

1st.—Obtain the Equivalent Radius (R) of the junction as existing:—

With 18′ switches:—$M = 18' \div \frac{3}{8}' = \underline{48}$

With 6′ $5\frac{1}{2}''$ space:—$W_4 = \underline{\underline{15' \ 6''}}$, as in previous example.

$$R = \frac{M \times (W_4^2 + L_4^2)}{2 \times (W_4 \times M - L_4)} \quad \text{(Approx. Rule 6d.)}$$

$$= \frac{48 \times (15{\cdot}5^2 + 166^2)}{2 \times (15{\cdot}5 \times 48 - 166)}$$

$$= \frac{48 \times (240{\cdot}25 + 27556)}{2 \times (744 - 166)}$$

$$= \frac{48 \times 27796}{2 \times 578}$$

$$= \underline{\underline{1154'}}$$

2nd.—Find the ideal switch length:—

S should be $\sqrt{R} \times {\cdot}581$

$= 33{\cdot}97 \times {\cdot}581$

$= \underline{\underline{19{\cdot}72'}}$

Assuming 21′ switches are the next length beyond 18′, which are available, we may try the use of the former and

base your calculations upon 21′ switches, and a reduced L_4 of 163′, then:—

3rd.—$M = S \div H = 21 \times \frac{8}{3} = \underline{\underline{56}}$

4th.— $R = \dfrac{56(15{\cdot}5^2 + 163^2)}{2(15{\cdot}5 \times 56 - 163)}$ (Approx. Rule 6d.)

$= \dfrac{56 \times 26809}{2 \times 705}$

$= \underline{\underline{1065'}}$

The crossing angles and leads may then be obtained as in the previous example, using a Radius of 1065′, but, as this radius is not quite exact, we may find that L_4 becomes slightly different to 163′. This difference will, however, be of no account.

It will be noticed that lengthening the switches, but not the lead, reduces the radius considerably in the above example.

We may test the switch length for this new Radius.

$\therefore S = \sqrt{1065} \times {\cdot}581$ (Rule XI.–3.)

$= 32{\cdot}63 \times {\cdot}581$

$= \underline{\underline{18{\cdot}96'}}$

As either 18′ or 21′ switches are fairly long, it is almost a matter of opinion whether it would be better to retain the 18′ switches, or increase them to 21′ in such a case as this.

Prob. 3. Required the easiest possible Equivalent Radius to which a junction may be laid in a double line, with a given Space (Sp) between tracks; the diamond crossing not to exceed a certain angle, say, 1 in 8.

EXAMPLE.—Space (Sp) = 6′ $5\frac{1}{2}''$ (gauge lines).

1st.—The diamond is N_3 which = 8.

$\therefore W_3 = G - H + Sp = 4'\ 8\frac{1}{2}'' - 4\frac{1}{2}'' + 6'\ 5\frac{1}{2}''$

$= \underline{\underline{10'\ 9\frac{1}{2}''}} = 10{\cdot}792'$

2nd.—Assume a switch length of 18′, then:—

$M = 18 \times \frac{8}{3} = \underline{\underline{48}}$ (Rule XI.–1.)

and $M^2 = \underline{\underline{2304}}$.

3rd.— $R = W \times \dfrac{(4 \times M^2 \times N^2) + M^2 + N^2}{2 \times (M^2 - N^2)}$ (Rule 2b)

$= 10{\cdot}792 \times \dfrac{(4 \times 2304 \times 64) + 2304 + 64}{2 \times (2304 - 64)}$

$= 10{\cdot}792 \times \dfrac{589824 + 2304 + 64}{2 \times 2240}$

$= 10{\cdot}792 \times \dfrac{592192}{4480}$

$= 10{\cdot}792 \times 132{\cdot}18$

$= \underline{\underline{1426'}}$

$= \underline{\underline{21 \text{ chains}, 40\text{ft}.}}$

To test the suitability of the switch length:—

$S = \sqrt{1426} \times {\cdot}581$ (Rule XI. 3)

$= 37{\cdot}76 \times {\cdot}581$

$= \underline{\underline{21{\cdot}94'}}$

Thus the 18' switch may be considered too short, and the Radius should be calculated afresh for a 21' switch.

1st.—$M = 21 \times \frac{8}{3} = \underline{\underline{56}}$

and $M^2 = \underline{\underline{3136}}$

2nd.—$R = 10{\cdot}792 \times \dfrac{(4 \times 3136 \times 64) + 3136 + 64}{2 \times (3136 - 64)}$ (Rule 2b.)

$= 10{\cdot}792 \times \dfrac{806016}{6144}$

$= \underline{\underline{1416'}}$

$= \underline{\underline{21 \text{ chains}, 30\text{ft}.}}$

The 21' switch may be considered near enough to the ideal length, because:—

$\sqrt{1416} \times {\cdot}581 = \underline{\underline{21{\cdot}86'}}$

Table 34 shows the Radii obtainable with various spaces between tracks, and a 1 in 8 diamond.

CHAPTER XVII.

THE DOUBLE TURNOUT.

A double turnout is an arrangement of two turnouts in which the switches of the first turnout are followed by those of the second turnout before a crossing occurs.

FIG. XVII-1.

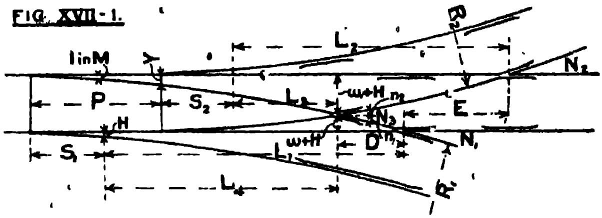


FIG. XVII-2.

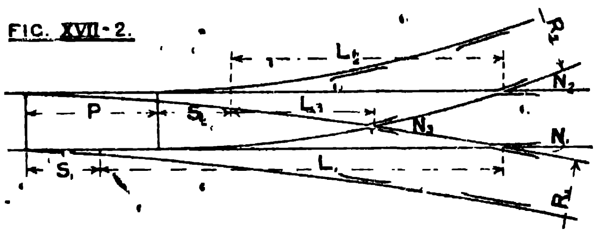


FIG. XVII-3.

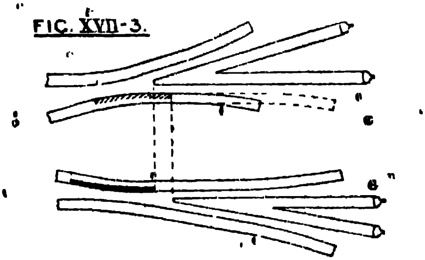
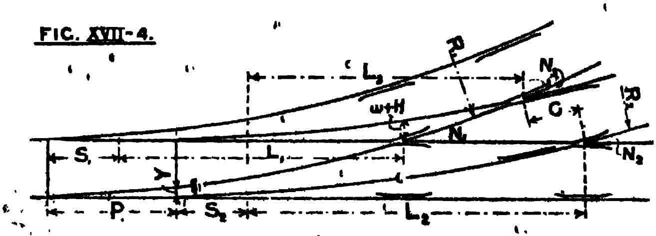


FIG. XVII-4.



Other names for the double turnout are "Tandem turn outs," and "Pannier leads."

Double turnouts have their disadvantages, such as the use of additional crossings, but when space is restricted, they often form the best way of acquiring the necessary accommodation.

The use of a succession of double turnouts for the gathering lines of sidings has been advocated, but has not been usually adopted except where it is necessary to crowd as much siding accommodation as possible on to an area of land.

FIG. XVII-5.

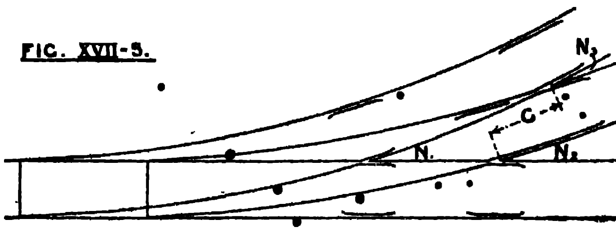


FIG. XVII-6.

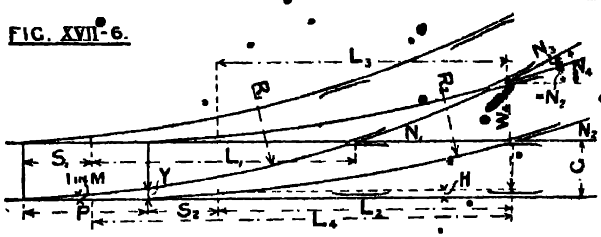


FIG. XVII-7.

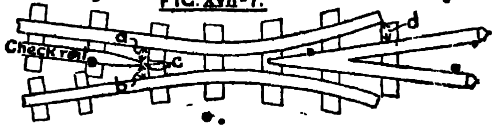
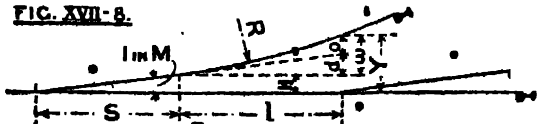


FIG. XVII-8.



The use of double turnouts should be avoided in main lines, and in any case, they are hardly permissible in a facing direction.

There are three general types of double turnout:—

Type 1 (Figs. 1 and 2).—In which the turnout tracks branch to opposite sides of the main. In this case, the third crossing, *i.e.*, where one turnout crosses the other, is in the four foot of the main.

Type 2 (Figs. 4, 5, and 6).—In which the turnout tracks branch to the same side of the main. In this case, the third crossing is outside of the four foot of the main.

Type 3.—In which the second turnout springs from the first. In this case the third crossing is on the main.

PRACTICAL DETAILS.

The usual arrangements of checking are shown by the diagrams.

After having found by calculation or otherwise, the position of the third crossing, the possibility of getting the check and wing rails in, must be examined.

Referring to Fig. 1, where the crossings are marked N_1, N_2, and N_3. If N_3 is too near to N_1, there will not be room for the right-hand wing rail of N_3, nor will there be space for the chairs of N_3.

The critical dimensions will generally be those marked $w + H$ in Figs. 1 and 4. 1′ 1½″ (gauge lines) may be taken as an absolute minimum for these dimensions; 1′ 4½″ being a better dimension to work to.

Again, if N_1 and N_2 are too near together, their check rails cannot be put in.

There is an arrangement which will ease checking and timbering difficulties; this is to have N_1 and N_2 opposite, or very nearly so, as shown in Fig. 2. Fig. 3 shows how the wing rail of one crossing acts as a check rail for the other in this case. The ideal lengths of check in front of crossing nose are not obtained, these lengths being etched black. Such arrangements, however, have answered well in practice, in siding work.

It will be noticed that the nose of the crossing of flatter angle, is shown further from the switches. It obtains better checking in this way.

Similar remarks apply to the case of Type 2, Figs. 4, 5, and 6.

In Fig. 4, the length C must be sufficient to insert the right-hand check of N_3. The wing of N_3 must be extended to check N_2.

In Fig. 5, the length C must allow of N_2 being checked. The extended wing of N_2 checks N_3.

In Fig. 6, N_2 and N_3 are opposite, and as N_3 is the flatter angle, the checking should be similar to Fig. 3, but reversed.

It is useful to know how far a check or wing rail can be taken into a converging space between two rails. Referring to Fig. 7; by pointing the check rail, its end may extend to where the space between gauge lines is a + b + c. a and b should not be much less than the usual end gap of a wing or check rail, such as at d. In a confined space a and b may be made $3\frac{1}{8}''$; noting that wheel flanges are usually observed to strike check rails where the gap is $2\frac{1}{4}''$. Then taking C as $\frac{3}{4}''$, $a+b+c=7''$, as a minimum.

In cases where one crossing is followed closely by another, the leg of the first crossing, made longer than usual if necessary, may be bent to form the wing of the second. This makes a strong arrangement, but the difficulty of removal in case of repairs makes it preferable to have the crossings far enough apart to allow of the leg and wing rail being separate.

CALCULATIONS.

Prob. 1 (Fig. 8). Distance between Points (P).

Having decided upon the position and length of the first switch, and the radius (R) of the curve leading from it, the distance P must be such as will give clearance (Y) between the gauge lines for the toe of the second switch. With the usual type of fittings, Y should not be less than $10\frac{1}{2}''$, though this may be reduced to $9''$ in special cases.

Referring to Fig. 8, Y may be looked upon as being equal to H + w, and P to S + l, where w and l are similar to the dimensions W and L in the General Rules for points and crossings.

When the Radius and Y, and therefore w, are known:

$$l = \sqrt{(w \times 2R) + \frac{R^2}{M^2}} - \left(\frac{R}{M}\right). \quad \text{(Rule 4d.)}$$

When the Radius and P, and therefore l, are known:—

$$w = \frac{l^2}{2R} + \frac{l}{M} \qquad \text{(Rule 11d.)}$$

The usual procedure in practice would be to choose a distance P such as would suit the timbering, etc., and then try by the latter of the above rules whether Y is sufficient; if not, P must be increased.

Table 35, which shows the clearance Y for various Radii and distances P, has been prepared by the use of Rule 11d.

Position of Third Crossing.

The symbols used are explained by the diagrams.

It must be understood that the switch lengths are known in all cases, that the main is straight, and that if one of the dimensions R, N, or L of a turnout with regard to the main, is known, the others may be obtained by the rules for single turnouts.

Though the following solutions by calculation are given for certain problems, the only method of practical use in most cases, is that of drawing to scale. This has been used in preparing Table 36.

Prob. 2 (Fig. 1). Double Turnout Type 1. Given: Particulars of First Turnout; distance between Points (P), and the width w + H between crossing N_3 and the main.

w + H may be taken as the minimum width considered necessary for wing rail and chairs of N_3.

The problem may be solved by the General Rules (Table 27).

EXAMPLE.—$R_1 = 600'$, S_1 and $S_2 = 15$, ∴ $M = 40$, $w + H = 1'\ 4\frac{1}{2}''$, ∴ $w = 1'\ 0'$. $P = 27'$.

1st.—Find w_1

$$w_1 + H + w + H = G$$
$$\therefore\ w_1 = G - w - 2H$$
$$w_1 = 4'\ 8\tfrac{1}{2}'' - 1'\ 0'' - 9''$$
$$w_1 = 2'\ 11\tfrac{1}{2}'' \text{ or } 2{\cdot}96'$$

2nd.— $$n_1 = M \times \sqrt{\frac{R_1}{R_1 + (2 \times w_1 \times M^2)}} \quad \text{(Rule 3c.)}$$

$$n_1 = 40 \times \sqrt{\frac{660}{660 + (2 \times 2{\cdot}96 \times 1600)}}$$

$$= 40 \times \sqrt{\frac{660}{660 + 9472}}$$

$$= 40 \times \frac{\sqrt{660}}{\sqrt{10132}}$$

$$= 40 \times \frac{25{\cdot}69}{100{\cdot}65}$$

$$= 40 \times {\cdot}255$$

$$n_1 = \underline{\underline{10{\cdot}20}}$$

3rd.— $$L_4 = w_1 \times \frac{2 \times M \times n_1}{M + n_1} \quad \text{(Rule 1c.)}$$

$$= 2{\cdot}96 \times \frac{2 \times 40 \times 10{\cdot}20}{40 + 10{\cdot}20}$$

$$L_4 = \underline{\underline{48{\cdot}11'}}$$

4th.—Find L_3

$$L_4 + S_1 = L_3 + S_2 + P$$

$$\therefore\ L_3 = L_4 + S_1 - S_2 - P$$

$$L_3 = 48{\cdot}11' + 15' - 15' - 27'$$

$$L_3 = \underline{\underline{21{\cdot}11'}}$$

5th.— $$n_2 = \frac{L_3 \times M}{(2 \times M \times w) - L_3} \quad \text{(Rule 5c.)}$$

$$= \frac{21{\cdot}11 \times 40}{(2 \times 40 \times 1) - 21{\cdot}11}$$

$$n_2 = \underline{\underline{14{\cdot}34}}$$

6th.—Angle N_3 is the sum of angles 1 in n_1 and 1 in n_2, *i.e.*:—

$$N_3 = \frac{n_1 \times n_2}{n_1 + n_2} \quad \text{(Rule XII.—24.)}$$

$$= \frac{10{\cdot}20 \times 14{\cdot}34}{24{\cdot}54}$$

$$N_3 = \underline{\underline{5{\cdot}96}}$$

7th.— $$R_2 = 2 \times w \times \frac{M^2 \times n_2^2}{M^2 - n_2^2} \qquad \text{(Rule 2c.)}$$

$$= 2 \times 1' \times \frac{40^2 \times 14{\cdot}34^2}{40^2 - 14{\cdot}34^2}$$

$$= 2 \times \frac{1600 \times 205{\cdot}64}{1600 - 205{\cdot}64}$$

$$= 2 \times \frac{329024}{1394{\cdot}36}$$

$$R_2 = \underline{\underline{471{\cdot}5'}}$$

8th.— $$N_2 = M \times \sqrt{\frac{R_2}{R_2 + (2 \times W \times M^2)}} \qquad \text{(Rule 3c.)}$$

$$= 40 \times \sqrt{\frac{471{\cdot}5}{471{\cdot}5 + (2 \times 4\tfrac{1}{3} \times 1600)}}$$

$$= 40 \times \frac{21{\cdot}71}{119{\cdot}7}$$

$$N_2 = \underline{\underline{7{\cdot}26}}$$

9th.— $$L_2 = 2 \times W \times \frac{M \times N_2}{M + N_2} \qquad \text{(Rule 1c.)}$$

$$= 2 \times 4\tfrac{1}{3} \times \frac{40 \times 7{\cdot}26}{40 + 7{\cdot}26}$$

$$= \frac{2516{\cdot}8}{47{\cdot}26}$$

$$L_2 = \underline{\underline{53{\cdot}25'}}$$

The Double Turnout of Type 2, with the same particulars given, as above, may be solved in a similar manner. The method of drawing to scale is more satisfactory, however, on account of the difficulty in finding suitable positions for N_2 and N_3, so that they may be checked.

Particulars of both Turnouts given.

It is possible to give formulæ to solve these problems, but these formulæ are so complex as to be of little practical use.

The cases where 2nd and 3rd crossings are opposite may, however, be conveniently solved in part by calculation.

Prob. 3 (Fig. 2). Double Turnout Type 1; second and third crossings opposite. Particulars of second Turnout given (because it is of the lesser radius); also distance (P) between points given.

It is easily seen that:—

$$L_1 = L_2 + S_2 + P - S_1$$

or if $S_1 = S_2$:—

$$L_1 = L_2 + P$$

Particulars of the first turnout ma[y be calculated] as in a single turnout when the Lead is known. From the data thus obtained, L_3 and N_3 are best obtained by drawing to scale.

Prob. 4 (Fig. 6). Double Turnout Type 2; second and third crossings opposite. Particulars of first Turnout given.

In order that the crossings N_2 and N_3 may be opposite, they must be situated where the first turnout has diverged from the main to the extent of the gauge.

The distance L_4 is therefore required. This may be calculated as in obtaining the L in the general problem, where W_4 is practically 2 G − H, or with the usual standards $W_4 = 9'\ 0\frac{1}{2}''$.

First find N_4, this being the imaginary angle the turnout makes with a line parallel to the main.

$$N_4 = M \times \sqrt{\frac{R - \frac{1}{2} W_4}{R + (2 \times W_4 \times M^2)}} \qquad \text{(Rule 3c.)}$$

$$\text{and, } L_4 = 2 \times W_4 \times \frac{M \times N_4}{M + N_4} \qquad \text{(Rule 1c.)}$$

After deciding upon distances P and S_2, we may measure L_2 and L_3 which are equal, or if $S_1 = S_2$, then :—

$$L_2 = L_3 = L_4 - P$$

N_2 and R_2 may be calculated as in a single turnout when the Lead is given.

Angle N_3 = difference between N_4 and N_2, *i.e.* :—

$$N_3 = \frac{N_2 \times N_4}{N_2 - N_4} \qquad \text{(Rule XII.—25.)}$$

CHAPTER XVIII.

THE THREE THROW.

The Three Throw is an arrangement of two turnouts with their switches lying alongside each other.

The device has several evident objections, and its use should not be permitted in main lines. In sidings to be laid upon a restricted area, it may occasionally happen that the use of three throws is the only way of obtaining the required standing room. Its use should be avoided wherever possible, and when an old set of three throws is in need of relaying, an endeavour should be made to supplant it by a double turnout.

There are two general types of three throw:—

TYPE 1.—With turnouts branching to different sides of the main (Fig. 2).

TYPE 2.—With turnouts branching to the same side of the main (Fig. 3).

Type 2 is especially objectionable because the outer switch has twice its usual deflecting angle.

PRACTICAL DETAILS.

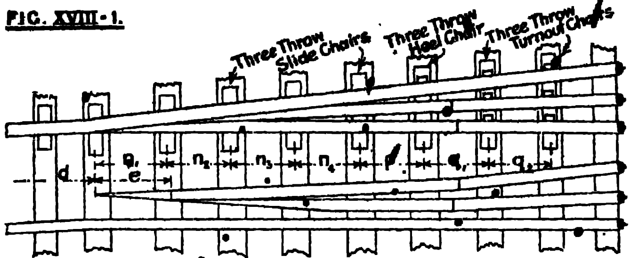
Switches for Three Throws.

One method of arranging the switches is shown in Fig. 1. The heel joints of two switches lie side by side, or in some designs a timber space apart. The toes of the switches are a timber space apart. A long and short switch are coupled, and work together. This method, where the two longer switches are nearer the centre of the four foot, is the one generally used.

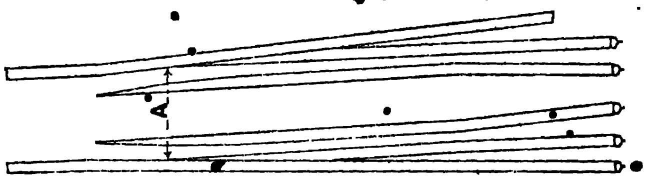
When the switches are set for either of the outer roads, the gauge is normal, but it is wide at the toe of the shorter switches when set for the middle road.

There are other methods of arranging the switches, but in no case can correct gauge be maintained for all roads.

FIG. XVIII-1.



Switches set for an outer road, gauge practically correct.



Switches set for Middle Road, gauge is wide at A.

FIG. XVIII-2.

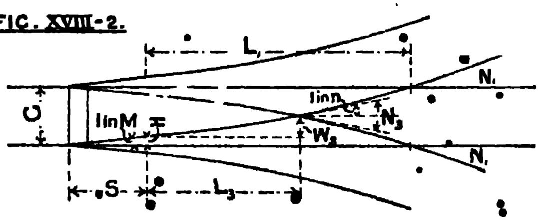
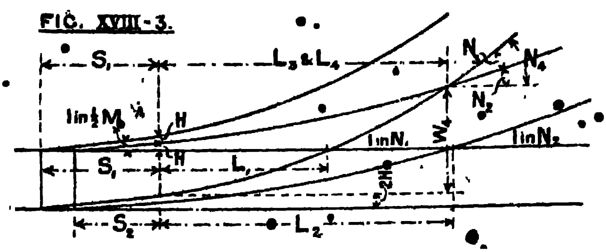


FIG. XVIII-3.



SWITCHES FOR THREE THROWS.

DETAIL DIMENSIONS.

Refer to Figs. XVIII.—1 and Figs. III.—15, 16 and 17.	Switch lengths.		
	........&........	........&........	........&........
Flangeway clearance at heel joint .... F			
Divergence at heel joint... H			
Length of Stock Rails...... a			
Toe of stock rail to toe of switch .... d			
Toe to toe of switches...... e			
Heel to heel of switches... f			
Distances apart of chairs (centre to centre):—			
Slide chairs .... n_1			
n_2			
n_3			
n_4			
n_5			
Last Slide and Heel chair.. p			
Heel chair and Turnout chair .... q_1			
Turnout chairs........... q_2			
Divergences (H) of chairs:—			
Heel chair ....H			
" " ....H_1			
Turnout chair No. 1 ... H			
" " " 1 ... H_1			
" " " 2 ... H			
" " " 2 ... H_1			

For list of Three Throw Switch Chairs see Chapter III.

CALCULATIONS.

Calculation is only feasible in solving the simpler problems, drawing to large scale being as a rule, more satisfactory.

TYPE 1.—In this type both turnouts may be considered as springing from the longer switch.

The length of the longer switch in feet should be about twice the angle number of the flatter crossing on the main.

The rules for each separate turnout with regard to the main, will be the same as for a single turnout. The special calculation will be to find the lead and angle for the middle or "crotch" crossing (N_3).

Where the turnouts are equal in lead and radius, and the main is straight, it will be obvious that N_3 will lie in the centre of the four foot, and that the crossings on the main will be opposite (see Fig. 2).

The General Rules of Table 27 may be applied in this case.

$$W_3 \text{ will be } \tfrac{1}{2}\, G - H$$

Or with the usual standards, $W_3 = 2'\ 4\tfrac{1}{4}'' - 4\tfrac{1}{2}''$

$$= \underline{\underline{1'\ 11\tfrac{3}{4}''}}$$

Angle n, which is half of the angle N_3, may then be obtained from Rule 3c and the Lead L_3 from Rule 1c, using the angle n. M will be the angle of the longer switch.

Table 37 contains examples of Three Throws in which the main line crossings are opposite. Approximate rules applying to this particular case are:—

$$N_3 = N_1 \times \cdot 726$$

$$L_3 = L_1 \times \cdot 62$$

TYPE 2.—In this type, the turnout which diverges further from the main may be considered as springing from the longer switch.

This switch, however, will have a divergence of twice the normal, or 2H; therefore the longest switches in use

should be employed. The turnout may be solved by the rules for single turnouts, but M will have half its usual value.

The turnout nearer the main springs from the shorter switch, which has the normal divergence, H.

It is possible, but not advisable, to solve the third crossing by the rules given for the double turnout, including the case where it is desired to have two crossings lying on the same timber, as in Fig. 3.

Alternative Calculation.

The calculations for the third crossing in either Type 1 or 2, may also be made on the assumption that one turnout springs from the other. Thus in Type 1 we should have to solve a single turnout of contra-flexure; the switch length being that of the longer switch; the divergence 2H; M one-half its usual value; and W being G − 2H.

In Type 2 the turnout will be of similar flexure, the switch length being that of the longer switch and the divergence, H. In this type, the above assumption is not sufficiently accurate to give the Lead, but it will suffice for finding the crossing angle.

CHAPTER XIX.

THE CROSSOVER ROAD.

In addition to the ordinary crossover road, the following notes will naturally apply to all cases of connections similarly made between two parallel or nearly parallel tracks; such as Crossover Junctions and Loop Connections.

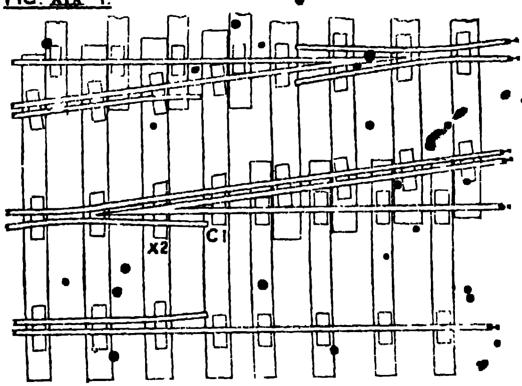
Suitable Radii for crossovers and various similar connections are mentioned in Chapter IX.

A Crossover Road consists of two turnouts joined with a connecting portion of track. The dimensions for the turnouts are dealt with in Chapter XV.

In the present discussion, turnouts of similar and contra flexure with the main, will be shortly referred to as the "similar" and "contra turnouts" respectively.

PRACTICAL DETAILS.

FIG. XIX-1.



One method of checking and timbering a crossover road is shown at the upper rail in Fig. 1, whilst "Through Checking" or checking by "Parallel Wing Rails" is shown at the lower rail.

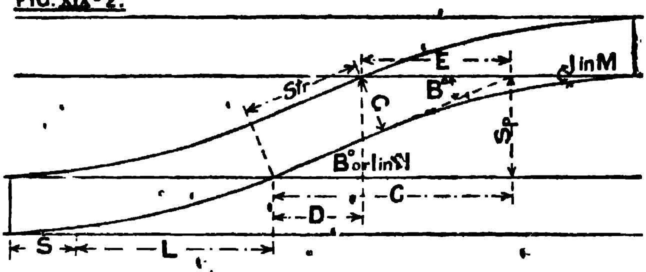
The latter method is to be preferred, but the chairs marked X 2 and C 1 will be of special patterns to suit either right or left hand crossings; unless the usual chairs are for wing rails parallel to the crossing.

In Chapter XII. it is advocated that the arrangements for timbering in a crossover should be studied when deciding upon a standard length for crossing legs.

CALCULATIONS.

Prob. 1 (Fig. 2). Crossover between straight and parallel mains at a given distance (Sp) apart, (gauge lines), and with a straight connecting portion between crossings. Turnouts alike. Length of switches and one turnout dimension given, namely, either Crossing Angle, Radius, or Lead. Required the distance apart (D) of crossing fine points and length overall of crossover.

FIG. XIX-2.



1st.—The turnouts may be solved by the General Rules (Tables 25, 26, or 27).

2nd.—A rule for D may be derived as follows:—

Referring to Fig. 2, produce the straight portion of the crossover as shown by the dotted line, then:—

$$D = C - E$$

and $\quad C = Sp \times \text{Cot } B$

and $\quad E = G \times \text{Cosec } B$

So $\quad D = (Sp \times \text{Cot } B) - (G \times \text{Cosec } B)$

Rule XIX.—1.

To convert this rule to C.L.M., so that trigonometry need not be used, substitute the values of Cot and Cosec B as given in Chapter XII., and we obtain:—

$$D = \left\{Sp \times \left(N - \frac{1}{4N}\right)\right\} - \left\{G \times \left(N + \frac{1}{4N}\right)\right\}$$

$$\therefore \quad D = SpN - \frac{Sp}{4N} - GN - \frac{G}{4N}$$

$$\therefore \quad D = N(Sp - G) - \frac{Sp + G}{4N}$$

Rule XIX.—2.

The $\frac{Sp+G}{4N}$ cannot be neglected unless only rough results are required.

To put the rule into words:—

To find Distance apart of crossings (D). From the Space deduct the Gauge and multiply by Number of crossing, then deduct the sum of Space and Gauge divided by four times No. of crossing.

EXAMPLE.—Required the length overall and the distance apart of crossings in a crossover with 1 in 8 crossings and 15′ switches. Straight mains, 8′ 5½″ apart (gauge lines).

1st.—When N = 8:—

$$L = 57'\ 9'' \qquad \text{(Rule 1c or Table 31.)}$$

$$\text{2nd.—}D = 8 \times (8{\cdot}46' - 4{\cdot}71') - \left(\frac{8{\cdot}46 + 4{\cdot}71}{4 \times 8}\right) \qquad \text{(By Rule XIX.—2.)}$$

(Taking 8′ 5½″ as 8·46′ and 4′ 8½″ as 4·71′).

$$= 8 \times 3{\cdot}75 - \frac{13{\cdot}17}{32}$$

$$= 30{\cdot}00 - {\cdot}41$$

$$= 29{\cdot}59 = \underline{\underline{29'\ 7''}}$$

3rd.—Length overall:—

$$= 2 \times (\text{Lead} + \text{Switch}) + D$$

$$= 2 \times (57'\ 9'' + 15'\ 0'') + 29'\ 7''$$

$$= \underline{\underline{175'\ 1''}}$$

Table 38 shows Distances D for various spaces, with 4′ 8½″ gauge.

EXAMPLE of the use of Table 38.

Required D for a space of 8′ 3″ (clear). Crossings 1 in 8½.

D for 6′ 0″ clear = 14′ 6½″

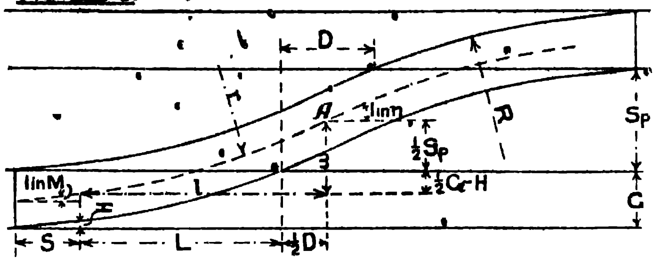
Add for 2′ 0″, 2 × 8′ 5⅝″ = 16′ 11¼″

Add for 3″, 3 × 8½″ = 2′ 1½″

D = 33′ 7¼″

Prob. 2 (Fig. 3). Crossover as in Prob. 1, but with the turnout curves continued through the crossings and directly reversing with no straight portion.

FIG. XIX-3.



This arrangement is objectionable. We may be tempted to adopt it when the available length is too small to allow of a straight between crossings, but what we save in length is very small unless the space is wide, and even then some portion of straight should be allowed.

First find the Turnout Lead (L) and Radius (R) if they are not given.

Considering the centre lines of the turnouts, it is obvious that the curves reverse at the point "A," which is central in the space and equi-distant from the crossings.

We may then find the Lead "l" for a width "w" with a radius "r," by the General Rules, and next:—

$$D = 2 \times (l - L).$$

EXAMPLE.—Crossings 1 in 8. Switches 15′. Space 8′ 5½″ (gauge lines).

1st.— $R = \underline{\underline{580'}}$ (Rule 2a or 2b or Table 31.)

$L = \underline{\underline{57{\cdot}77'}}$ (Rule 1a or 1b or Table 31.)

2nd.— $r = R - \frac{1}{2} G$

$= 580 - 2{\cdot}35 = \underline{\underline{577{\cdot}65'}}$

3rd.— $w = \frac{1}{2} (Sp + G) - H$

$= \frac{1}{2} (8'\ 5\frac{1}{2}'' + 4'\ 8\frac{1}{2}'') - 4\frac{1}{2}''$

$= \underline{\underline{6'\ 2\frac{1}{4}''}} = \underline{\underline{6{\cdot}21'}}$

4th.— $n = \sqrt{\frac{M^2(2r - w)}{2r + 4wM^2 + w}}$ (Rule 3b.)

$$= \sqrt{\frac{1600 \times (1155 \cdot 3 - 6 \cdot 21)}{1155 \cdot 3 + (4 \times 6 \cdot 21 \times 1600) + 6 \cdot 21}}$$

$$= \sqrt{\frac{1838544}{40905 \cdot 5}}$$

$$= \sqrt{44 \cdot 94}$$

$$n = \underline{\underline{6 \cdot 704}}$$

5th.— $l = w \times \frac{2Mn}{M + n}$ (Rule 1c.)

$$= 6 \cdot 21 \times \frac{2 \times 40 \times 6 \cdot 704}{46 \cdot 704}$$

$$= \underline{\underline{71 \cdot 31'}}$$

6th.— $D = 2(l - L)$

$$= 2(71 \cdot 31' - 57 \cdot 77')$$

$$= \underline{\underline{27 \cdot 08'}}$$

The saving over a straight between crossings in this case will be about 2′ 6″.

It may be noticed that the above solution is not quite precise, but it is near enough for all practical purposes.

Table 39 gives the distances for crossings with reverse curves between them, of the same radius as the crossings would have on a turnout. The figures in the table will show a slight discrepancy with results arrived at by the above rule because the former have been obtained by a geometrical method.

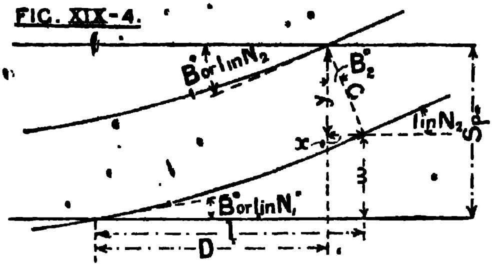
Prob. 3 (Fig. 4). Crossover between straight or curved parallel mains, with given crossings of different angle.

There is a possibility of this case arising when crossings of the same angle are not at hand.

A curve will occur between the crossings, and the greater the difference in angle, the sharper the curve.

We will first assume the mains straight, and the curve to extend from one crossing point to the other.

FIG. XIX-4.



A rule for "D" may be derived by using the same principle as in a turnout. Instead of the switch angle, we must take the angle of the flatter (or first) crossing and call this 1 in N_1, and the other crossing 1 in N_2.

$$y = G \times \text{Cos } B_2$$

or in C.L.M. $y = G - \frac{G}{2N_2^2}$ (approx.) *

$$w = Sp - y$$

$$= Sp - G + \frac{G}{2N_2^2}$$

$$l = w \times \frac{2N_1 \times N_2}{N_1 + N_2} \quad \text{from Rule 1c.}$$

$$= \left(Sp - G + \frac{G}{2N_2^2}\right) \times \left(\frac{2N_1 \times N_2}{N_1 + N_2}\right)$$

$$x = G \times \text{Sin } B_2$$

$$= \frac{G}{N_2} \quad \text{(approx.)}$$

$$D = l - x$$

$$= \left\{\left(Sp - G + \frac{G}{2N_2^2}\right) \times \left(\frac{2N_1 \times N_2}{N_1 + N_2}\right)\right\} - \frac{G}{N_2}$$

Rule XIX.—3.

If the mains are curved it will be evident from the remarks upon "Equivalent Radius" in Chapter XIII., that the same rule for "D" will apply with sufficient accuracy.

EXAMPLE.—Required the distance "D" between a 1 in 9 (N_1) and a 1 in 7 (N_2) crossing with 8′ 5½″ space between mains (gauge lines).

* Because from Rule XII.—15, Cos $B = 1 - \frac{1}{2N^2}$ (approx.)

$$D=\left\{\left(8{\cdot}46'-4{\cdot}71'+\frac{4{\cdot}71}{98}\right)\times\left(\frac{2\times 9\times 7}{9+7}\right)\right\}-\frac{4{\cdot}71}{7}$$

By Rule XIX.—3.

$$=\{(8{\cdot}46-4{\cdot}71+{\cdot}05)\times(7{\cdot}875)\}-{\cdot}67$$
$$=(3{\cdot}80\times 7{\cdot}875)-{\cdot}67$$
$$=29{\cdot}92-{\cdot}67$$
$$=\underline{\underline{29{\cdot}25'}}=\underline{\underline{29'\ 3''}}$$

A rough rule applying to this case is:—

$$D=\left\{(Sp-G)\times N_3\right\}-\frac{G}{N_2}\qquad \textbf{Rule XIX.—4.}$$

where N_3 = the average of N_1 and N_2; or in words: From the Space deduct the Gauge and multiply by the average of the crossing numbers, then deduct the Gauge divided by the second crossing number

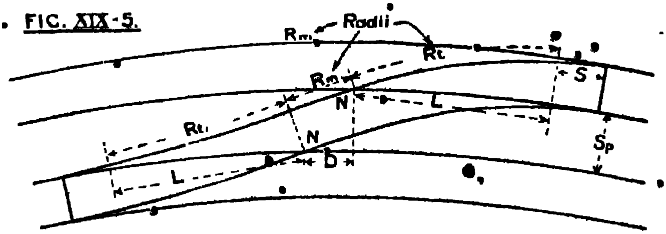
In the above example:—

$$D=\left\{(8'\ 5\tfrac{1}{2}''-4'\ 8\tfrac{1}{2}'')\times\frac{9+7}{2}\right\}-\frac{4'\ 8\tfrac{1}{2}''}{7}$$
$$=(3'\ 9''\times 8)-8''$$
$$=30'\ 0''-8''$$
$$\underline{\underline{29'\ 4''}}$$

The approx. rule will, however, show a greater discrepancy with wider angles or with greater differences between the angles.

Prob. 4 (Fig. 5). Crossover between curved parallel mains of Radius "R_m" and at a given distance (Sp) apart. Crossings to be of same angle. Required the Distance (D) apart of crossings, length of crossover overall, and the Radii of the turnout curves.

FIG. XIX-5.



This arrangement is only suitable when the main curve is fairly flat.

For the crossings to be of the same angle it is evident that the track between them must be curved to practically the same radius as the mains.

The distance "D" will be the same as on straight mains with the same crossing angles and space.

The two turnouts will be equal in lead but different in radius, the sharper curve being in the similar turnout.

Generally, the procedure will be to first decide upon a minimum radius for the similar turnout.

EXAMPLE.—Main 21 chains $= R_m$. Turnout 7 chains $= R_t$. Space 6′ 5½″. Switches 15′.

1st.—Obtain Equivalent Radius (R_e):—

$$R_e = \frac{21 \times 7}{21 - 7} \qquad \text{(Rule XIII.–4.)}$$

$$= \frac{147}{14} = 10\tfrac{1}{2} \text{ chains} = \underline{\underline{693'}}$$

2nd.—Obtain crossing for this Radius, from Tables or General Rules

$$\text{When } R = 693' \text{ and } S = 15'$$

$$N = \underline{\underline{8\tfrac{3}{4}}} \qquad \text{(Table 31.)}$$

This will be the angle of the two crossings and:—

3rd.— $D = \underline{\underline{15'\ 0''}}$ (Table 38 or Rule XIX.–2.)

4th.—For Radius of other turnout:—

$$R_{t_1} = \frac{R_m \times R_e}{R_m - R_e} \qquad \text{Rule XIII.—2.}$$

$$= \frac{21 \times 10{\cdot}5}{21 - 10{\cdot}5} = \underline{\underline{21 \text{ chains.}}}$$

5th.—The leads will be equal and:—

$$L = \underline{\underline{62'\ 2''}} \qquad \text{(Table 31.)}$$

6th.—Length overall:—

$$2(L + S) + D$$

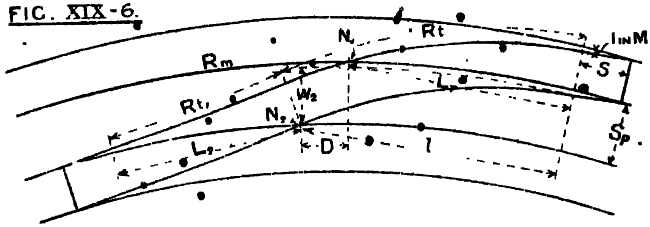
$$= 2(62'\ 2'' + 15') + 15'\ 0''$$

$$= \underline{\underline{169'\ 4''}}$$

When the mains are sharply curved the different radii of the inner and outer rails should be allowed for.

Prob. 5 (Fig. 6). Crossover between two curved parallel mains of Radius "R" and at a given distance (Sp) apart. Particulars of Similar Turnout given, and its curve to extend to the crossing (N_2) of the Contra Turnout. Required the Particulars of Contra Turnout and Distance (D) apart of crossings.

FIG. XIX-6.



When the curve of the mains is fairly sharp, the arrangement of the previous problem, *i.e.*, crossings of equal angle, causes the lead of the contra turnout to become unnecessarily long.

To reduce N_2, and therefore its lead, we may allow the curve of the Similar Turnout to extend to N_2.

The calculation is then similar to that of obtaining the second crossing in a double line junction.

EXAMPLE.—Mains; 14 chains $= R_m$ S. Turnout 7 chains $= R_t$. Space 6′ 5½″. Switches 15′.

1st.— $R_e = \dfrac{R_m \times R_t}{R_m - R_t}$ (Rule XIII.—4.)

$$= \frac{14 \times 7}{14 - 7} = \underline{\underline{14 \text{ chains}}} = \underline{\underline{924'}}$$

2nd.— $N_1 = \underline{\underline{10}}$ (Table 31 or Rule 3c.)

$L_1 = \underline{\underline{69'\ 3''}}$ (Table 31 or Rule 1c.)

3rd.— $N_2 = M \times \sqrt{\dfrac{R_e - G - \frac{1}{2}W_2}{R_e - G + (2 \times W_2 \times M^2)}}$ (Rule 3c.)

and $W_2 = Sp - H$

$$= 6'\ 5\tfrac{1}{2}'' - 4\tfrac{1}{2}'' = 6'\ 1'' = 6{\cdot}08'$$

$$\therefore N_2 = 40 \times \sqrt{\frac{924 - 4{\cdot}71 - 3{\cdot}04}{924 - 4{\cdot}71 + (2 \times 6{\cdot}08 \times 1600)}}$$

$$= 40 \times \sqrt{\frac{916{\cdot}25}{20375}}$$

$$= 40 \times {\cdot}212$$

$$= \underline{\underline{8{\cdot}48}} \text{ say } \underline{\underline{8\tfrac{1}{2}}}.$$

This is Crossing No. for Contra Turnout.

4th.—Find l.

$$l = 2 \times W \times \frac{M \times N_2}{M + N_2} \qquad \text{(Rule 1c.)}$$

$$= \frac{2 \times 6{\cdot}08 \times 40 \times 8{\cdot}5}{48{\cdot}5}$$

$$= \underline{\underline{85{\cdot}24'}} \text{ say } \underline{\underline{85'\ 3''}}$$

5th.— $D = l - L_1$

$$= 85{\cdot}24 - 69{\cdot}25$$

$$= \underline{\underline{16'}}$$

6th.—As regards the Contra Turnout:—

When $N_2 = 8\frac{1}{2}$:—

$R_e = \underline{\underline{658'}}$ say $\underline{\underline{10 \text{ chains}}}$; Table 31 or Rule 2c.)

and $L_2 = \underline{\underline{60'\ 9''}}$ (Table 31 or Rule 1c.)

7th.— $$R_{t\,1} = \frac{R_m \times R_e}{R_m - R_e} \qquad \text{(Rule XIII.—2.)}$$

$$= \frac{14 \times 10}{14 - 10} = \underline{\underline{35 \text{ chains}}}; \text{ approx.}$$

8th.—Length overall of crossover:—

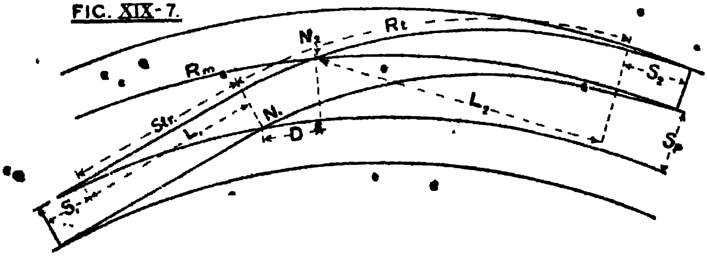
$$= L_1 + D + L_2 + 2S$$

$$= 69'\ 3'' + 16'\ 0'' + 60'\ 9'' + 30'\ 0''$$

$$= \underline{\underline{176'\ 0''}}$$

Prob. 6 (Fig. 7). Crossover between curved parallel mains of Radius "R." Radius of Similar Turnout given, and the Contra Turnout to be Straight. Required the Particulars of the Turnouts and Distance (D) apart of crossings.

FIG. XIX-7.



As the Radius of the mains decreases, the Contra Turnout may be flattened, and it eventually becomes reasonable to make it Straight as in this case.

EXAMPLE.—Mains, 10 chains $= R_m$. Similar Turnout, 7 chains $= R_t$. Space $= 6'\ 5\frac{1}{2}''$.

1st.—With the Contra Turnout Straight, and the Main 10 chains radius, the Lead and Crossing will be for all practical purposes the same as if the reverse were the case, *i.e.*:—

$$\text{With } S_1 = \underline{\underline{15'}} \qquad \text{(Table 29.)}$$
$$N_1 = \underline{\underline{8\tfrac{1}{2}}} \qquad \text{(Table 31.)}$$
$$L_1 = \underline{\underline{60'\ 9''}} \qquad \text{(Table 31.)}$$

2nd.—As regards the Similar Turnout:—

$$R_e = \frac{10 \times 7}{10 - 7} = 23\tfrac{1}{3} \text{ chains} = \underline{\underline{1540'}}$$
$$S_2 = \sqrt{R_e} \times \cdot 581 \qquad \text{Rule XI.–3.}$$
$$= \sqrt{1540} \times \cdot 581$$
$$= \underline{\underline{22{\cdot}80'}}$$

or say $S_2 = \underline{\underline{21'}}$ (See Chapter XI.)

Consulting Table 31; when $S = 21'$ and $R = 1551'$:—

$$N_2 = \underline{\underline{13}}$$
$$L_2 = \underline{\underline{91'\ 5''}}$$

3rd.—By Rule XIX.–3:—

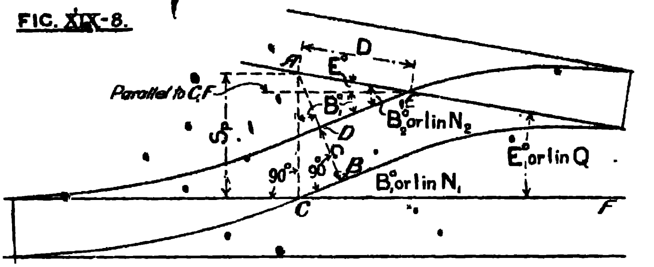
$$D = \left\{\left(6{\cdot}46 - 4{\cdot}71 + \frac{4{\cdot}71}{2 \times 13^2}\right) \times \left(\frac{2 \times 8{\cdot}5 \times 13}{8{\cdot}5 + 13}\right)\right\} - \frac{4{\cdot}71}{13}$$
$$= (1{\cdot}75 + \cdot 014) \times (10{\cdot}28) - \cdot 36$$
$$= \underline{\underline{17{\cdot}77'}} \text{ say } \underline{\underline{17'\ 9''}}$$

Prob. 7 (Figs. 8 and 9). Crossover between two straight mains which are inclined towards each other at a given angle (E°) or 1 in Q by C.L. Measure.

To make a defined problem we must have given: the Angle of the first crossing (B_1 or 1 in N_1) and the Space (Sp) between mains at some point, preferably at B_1, on the square. We shall then require the Angle of the second crossing (B_2 or 1 in N_2), and the Distance apart of crossings (D) which will be taken as shown on the figures.

Case 1 (Fig. 8). When the mains converge from B_1 to B_2.

FIG. XIX-8.



Following the angles in the figure, we see that:—

$$B_2 = B_1 + E$$

Converting this to C. L. Measure we get:—

$$N_2 = \frac{N_1 \times Q}{N_1 + Q}$$ **Rule XIX.—5.** (From Rule XII.—24.)

To derive a rule for D; drop a perpendicular $A\,B$ and then from the triangle $A\,B\,C$,

$$A\,D + G = Sp \times \text{Cos } B_1$$

$$\therefore A\,D = Sp \text{ Cos } B_1 - G$$

and from the triangle $A\,D\,E$,

$$D = AD \times \text{Cosec } B_2$$

$$\therefore D = (Sp \text{ Cos } B_1 - G) \times \text{Cosec } B_2$$

Rule XIX.–6.

Which converted to C. L. Measure becomes:—

$$D = \left(Sp - \frac{Sp}{2N_1^2} - G\right)^* \times \left(N_2 + \frac{I}{4N_2}\right)$$ **Rule XIX.–7.**

EXAMPLE.—Angle between Mains 1 in $31\frac{1}{2}$. $N_1 = 9$. $Sp = 6'\ 5\frac{1}{2}''$.

1st.— $N_2 = \dfrac{9 \times 31\frac{1}{2}}{9 + 31\frac{1}{2}} = 7$ (Rule XIX.–5.)

2nd.— $D = (6{\cdot}46 - \dfrac{6{\cdot}46}{162} - 4{\cdot}71) \times (7 + \frac{1}{28})$ (Rule XIX.–7.)

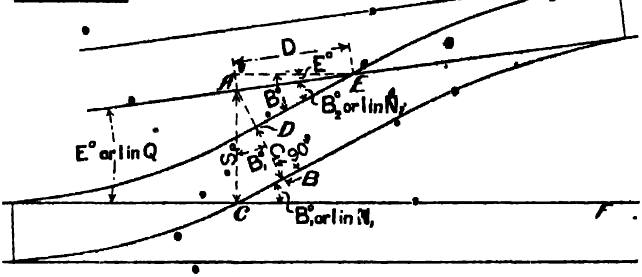
$$= 1{\cdot}71 \times 7\tfrac{1}{28}$$

$$= \underline{\underline{12{\cdot}03'}}$$

* See page 240

Case 2 (Fig. 9). When the mains diverge from B_1 to B_2.

FIG. XIX-9.



Working on similar lines we get :—

$$B_2 = B_1 - E$$

or, $N_2 = \dfrac{N_1 \times Q}{Q - N_1}$ **Rule XIX.-8.** (From Rule XII.-25.)

The same rules for D apply as in Case 1.

Example.—Angle between mains 1 in $31\frac{1}{2}$, $N_1 = 7$, Sp.. $= 6' \; 5\frac{1}{2}''$

1st.— $N_2 = \dfrac{7 \times 31\frac{1}{2}}{31\frac{1}{2} - 7} = 9$ (Rule XIX.-8.)

2nd.— $D = \left(6{\cdot}46 - \dfrac{6{\cdot}46}{98} - 4{\cdot}71\right) \times \left(9 + \dfrac{\cdot 1}{36}\right)$ (Rule XIX.-7.)

$= 1{\cdot}68 \times 9\frac{1}{36}$

$= \underline{\underline{15{\cdot}17'}}$

CHAPTER XX.

THE DIAMOND AND THE OBTUSE CROSSING.

The technical terms in connection with this subject will be used in accordance with the following definitions.

Diamond.

The diamond-shaped portion of track formed by one track crossing another, and comprising two acute or vee crossings and a pair of obtuse or diamond crossings.

Obtuse Crossing or Diamond Crossing.

One of the pair or "set" of crossings formed where the two rails of one track cross the two rails of another track.

The usual naming of the parts of the obtuse crossing is shown by Fig. XX.-1.

Other notes applying to the "diamond" occur in the chapters on the Double Line Junction and the Slip Road

It must first be repeated that roads through diamond crossings should be as straight as practicable under the circumstances.

Danger lies in the gap between the diamond crossing points, and from this arises the Board of Trade Rule limiting the fixed diamond at an angle of 1 in 8. When the diamond is on the curve, severe wear occurs on the diamond points and wing rail, and the tendency for the wheels to get to the wrong side of the point is increased.

There are at least two mechanical ways of remedying this tendency. One is the use of "switch diamonds," which are worked like a set of switches, and in running lines, all the requirements applying to facing points are rendered necessary. By this device diamonds flatter than 1 in 8 are introduced in many cases.

Another device especially useful in diamonds on the curve is the "Williams Diamond Point Protector," in which blocks are caused to rise in the gaps and close them.

Constructional details of these appliances are beyond the scope of this work. and the reader is referred to those firms or railways who manufacture them.

The measurement of the angle of an obtuse crossing will naturally be taken by the same method of measurement as used in the case of the acute crossing; in this work Centre Line Measure being used.

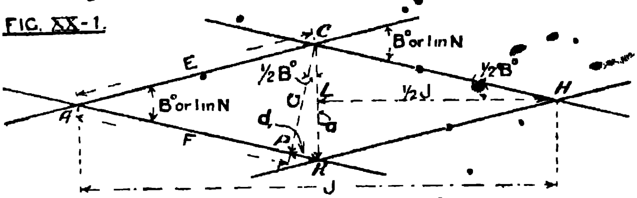
The angle of an obtuse crossing laid in the track, may be taken by marking two points, one on either side of the elbow, where the gauge lines of elbow and point rails are 6 inches apart. The distance between these points in feet will give the Number of the Crossing.

CALCULATIONS.

These will appertain to the problems in connection with the dimensions of the diamond-shaped figure formed by the intersecting tracks.

Prob. 1 (Fig. XX.-1). Given the angle of the Diamond Crossing (B or 1 in N) between two straight tracks. Required the length of the diamond sides (E); the distance between Vee and Diamond Crossings measured along the main (F); the distances d and d_1, and the distance apart of Vee Crossings (J).

FIG. XX-1.



It is obvious that the Vees are of the same angle as the diamond, and referring to Fig. XX.-1, by elementary trigonometry, it will be seen that:—

$E = G \times \text{Cosec } B$ **Rule XX.—1.**

$F = G \times \text{Cot } B$ **Rule XX.—2.**

$d = G \times \text{Sec } \frac{1}{2} B$ **Rule XX.—3.**

$d_1 = G \times \text{Tan } \frac{1}{2} B$ **Rule XX.—4.**

$\frac{1}{2} J = N \times d$

$\therefore J = 2 \times N \times G \times \text{Sec } \frac{1}{2} B$ **Rule XX.—5.**

Converting to C.L.M., see Chapter XII., we get:—

$E = G \times \left(N + \frac{1}{4N} \right)$ **Rule XX.—6.**

$F = G \times \left(N - \frac{1}{4N} \right)$ **Rule XX.—7.**

$$d = G + (G \div 8N^2)^* \qquad \textbf{Rule XX.—8.}$$

$$d_1 = G \div 2N \qquad \textbf{Rule XX.—9.}$$

$$J = 2 \times N \times G \times \left(1 + \frac{1}{8N^2}\right)^* \qquad \textbf{Rule XX.—10.}$$

EXAMPLE.—1 in 8 Crossings, 4′ 8½″ Gauge.

1st.—$E = 4' \, 8\tfrac{1}{2}'' \times \left(8 + \frac{1}{4 \times 8}\right)$ Rule XX.—6.

$$= 4' \, 8\tfrac{1}{2}'' \times 8\tfrac{1}{32}$$

$$= 37' \, 8'' + 1\tfrac{3}{4}'' = \underline{\underline{37' \, 9\tfrac{3}{4}''}}$$

or in decimals $E = 4{\cdot}708' \times 8{\cdot}03125$

$$= \underline{\underline{37{\cdot}81'}}$$

2nd.—$F = 4' \, 8\tfrac{1}{2}'' \times \left(8 - \frac{1}{4 \times 8}\right)$ (Rule XX.—7.)

$$= 4' \, 8\tfrac{1}{2}'' \times 7\tfrac{29}{32}$$

$$= 37' \, 8'' - 1\tfrac{3}{4}'' = \underline{\underline{37' \, 6\tfrac{1}{4}''}}$$

3rd.—$d = 4' \, 8\tfrac{1}{2}'' + \{4' \, 8\tfrac{1}{2}'' \div (8 \times 8 \times 8)\}$ (Rule XX.—8.)

$$= 4' \, 8\tfrac{1}{2}'' + (4' \, 8\tfrac{1}{2}'' \div 512)$$

$$= \underline{\underline{4' \, 8\tfrac{5}{8}''}}$$

4th.—$d_1 = 4' \, 8\tfrac{1}{2}'' \div 16$ (Rule XX.—9.)

$$= \underline{\underline{3\tfrac{1}{2}''}}$$

5th.—$J = 2 \times 8 \times 4' \, 8\tfrac{1}{2}'' \times \left(1 + \frac{1}{8 \times 64}\right)$ (Rule XX.—10.)

$$= 75' \, 4'' \times 1\tfrac{1}{512}$$

$$= 75' \, 4'' + \frac{75' \, 4''}{512}$$

$$= 75' \, 4'' + 1\tfrac{3}{4}''$$

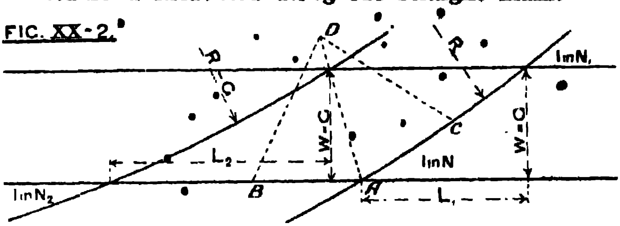
$$= \underline{\underline{75' \, 5\tfrac{3}{4}''}}$$

Distances E and F for various angles are given in Table 38.
,, d ,, d_1 ,, ,, ,, ,, 44.
,, J ,, ,, ,, ,, 45.

* $1 + \frac{1}{8N^2}$ is a near approximation to Sec. ½ B, and may be arrived at by a study of Fig. XII.–4.

Prob. 2 (Fig. XX.–2). Given the angle (1 in N) of the diamond crossing between a straight main and a curved through road of given Radius (R) on outer rail. Required the angles (N_1 and N_2) of the Vee crossings and their distances (L_1 and L_2) from the diamond measured along the straight main.

FIG. XX-2.



One way of solving this problem is by the system described for Double Junctions, but as a preliminary, widths W must be obtained from an imaginary line parallel to the main.

Otherwise, the problem may be solved by the General Rules (Table 27), but in place of the switch angle number "M," we must substitute N. The W will be equal to the Gauge.

Then if N_1 denotes the crossing of wider angle than N, and L_1 its distance,

$$N_1 = N \times \sqrt{\frac{R - \frac{1}{2}G}{R + (2 \times G \times N^2) + \frac{1}{2}G}}$$ **Rule XX.—11.** (From Rule 3c.)

$$\text{and } L_1 = 2\,C \times \frac{N \times N_1}{N_1 + N}$$ **Rule XX.—12.** (From Rule 1c.)

To find the Number (N_2) of the crossing of flatter angle than N, the Radius to be taken will be R – G, and we get:—

$$N_2 = N \times \sqrt{\frac{R - G + \frac{1}{2}G}{R - G - (2 \times G \times N^2) - \frac{1}{2}G}} \text{ *}$$

$$= N \times \sqrt{\frac{R - \frac{1}{2}G}{R - (2 \times G \times N^2) - 1\frac{1}{2}G}}$$ **Rule XX.—13.**

EXAMPLE.—N = 6 R = 800′

* Obtained by first stating N in terms of N_2 and then transposing to give N_2.

1st.—
$$N_1=6\times\sqrt{\frac{800-2\cdot35}{800+(2\times4\cdot708\times36)+2\cdot35}} \quad \text{(Rule XX.—11.)}$$
$$=6\times\sqrt{\frac{797\cdot65}{800+338\cdot98+2\cdot35}}$$
$$=6\times\sqrt{\cdot6989}$$
$$=6\times\ \cdot836$$
$$=\underline{\underline{5\cdot016}}$$

2nd.—
$$L_1=2\times4\cdot708\times\frac{6\times5\cdot016}{6+5\cdot016} \quad \text{(Rule XX.—12.)}$$
$$=\frac{283\cdot384}{11\cdot016}$$
$$=\underline{\underline{25\cdot72'}}$$

3rd.—
$$N_2=6\times\sqrt{\frac{800-2\cdot35}{800-(2\times4\cdot708\times36)-7\cdot06}} \quad \text{(Rule XX.—13.)}$$
$$=6\times\sqrt{\frac{797\cdot65}{453\cdot96}}$$
$$=6\times\sqrt{1\cdot7571}$$
$$=6\times1\cdot326$$
$$=\underline{\underline{7\cdot956}}$$

4th.—
$$L_2=2\times4\cdot708\times\frac{7\cdot956\times6}{7\cdot956+6} \quad \text{(Rule XX.—12.)}$$
$$=\underline{\underline{32\cdot21'}}$$

Prob. 3. Given the angle (1 in N) of the diamond crossing between two curved tracks of given Radii. Required the angles of the Vee crossings and their distances from the diamond crossing.

This problem may be solved in a similar manner to the above by first obtaining the Equiv. Radius and using this as the R in the Rules. The distances L, however, to be exact, should be measured along a tangent to the main at the diamond crossing.

If in this problem, the length of the Diamond Sides are required, solutions may be obtained by calculation such as given by W. L. Webb and other American writers, but the only use of such intricate calculations would be as exercises for advanced mathematical students. In practice, the diamond sides would be measured from a drawing or lining out, either after calculating the distances as mentioned above or from the original data.

It may be noted that if the curves are of the same radius and flexure, the diamond sides will be equal and the Vee angles equal to the diamond angle.

Where there is any curve in the roads, after the position of one of the diamond crossings is defined, the position of the opposite one should be fixed by bisecting the obtuse angle as described in Chapter V. and repeated in Fig. XX.–2, but the equal base lines *A B* and *A C* should not exceed the gauge.

This procedure is sufficiently accurate in all practical cases, strictly, however, the two crossing points should lie on line *H J*. Fig. XX.–3, which is the bisector of the angle *G E F* formed by two tangents *E G* and *E F* to the centre lines of the four foot at their intersection *E*.

FIG. XX-3.

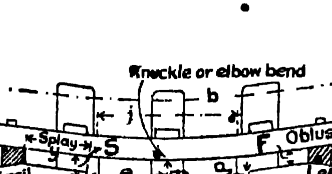


FIG. XX-4.

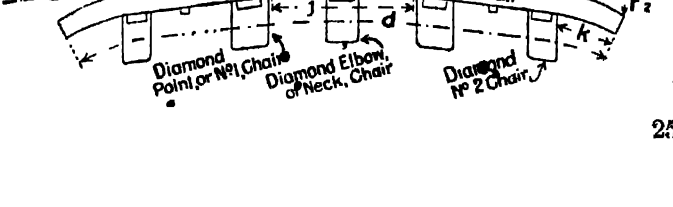


FIG. XX-5.

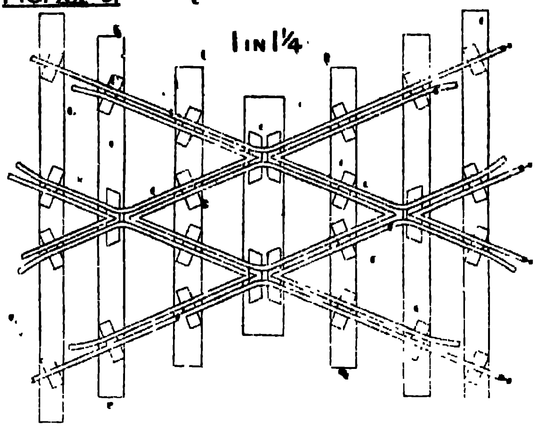


FIG. XX-6.

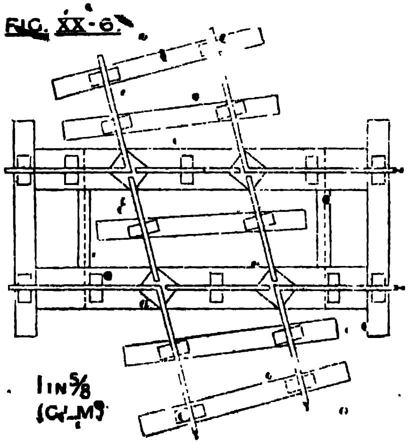
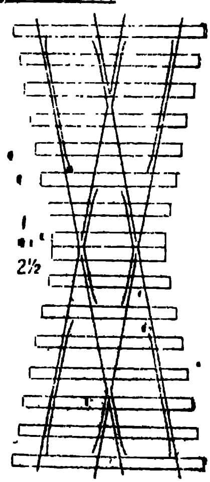


FIG. XX-7.



DETAIL DIMENSIONS OF DIAMOND CROSSINGS.
(Figs. XX.—4, 5, 6 and 7.)

The preliminary remarks made with regard to the details of Vee crossings apply to the following:—

Flattest angle of fixed diamond crossing to be used.	Usual. The Board of Trade limit of 1 in 8.
Angles available from stock.	
Angles of switch diamonds which may be used.	
Angles in which the point rail forms the wing rail of the Vee as in Fig. 5.	
Length of diamond point rail (a).	
Length of elbow or wing rail (b).	
Length of obtuse check rail (d).	
Width of nose of point rail (p).	
Normal Flangeway Clearance from point rail to elbow and check rails (F).	
Clearance at ends of check rail (F_2).	
Distance of nose of point rail from centre of elbow bend (e).	$e = (p+F) \times N$. Rule XX.—14. or in words:— To width of nose add flangeway and multiply by Number of crossing.
Distance apart of points (2 × e.)	See Table [illegible].

(continued.)

DETAIL DIMENSIONS OF DIAMOND CROSSINGS.—*continued.*

Gap between wing and check rails at elbow (g)................	
Length of wing and check rail elbow bends (j).	j = (g − F) × 4 × N. **Rule XX.–15.** **or in words: Deduct clearance from gap and multiply by four times Number of crossing**
Length of splay of point rail (y). (See Table 43).	**y = (Width of rail − p) × N. Rule XX.—16. or in words:— From width of rail deduct width of nose and multiply by Number of crossing.**
Length of bend at end of check rail (k).	

CHAIRING OF DIAMONDS.

Refer also to "List of Chairs" Chap. III.

Angles at which a Twin Diamond Point Chair ( )* is used to receive both diamond points as in Fig. 6..............	Usually 1 in ½ (a right angle) to 1 in ¾.
Angles at which two Diamond Point Chairs ( ) but no other special chairs beyond Check Chairs ( ) are used, as in Fig. XX.–5...........................	
Angles at which two Diamond No. 2 Chairs ( ) are used as in Fig. 4..........	
Angles at which Diamond Elbow Chair ( ) is used as in Fig. 4	
Angles at which in yard work, no check or wing rails need be used and special timbering may be introduced as suggested in Fig. 6.	Usually 1 in ½ C.L.M. (a right angle) to 1 in ¾ C.L.M. (67°–23').
Angles at which the obtuse check rail may cease to extend to the Vee as it does in Fig. 5 and usual arrangements introduced as in Fig. 7.	Usually about 1 in 2½.
Position of crossing blocks	

*** The spaces in brackets are for the reception of the distinguishing marks on the chairs.**

DIAMOND CHAIRS.—SPREAD, POSITION, ETC.
(Refer to Chapter III. and Figs. III. 7, 11, and 6.)

Spread (S) = Width between gauge lines of diamond point rail and elbow rail, at centre of chair in question.

Distance of chair (centre) from fine point of obtuse crossing (centre of elbow bend) is S multiplied by No of crossing. The S marked on Fig III. 7 applies to a Vee crossing; to apply to a Diamond, F must be added to it (see Fig. XX.-4).

Section lbs. per yard.	Chairs for Angles of .—	SPREAD (S).		
		Point Chair.	Chair No. 2.	Chair No 3.

CHAPTER XXI.

SCISSORS CROSSOVER ROADS.

A Set of Scissors Crossover Roads may be defined as a pair of crossover roads which intersect each other. They are sometimes named " Double Crossovers."

The use of "Scissors" is not advisable where there is sufficient room to put in two separate crossovers, because more crossings are needed and heavier maintenance is entailed.

Types of Scissors Crossovers.

Type 1, Fig. 1, page 262 (Diamond crossings in 6ft.).—This is useful when the length taken by the crossovers in both mains, *i.e.*, the length overall, must be at a minimum.

The portion through the crossings should be straight, though when the mains are curved, this portion may have to be curved to save distance in crossing the six foot.

Type 2, Fig. 2, page 264 (One Diamond crossing in 4ft., points to outside of crossings).—This may be used when there is more length available in one main than the other, and where the space between mains will not allow for chairs in Type 1

Type 3, Fig. 3, page 267 (One Diamond crossing in 4ft., points to inside of crossings).—This gives a minimum length taken up in one of the mains. Only four Vee crossings are needed with this type.

Tables 40, 41, and 42 contain examples of Scissors Crossovers of these three types.

Practical Details.

In the following notes, when a wing rail is mentioned as acting as a check rail, it must be of the type known as a "parallel wing."

Type 1.—As regards chairing, the critical dimensions are those marked x (*Fig. 1*) which should be equal.

The minimum for x may be as little as 1′ 0¼″ (gauge lines), but usually 1′ 1¼″ should be allowed. Thus a scissors of this type usually needs a space between mains of 6′ 11½″ (gauge lines) or 6′ 3″ clear. If the clear space is under 6′ 4″ special chairs will probably be required.

An idea of the checking is given by Fig. 4.

If the space between mains is widened, the checking length marked c will diminish, as shown in Fig. 8, and the space must not be increased so that c becomes too small for safety.

FIG. XXI-4.

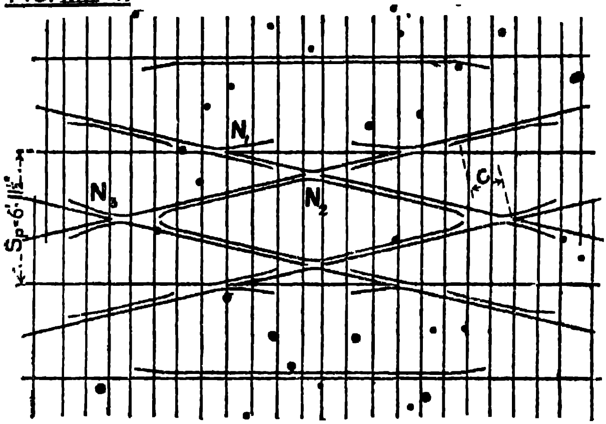
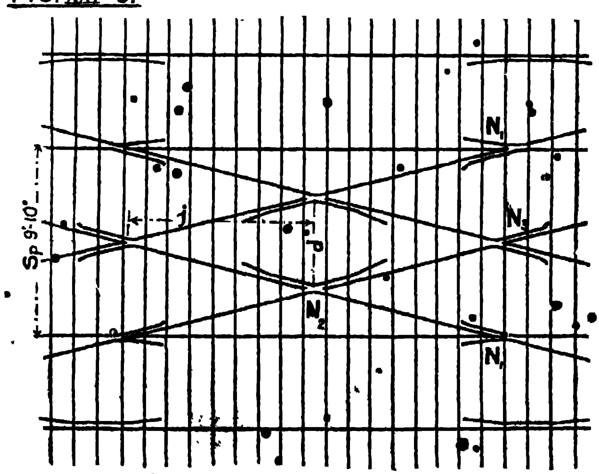


FIG. XXI-5.



With $8\frac{1}{2}$ crossings and d = 7″ and c = 2′ 9″, the maximum space (Sp) will be found to be about 7′ 8″ (gauge lines). If the space is further increased, it must be increased so much as will enable the wing rails to act as check rails as shown in Fig. 5, and in detail in Fig. 9. Efficient checking can be obtained in this way with a space of 9′ 10″ (gauge lines) or 9′ $4\frac{1}{2}$″ clear. With a space wider than 9′ 10″ there will be no checking difficulties.

FIG. XXI-6.

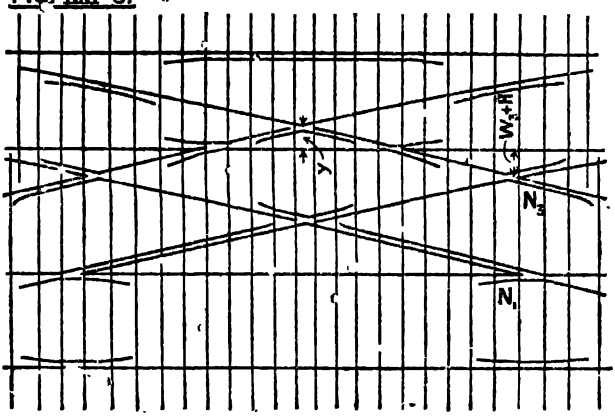
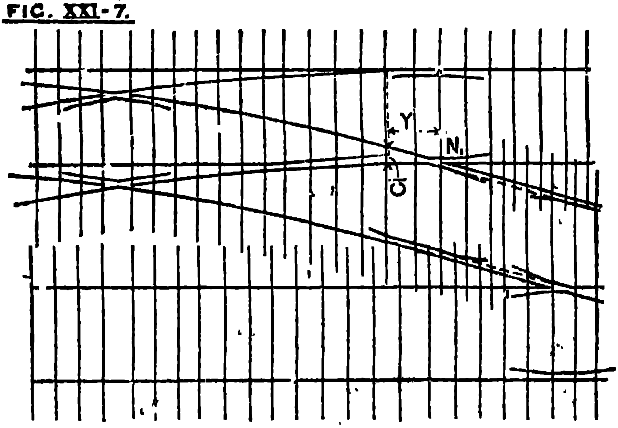


FIG. XXI-7.



With a fairly narrow space, such as 6′ 6″ between mains, the two acute crossings N_1 and the elbow rail of the diamond are made in one piece. This is known as a "triple crossing" or sometimes as a "fiddle back."

Type 2.—The critical dimensions as regards chairing will be y and $W_3 + H$ (Fig. 2); the usual minimum for either being 1′ 1½″ (gauge lines), but 1′ 4½″ is advisable.

FIG. XXI-8.

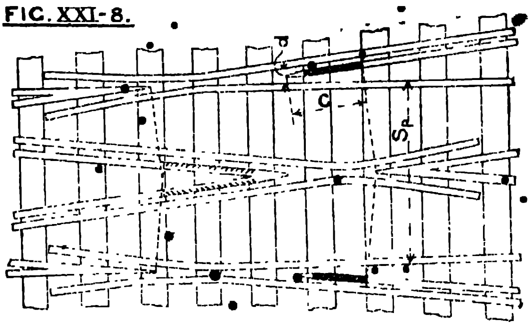
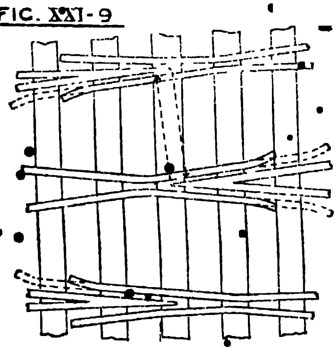


FIG. XXI-9



The checking is shown in Fig. 6. It will be noticed that crossings N_1 and N_3 have been kept opposite to each other, to ease checking difficulties. Fig. 9 will serve to show how the wings act as checks in this case. If N_1 and N_3 are not opposite, N_3 must be one or more timber spaces nearer the diamond crossing, or checking will fail.

Type 3, Fig. 7.—The dimension Cl must allow for throw and clearance of switch toe as in slip roads, 9″ being a usual minimum.

The necessary length of check in front of crossing N_1 must also be allowed for.

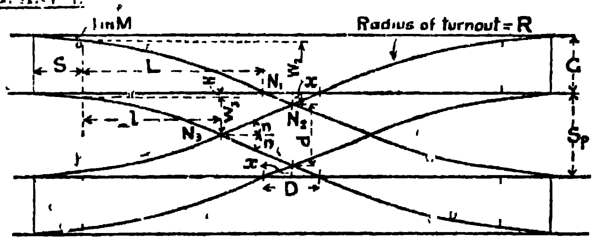
This type is limited in its application, because unless we use short switches to throw the diamond crossing well into the 4ft., its angle becomes flatter than 1 in 8.

CALCULATIONS.

In all types of scissors the rules and tables for single turnouts and crossover roads apply to the particulars in connection with the four crossings on the mains.

Type 1, Fig. 1.—The dimensions additional to those of the turnouts are:—Angle of Obtuse crossing (N_2); Angle of Acute crossing in six foot (N_3); Lengths l and D.

FIG. XXI-1.



Example.—Main lines straight; Cross roads straight across six foot; Angle of Turnout crossings (N_1) $7\frac{1}{4}$; Switches 12′. Space (gauge lines) 6′ $9\frac{1}{2}$″ (Sp).

1st.—When $N_1 = 7\frac{1}{4}$:—

R = 483′ (Table 31.)
L = 51′ 2″ (Table 31.)
D = 14′ 9″ (Table 38) or Rule XIX.–2

2nd.—Find d.

Because the track is straight across the six foot, the angle of the obtuse ($B°_2$) is twice the angle of the acute crossings ($B°_1$).

$$d = G \times \text{Sec } \tfrac{1}{2} B_2 \qquad \text{(Rule XX.–3.)}$$
$$= G \times \text{Sec } B_1$$
$$= G \times \left(1 + \frac{1}{2N_1^2}\right) \qquad \text{(approx.)}$$
$$= G + \frac{G}{2 \times N_1^2} \qquad \textbf{Rule XXI. 1.}$$

In the example; $d = 4{\cdot}708 + \dfrac{4{\cdot}708}{2 \times 7{\cdot}25^2}$

$$= 4{\cdot}708 + \frac{4{\cdot}708}{105{\cdot}12}$$
$$= 4{\cdot}708 + {\cdot}045$$
$$= \underline{\underline{4{\cdot}753'}}$$

3rd.— $$x = \frac{Sp - d}{2} \qquad \textbf{Rule XXI.—2.}$$
$$= \frac{6{\cdot}792 - 4{\cdot}753}{2}$$
$$= \underline{\underline{1{\cdot}019}} = 1'\ 0\tfrac{1}{4}''$$

4th.— $$W_3 = \tfrac{1}{2}\ Sp - H \qquad \textbf{Rule XXI.—3.}$$
$$= \tfrac{1}{2}\ (6'\ 9\tfrac{1}{2}'') - 4\tfrac{1}{2}''$$
$$= \underline{\underline{3'\ 0\tfrac{1}{4}''}} = 3{\cdot}0208$$

5th.— $$n = M \times \sqrt{\frac{R - G - \frac{1}{2} W_3}{R - G + (2 \times W_3 \times M^2)}} \qquad \text{(Rule 3c.)}$$
$$= 32 \times \sqrt{\frac{483 - 4{\cdot}71 - 1{\cdot}51}{478{\cdot}3 + (2 \times 3{\cdot}0208 \times 1024)}}$$
$$= 32 \times \sqrt{\frac{477}{478{\cdot}3 + 6186{\cdot}6}}$$
$$= 32 \times \frac{\sqrt{477}}{\sqrt{6664{\cdot}9}}$$
$$= \frac{32 \times 21{\cdot}84}{81{\cdot}64}$$
$$= \underline{\underline{8{\cdot}56}}$$

* $\text{Sec } B = \dfrac{1}{\text{Cos } B}$ and then from Rule XII.–15, $\text{Sec } B = 1 + \dfrac{1}{2N^2}$ (approx.)

6th.— $N_3 = n \div 2$ (approx.) (from Rule XII.–24.)

$$= 8.56 \div 2$$

$$= \underline{\underline{4.28}}$$

7th.— $$l = W_3 \times \frac{2 \times M \times n}{M + n} \quad \text{(Rule 1c.)}$$

$$= 3.0208 \times \frac{2 \times 32 \times 8.56}{40.56}$$

$$= \frac{1654.916}{40.56}$$

$$\underline{\underline{40.80}} = 40'\ 9\tfrac{1}{2}''$$

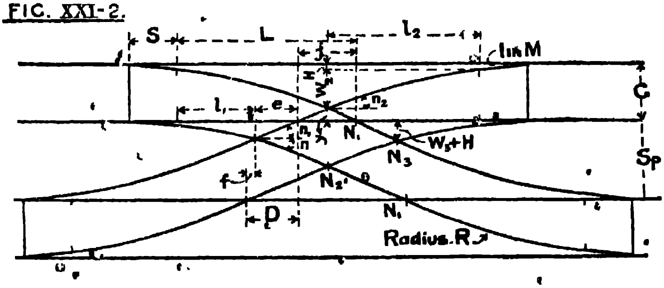
8th.— $N_2 = N_1 \div 2$ (approx.)

$$= 7.25 \div 2$$

$$= \underline{\underline{3.62}}$$

Type 2, Fig. 2.—For the length of the scissors overall to be at a minimum, x must be as small as possible. In order to simplify the timbering and checking, however, an arrangement often adopted is for the crossing in the six foot (N_3) to lie opposite one of the crossings on the main (N_1).

FIG. XXI-2.



Example.—Main lines straight; cross roads straight between mains. Angle of Turnout crossings (N_1), 1 in $8\frac{1}{2}$. Switches 15ft. Space (Sp) 6′ $5\frac{1}{2}''$ (gauge lines). Crossing in six foot (N) to lie opposite a crossing on main (N_1).

1st.—When $N_1 = 8\frac{1}{2}$ then:—

$R = 658'$ (Table 31.)

$L = 60'\ 9''$ (Table 31.)

$D = 14'\ 6\frac{1}{2}''$ (Table 38) or Rule XIX. 2

2nd.—To cause N_3 and N_1 to lie on the same timber and yet to keep N_3 slightly nearer to the obtuse crossing to improve the checking, make $W_3 + H = 1'\ 7'' = 1{\cdot}583'$ as a trial.

$$W_3 = 1'\ 7'' - 4\tfrac{1}{2}''$$
$$= 1'\ 2\tfrac{1}{2}'' = 1{\cdot}208'$$

3rd.—Because the tracks are straight:—

$$c = (W_3 + H) \times N_1 \text{ (in Right Angle Measure)}$$
$$= 1{\cdot}583 \times 8{\cdot}471 \text{ (see Table 30.)}$$
$$= 13{\cdot}41' = 13'\ 4\tfrac{7}{8}''$$

4th.— $$f = D - c$$
$$= 14'\ 6\tfrac{1}{2}'' - 13'\ 4\tfrac{7}{8}''$$
$$= 1'\ 1\tfrac{5}{8}''$$

This is the distance between fine points of N_3 and N_1. Allowing for fine point distances, it will be seen that the blunt noses will lie upon the same timber, and yet N_3 will be nearer to the obtuse than N_1. The $W_3 + H$ of $1'\ 7''$ will therefore be suitable.

5th.— $$n = M \times \sqrt{\frac{R - G}{R - G + (2 \times W_3 \times M^2)}} \quad \text{(Rule 3c.)}$$

$$= 40 \times \sqrt{\frac{653}{653 + (2 \times 1{\cdot}208 \times 1600)}}$$

$$= 40 \times \frac{\sqrt{653}}{\sqrt{4519}}$$

$$= 40 \times \frac{25{\cdot}55}{67{\cdot}22}$$

$$= 15{\cdot}204$$

6th. $l_1 = 2W_3 \times \frac{M \times n}{M + n}$ (Rule 1c.)

$= \frac{2 \times 1 \cdot 208 \times 40 \times 15 \cdot 204}{55 \cdot 204}$

$= \frac{1469 \cdot 314}{55 \cdot 204}$

$= \underline{\underline{26 \cdot 62}} = 26' \ 7\frac{1}{2}''$

7th.— $n_1 = N_1 = 8 \cdot 50$

and $N_3 = \frac{n \times n_1}{n + n_1}$ (Rule XII.–24.)

$= \frac{15 \cdot 20 \times 8 \cdot 50}{23 \cdot 70}$

$= \underline{\underline{5 \cdot 45}}$

8th.— $j = L - c - l_1$

$= 60 \cdot 75 - 13 \cdot 41 - 26 \cdot 62$

$= \underline{\underline{20 \cdot 72}} = 20' \ 8\frac{1}{2}''$

9th.— $l_2 = L - \frac{1}{2} j$

$= 60 \cdot 75 - 10 \cdot 36$

$= \underline{\underline{50 \cdot 39'}} = 50' \ 4\frac{1}{2}''$

10th.— $W_2 = \frac{l_2^2}{2R} + \frac{l_2}{M}$ (Rule 11d.)

$= \frac{50 \cdot 4^2}{1316} + \frac{50 \cdot 4}{40}$

$= 1 \cdot 930 + 1 \cdot 260$

$= \underline{\underline{3 \cdot 190'}}$

11th.— $y = G - W_2 - H$

$= 4 \cdot 708 - 3 \cdot 190 - \cdot 375$

$= \underline{\underline{1 \cdot 143'}} = 1' \ 1\frac{3}{4}''$ (which will just suffice.)

12th.— $n_2 = M \times \sqrt{\dfrac{R}{R + (2 \times W_2 \times M^2)}}$ (Rule 3c.)

$= 40 \times \sqrt{\dfrac{658}{658 + (2 \times 3{\cdot}190 \times 1600)}}$

$= 40 \times \dfrac{\sqrt{658}}{\sqrt{10866}}$

$= \dfrac{40 \times 25{\cdot}65}{104{\cdot}2}$

$= \underline{\underline{9{\cdot}84}}$

13th.— $N_2 = n_2 \div 2$ (approx.)

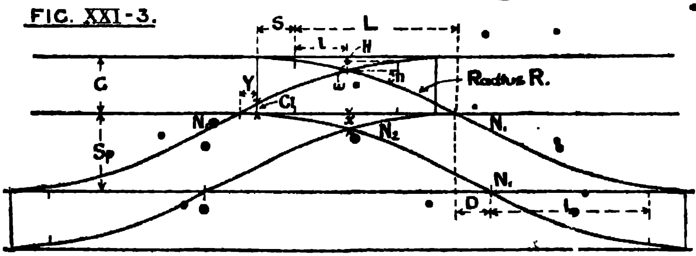
$= 9{\cdot}84 \div 2$

$= \underline{\underline{4{\cdot}92}}$

Type 3, Fig. 3.—In this type, as previously mentioned, there are difficulties in keeping the diamond crossing to a limit of 1 in 8. It will be found that the switches must either be short or curved, so that they leave the main with more divergence than is usual in main line work. Also the turnout curves have to be rather small in radius.

Table 42 will give an idea of the results to be expected.

FIG. XXI-3.



EXAMPLE.—Turnout Radii 9 chains = 594′, Switch 12′, H $4\frac{1}{2}''$; therefore M = 32; Cl 9″; Sp 6′ $5\frac{1}{2}''$ (gauge lines).

1st.— $N_1 = \underline{\underline{8{\cdot}00}}$ (See Table 31.)

2nd.— $L = \underline{\underline{55' \ 5''}}$ (See Table 31.)

3rd.— $D = \underline{\underline{13' \ 7\frac{1}{2}''}}$ (See Table 38.)

4th.— $Y = Cl \times N_1$

$= 9'' \times 8 = \underline{\underline{6' \ 0''}}$

5th.— $$l = \frac{L - S - Y}{2}$$

$$= \frac{55' \, 5'' - 12' - 6'}{2}$$

$$= \underline{\underline{18' \, 8\tfrac{1}{2}''}} = \underline{\underline{18{\cdot}71'}}$$

6th.— $$w = \frac{l^2}{2R} + \frac{l}{M}$$ (Rule 11d.)

$$= \frac{18{\cdot}71^2}{1188} + \frac{18{\cdot}71}{32}$$

$$= {\cdot}295 + {\cdot}585$$

$$= \underline{\underline{{\cdot}880}} = \underline{\underline{10\tfrac{1}{2}''}}$$

7th.— $$x = w + H$$

$$= 10\tfrac{1}{2}'' + 4\tfrac{1}{2}'' = \underline{\underline{1' \, 3''}}$$

8th.— $$n = M \times \frac{\sqrt{R}}{\sqrt{R + (2 \times w \times M^2)}}$$ (Rule 3c.)

$$= 32 \times \frac{\sqrt{594}}{\sqrt{594 + (2 \times {\cdot}880 \times 1024)}}$$

$$= \underline{\underline{15{\cdot}93}}$$

9th.— $$N = \frac{n}{2} \text{ (approx.)}$$

$$= \frac{15{\cdot}93}{2} = 1 \text{ in } \underline{\underline{7{\cdot}96}}$$

CHAPTER XXII.

THE SLIP ROAD.

A Slip Road is a connecting track between two other tracks at a place where they cross each other

A Double Slip Road connects two intersecting tracks in both directions.

The use of slip roads in a facing direction on main lines should be avoided on account of the difficulty in providing for locking bars, etc , and also because of the sharp curve of the slip rail. The Radii obtainable with various angles of crossings will be found in Table 45

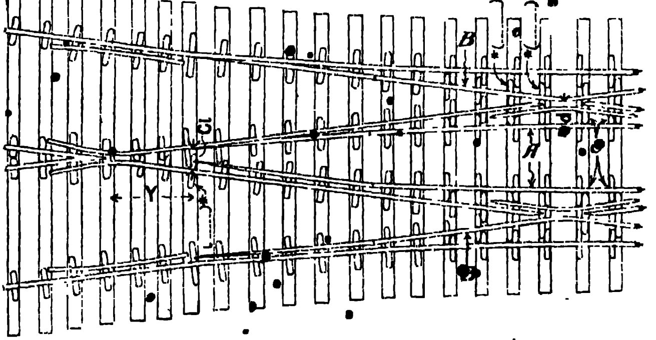
Straight intersecting tracks are best suited for the insertion of slips, and naturally the flatter the crossing angle, the easier the curve of the slip.

If the main track is curved, the cross track should, if possible, be curved to the same extent and flexure. This in the case of a series of slips being required in several parallel mains, will keep the crossing angles constant and the slips of similar dimensions The slips on the inside of the curve will naturally be of less radius than those on the outside of the curve.

PRACTICAL DETAILS.

Fig XXII. 1 shows typical details of double slips in a diamond of 1 in 8.

FIG. XXII-1.



The clearance (Cl) between gauge lines at the switch toe should allow for throw of switch and clearance of open switch from opposite rail. 9ins. is the usual minimum for Cl, though 8ins. may sometimes be made to suffice. Generally Cl should be greater in double than single slips, to allow room for the two slide chairs needed.

The distance Y should allow for suitable timber spacing and for a sufficient length of check rail opposite to the Vee crossing.

The switch length is largely a matter of practical choice, but it should approximately suit the radius as explained in Chapter XI.

The slip rail (*A*), lying between the switches, may generally be made of a stock length, whilst the rail *B* may possibly be one of the special short rails for laying on the inside of curves.

The dimension d between diamond crossing and slip rail (gauge lines) should not usually be less than 1′ 2″. This will allow space for the chairs at *C* which are liable to become foul of the slip rail.

A valuable purpose would be served by designing model arrangements of diamonds with slips, to give good chairing and a minimum of rail cutting.

Special chairs are needed in some cases for use in slip roads. These are marked * on Fig 1, brackets being left to receive the distinguishing marks on such chairs.

CALCULATIONS.

The dimensions concerned and the symbols used will be as follows:—

- B. The Angle of the Diamond and Vee crossings in degrees.
- N. The Number of the same angles (by C.L.M.).
- J. Direct distance apart of Vee crossings (fine points).
- Y. Distance from toe of switches to Vee crossing (fine point).
- Cl. Clearance between gauge lines at toe of switches.
- S. Switch length.
- H. Heel divergence of switch.
- M. Number of switch angle by C.L.M. $M = S \div H$

A°. Switch angle in degrees.

l. Length of slip rail between heels of switches (outer rail of curve).

L. Length of slip road overall.

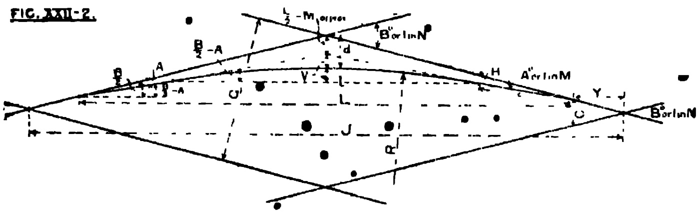
R. Radius of outer rail of slip.

d. Cross distance of diamond crossing from slip rail (gauge lines).

V. Versed sine of slip rail, *i.e.*, on a chord equal to l.

Taking into account the short length of the curve and the fairly flat angles we are dealing with, certain allowable approximations are made.

FIG. XXII-2.



Prob. 1. Given the angle B° or 1 in N between two straight tracks. Required the best practicable slip road which may be laid in the diamond.

EXAMPLE.—N = 8. Cl not less than 9″. S. 15′, *i.e.*, M = 40.

1st.— $J = 2 \times N \times G \times \left(1 + \frac{1}{8N^2}\right)$ (Rule XX.—10.)

$= 2 \times 8 \times 4'\ 8\frac{1}{2}'' \times \left(1 + \frac{1}{512}\right)$

$= \underline{\underline{75'\ 5\frac{3}{4}''}}$ (see Problem XX.—1.)

2nd.— $Y = Cl \times N$ **(Rule XXII.—1.)**

$= 9'' \times 8$

$= \underline{\underline{6'\ 0''}}$

3rd.— $l = J - 2Y - 2S$ **(Rule XXII.—2.)**

$= 75'\ 6'' - 12'\ 0'' - 30'\ 0''$

$= \underline{\underline{33'\ 6''}}$

If we have 33′ rails in stock, l may be made 33′, which will make the slip length overall (L) 63′. Y and Cl must then be calculated afresh on this basis:—

4th.— $Y = \frac{1}{2} J - \frac{1}{2} l - S$ **(Rule XXII.—3.)**

$= 37'\ 9'' - 16'\ 6'' - 15'$

$= \underline{\underline{6'\ 3''}}$.

5th.— $Cl = Y \div N$ **(Rule XXII.—4.)**

$= 6'\ 3'' \div 8$

$= \underline{\underline{9\frac{3}{8}''}}$

6th.— $R = \frac{1}{2} l \div \text{Sin}\left(\frac{B}{2} - A\right)$ **(Rule XXII.—5)** (from Rule C 31).

which converted to C L.M. becomes:—

$$R = \frac{l \times N \times M}{M - 2N} \text{ (approx.)}$$ **(Rule XXII.—6.)**

In the example $R = \frac{33 \times 8 \times 40}{40 - 16}$

$= \frac{10560}{24}$

$= \underline{\underline{440'}}$

7th.— $V = l^2 \div 8R$ (from Rule IX.—1.)

$= 33^2 \div (8 \times 440)$

$= 1089 \div 3520$

$= \underline{\underline{\cdot 309'}} = \underline{\underline{3\frac{3}{4}''}}$

8th.— $d = \left(\frac{L}{2} \div M\right) + V$ (approx.) **(Rule XXII.—7.)**

$= \left(\frac{63}{2} \times \frac{1}{40}\right) + \cdot 309'$

$= \frac{63}{80} + \cdot 309'$

$= \cdot 788' + \cdot 309'$

$= \underline{\underline{1 \cdot 097'}} = \underline{\underline{1'\ 1\frac{1}{4}''}}$

To be more accurate $\left(\frac{L}{2} \div M\right)$ should be multiplied by $\text{Sec}\,\frac{B}{2}$ or $1\frac{1}{8N^2}$

The use of shorter switches than the 15′ of the above example will give a greater "d" and thus more space for the chairs at "C" (Fig. 1). Short switches, however, have the objections noted in Chapter XI.

To retain long switches we may (1) cut away the chair jaws or the bottom flange of the rail at "C". (2) Pack out the switch heel to obtain more divergence. (3) Use curved switches, which are especially useful in slips as they give more divergence for their length.

THE OUTSIDE SLIP ROAD.

When the switches of a slip road lie outside of the Vee crossings of the diamond, it is known as an Outside Slip Road.

Being rather complicated it is not commonly used, but it provides a means of forming a slip when the diamond angle is too wide to allow of an inside slip.

PRACTICAL DETAILS.

Photo No 8 gives an idea of the checking and chairing required.

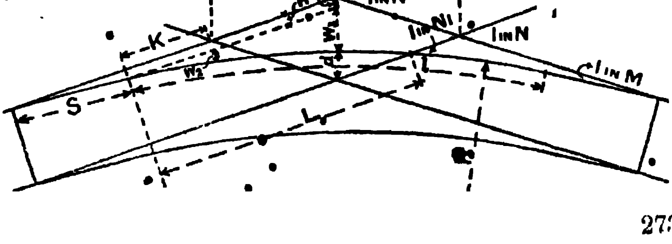
The minimum for the distances "W_2, H" and "d" (Fig. XXII 3) should be about 1′ 2″. The best arrangement is for them to be nearly equal.

CALCULATIONS.

The calculations form an interesting application of the General Rules combined with those for Slip roads.

Prob. 2. Required the particulars of an outside slip road of Radius (R) = 400′, to connect two straight tracks crossing at an angle of 1 in 4 (N); Switches 15′ *i.e.* M = 40.

FIG. XXII - 3.



1st.— $J = 2 \times N \times G \times \left(1 + \frac{1}{8N^2}\right)$ (Rule XX.—10.)

$= 2 \times 4 \times 4' \; 8\frac{1}{2}'' \times 1\frac{1}{128}$

$= 37' \; 8'' + 3\frac{1}{2}''$

$= 37' \; 11\frac{1}{2}''$ say $38'$

2nd.—Find length of slip between heels of switches (l). For this purpose Rule XXII.–6 must be transposed and we obtain :—

$$l = \frac{R(M-2N)}{N \times M} \quad \text{(Rule XXII.—8.)}$$

$$= \frac{400 \times (40-8)}{4 \times 40}$$

$$= \frac{400 \times 32}{4 \times 40}$$

$$= 80'.$$

3rd.—N_1 and L may be found by the rules for the ordinary turnout, and thus :—

$N_1 = 6\frac{3}{4}$ by Rule 3c.

$L = 49' \; 6'$ by Rule 1c.

4th.— $K = \frac{l-J}{2}$ (approx.) (Rule XXII.—9.)

$$= \frac{80' \; 0'' - 38' \; 0''}{2}$$

$$= 21' \; 0''$$

5th.—Find the width W_2 for a lead $= K$.

$$W_2 = \frac{K^2}{2R} + \frac{K}{M} \quad \text{(Rule 11d.)}$$

$$= \frac{21^2}{2 \times 400} + \frac{21}{40}$$

$$= \frac{441}{800} + \frac{21}{40}$$

$$= \cdot551' + \cdot525'$$

$$= 1\cdot076' = 1' \; 1''$$

and $W_2 + H = 1' \; 1'' + 4\frac{1}{2}'' = 1' \; 5\frac{1}{2}''$

6th.—Find the Width W_3 for a lead $=\frac{1}{2}l$.

$$W_3 = \frac{(\frac{1}{2}l)^2}{2R} + \frac{\frac{1}{2}l}{M} \qquad \text{(Rule 11d.)}$$

$$= \frac{40^2}{2 \times 400} + \frac{40}{40}$$

$$= 2{\cdot}00' + 1{\cdot}00' = \underline{\underline{3{\cdot}00'}}$$

7th.— $d = G \text{ (practically)} - W_3 - H$

$$= 4'\ 8\tfrac{1}{2}'' - 3'\ 0'' - 4\tfrac{1}{2}''$$

$$= \underline{\underline{1'\ 4''}}$$

It may be noted that if in the example R had been taken as 462′ (7 chains) with 15′ switches, d would work out to $10\frac{3}{4}''$, which would be too small.

In setting out the slip, the switch heels should first be located by measuring the distance $\frac{1}{2}l$ on either side of the centre of the diamond. The remaining procedure easily follows.

CHAPTER XXIII.

SIDING GROUPS.

The planning of Siding Groups with their "Gathering Lines," or "Ladder Tracks," hardly comes within the scope of this work, but brief notes upon some of the dimensional questions involved will not be out of place.

The designer may be working under one of two leading conditions.

(1) When the general lay-out of the sidings is determined by the shape of the land available, and it is required to gain as much siding standing room on the given area as possible, regardless of regular or ideal alignment of the tracks.

(2) When the siding groups may be designed practically independently of the quantity and shape of the land required or of local obstacles.

In this case the designer has a free hand and the objects in planning the lay-out will be:—

(*a*) Easy and regular running on all routes, using curves of a constant radius wherever possible, but increasing the radius to compensate for the effect of gradients, where necessary (see Chapter II.)

(*b*) The maximum amount of standing room obtainable with the track materials used, compatible with the requirement of easy running.

(*c*) Repetition of turnouts, slip roads, etc., of the same dimensions, and of a model type in which cutting of rails is avoided, and good chairing and timbering attained.

When working under Condition (1), calculation is of little use, and it will generally be a matter of making repeated trials by drawing or lining out, until the best arrangement is arrived at.

When Condition (2) obtains, calculation may be applied to almost all the component parts of the scheme.

Figs. XXIII.-1, 2, and 3 show various systems of arranging gathering lines and connections, the approximate angle which the gathering line makes with the main and sidings being obtainable from the scales drawn upon the diagrams.

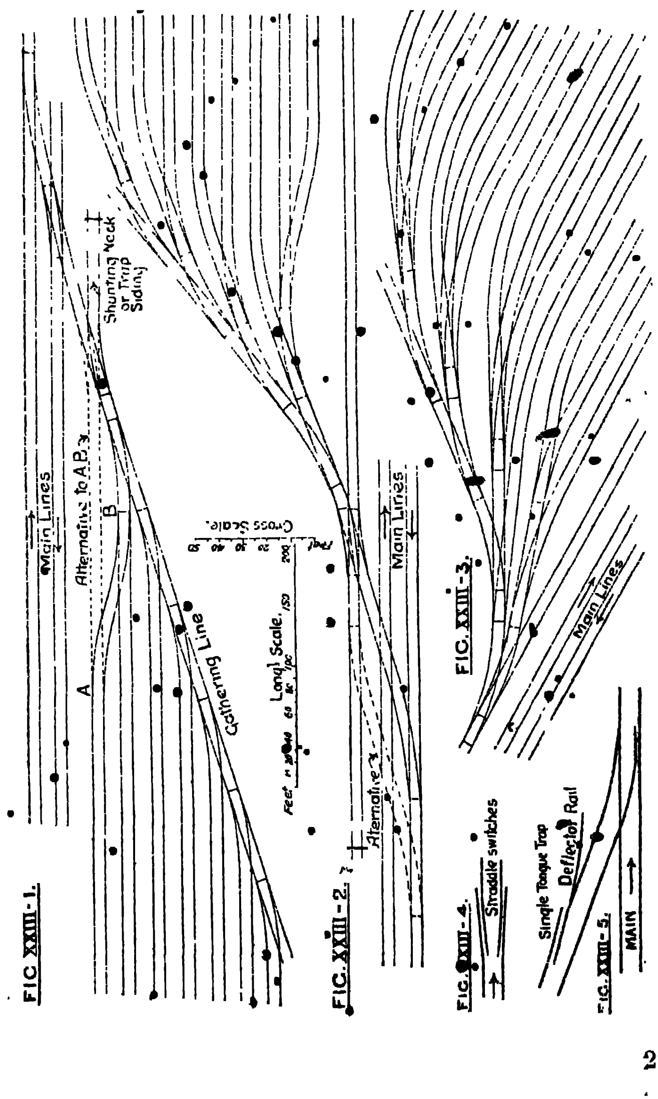


FIG. XXIII-1.
FIG. XXIII-2.
FIG. XXIII-3.
FIG. XXIII-4.
FIG. XXIII-5.

If the main and the sidings parallel thereto are on the curve, it will usually be found that the gathering line should have the same curve, and the same angle between it and the main as with straight tracks in a similar design.

In designing, the following method of procedure is recommended:—

Draw the gathering line with the particular arrangement and type of turnouts which it has been decided to use, upon tracing paper. Upon another sheet of tracing paper draw the main line and the series of parallel sidings, spaced as required. By placing one plan over the other, it will be seen how best to fit them together, and scheme in the special connections needed at the commencement of the gathering line, including those to the main, to the trap siding, and to the first one or two sidings.

Safety or Trap Points and Sidings.

The Board of Trade Rules state that "Safety Points" must be provided upon goods lines and sidings at their junctions with passenger lines.

These points are intended to prevent vehicles passing from the siding on to or foul of the main line without permission of the signalman.

Where there is sufficient space, the Safety Points lead into a "Safety Siding" otherwise named a "Trap," "Runaway," or "Catch Siding" This siding will often prove useful for other purposes if made long enough to receive an engine.

In all cases, the points of traps should be situated as near as practicable to the outlet crossing on the main so as to reduce the possibility of vehicles being left by inadvertence, either foul of, or unprotected from the main line. At the same time the trap or trap siding must be in such a position that it derails or leads runaway vehicles clear of the main.

A device intended as a substitute for Safety Points is the "*Derailer,*" consisting of a specially shaped block which may be moved on to or away from the rail by a lever in the signal box. The advantages claimed for this device are that a length of siding standing room may be saved, at least equal to the length of the switch which would be necessary with trap points; also that a vehicle entering into the siding will pass over it without derailment or damage to the permanent way, even though by mistake the derailer has been left on the rail.

The usual arrangement for a trap siding is shown on Fig. 1, where a turnout leads to a buffer stop. The space between the trap siding and the main must not be less than the standard space between running lines, usually 6 ft.

Instead of buffer stops, a "Drag" is sometimes provided, formed by covering some length of the rail or forming a bank with sand or ashes. This method has been found to be quite as efficient as a buffer stop. *

A set of double slips will often provide a convenient lay-out for the trap and other connections at the entrance of a siding group (see Fig. 2). There is an objection to this, however, in that the switches on the trap road will not be set at the same time to suit wheels trailing over them into the trap from both directions.

When there is not room for a siding, and Trap Points are used, these may be formed of either a single switch, a pair of switches, or a pair of "Straddle" switches. The last device is shown in Fig. 4; the wheels are assumed to travel along the back rails until the gauge becomes too wide and they drop off.

The Single Tongue Trap (Fig. 5), however, has been found by experiment to be the most satisfactory for general adoption. †

It may be advisable to extend the trap switch by a rail curved so as to turn vehicles away from the main, or to provide a "deflector rail" as recommended in the articles referred to in the footnote.

*** See the Railway Gazette, October 15th, 1920, for illustrated account of experiments by the G. W. Railway.**

† See the Railway Gazette, October 15th, 1920, and August 26th, 1921, for illustrated account of experiments by the G. W. Railway, with recommendations.

CHAPTER XXIV.

SYSTEM OF FACTORS OR SIMPLIFIED RULES FOR POINTS AND CROSSING PROBLEMS.

The General Rules explained in Chapter XIII. and collected in Tables 25, 26, and 27, are intended for use when preparing Reference Tables and Lists of Standard Dimensions, and in other cases where exact results are desired. During the course of work in practice, however, we shall often find the need of a set of rules which will give results rapidly and yet with a fair degree of accuracy. This chapter will explain a method of obtaining and applying such rules.

The same symbols will be used as in the General Rules

The system as a whole will be named the *System of Factors*, because by it Factors or Numbers are obtained by which we may multiply or divide some given dimension in a turnout or other arrangement to find the required dimensions.

As an example, if it is desired that a certain turnout must have a 1 in 8 crossing, it can be stated that under certain conditions:—

The Lead will be $8 \times 7{\cdot}14 = 57{\cdot}12'$.

The Radius will be 8 squared $\times 9{\cdot}09 = 582'$.

The Factors in this case being 7·14 and 9·09.

These Factors give simple rules which are invaluable in scheming, planning, and comparing different arrangements either in the office or on the ground.

They are also useful when making out orders for the materials, including Switch lengths, Crossing Angles, etc., from a small scale general plan.

We must first of all know the particular Standards to which the Factors are to apply. These are:—

Gauge.

Switch Heel Divergence.

Distances between Tracks.

The examples will apply to the 4′ 8½″ gauge and 4½″ Heel Divergence, though as regards simple Turnouts a Heel Divergence of 4¾″ will be included in the Tables.

To find Factors for the Single Turnout (Table 46).

For Factors F_1 choose various values for the relation between S and N, such as are likely to occur in practice.

This relation was discussed in the Chapter upon the Switch, the ideal there recommended being $S = 1\frac{3}{4} \times N$.

From the F_1's obtain corresponding relations between M and N, these will be the F_2's.

Then take Rules No. 1c and No. 2c, Table 27, and in these, instead of M, substitute its value in terms of N, thus obtaining Factors F_3 and F_4.

The remaining Factors may be deduced from these now arrived at. The Factors F_1, F_2, F_3 and F_4 will be named "Guiding Factors" and the remainder "Working Factors."

The Versed sine of the Lead Curve for a fixed relation between S and N will be constant; for example, no matter what values S and N may have, whenever $S \div N = 1\frac{3}{4}$, V will be $8\frac{3}{4}''$.

General Directions for using Factors for Single Turnouts (Tables 46 and 47).

Take the dimension given in the problem, this will either be N, R, or L; if the Switch is not given choose a suitable length for it.

The Tables are well suited for this purpose, for suppose we wish the Switch to be, for example, $1\frac{3}{4} \times N$. Even though we may not know N we can obtain S from the R or the L by following the line in Table 46 beginning with $F_1 = 1\frac{3}{4}$ and find that :—

$$S \div \sqrt{R} = \cdot581$$

$$\therefore S = \sqrt{R} \times \cdot581$$

$$\text{otherwise } L \div S = 4\cdot08$$

$$\therefore S = L \div 4\cdot08$$

Thus by multiplying $\sqrt{R}$ by $\cdot581$ or by dividing L by $4\cdot08$, according to whether R or L is given, we shall obtain an ideal length of Switch. This, however, may not be a stock length, so therefore we must choose the nearest stock length to the ideal, generally preferring one which is longer to one which is considerably shorter.

We may now proceed as if the Switch had been given in the first place.

We will assume that a Radius of 660′ and a 15′ Switch are given.

First obtain a Guiding Factor F_5 thus:—

$$\begin{aligned} F_5' &= S \div \sqrt{R} \\ &= 15 \div \sqrt{660} \\ &= 15 \div 25{\cdot}69 \\ &= {\cdot}584 \end{aligned}$$

Looking down the column of F_5's we find that the nearest one to this is ·581. We may therefore use the Working Factors in this line of the Table and so:—

$$\begin{aligned} N &= \sqrt{R} \div F_6 \\ &= 25{\cdot}69 \div 3{\cdot}02 \\ &= 8{\cdot}51 \end{aligned}$$

$$\begin{aligned} \text{and } L &= \sqrt{R} \times F_7 \\ &= 25{\cdot}69 \times 2{\cdot}37 \\ &= 60{\cdot}88' \end{aligned}$$

If the Crossing Number is given, obtain a Guiding Factor F_1 by dividing S by N; if the Lead is given obtain a Guiding Factor F_8 by dividing L by S, and then in either case proceed in a similar manner to the above.

Factors may be applied when the main line is curved by using the principle of "Equivalent Radius from the Straight," as explained in Chapter XIII.

The relation between S and N should generally be the same as with a straight main, but we may note that with curves of similar flexure we may often have to make S shorter, and with curves of contra flexure, longer than the ideal.

To find Factors for Various Widths (W). Table 48.

The System of Factors with its simplification of calculations may be applied to other cases than the Turnout.

The reader should first peruse the opening remarks in Chapter XIII., he will then see that the proposal is to find Factors applying to other Widths (W) than the fixed Width (G − H) of the Turnout.

We shall not need to obtain as many Factors as in the case of the simple Turnout, the only problems likely to occur being as enumerated in the first two columns below.

Given (in addition to the Switch).	To find.	Rule.
R and W	N	$\sqrt{R}^* \div F_6$
Ditto.	L	$\sqrt{R}^* \times F_7$
N and W	$\sqrt{R}^*$	$N \times F_6$
L and W	$\sqrt{R}^*$	$L \div F_7$
R and N	W	W is opposite F_6 in table, and $F_6 = \sqrt{R} \div N$

In the first column of Table 48 are placed the various Widths (W) which will occur in Junction, Double Turnout, etc., problems.

The second column shows the kind of problem to which the W's apply, reference being made to the various Diagrams.

As in the case of Single Turnout Factors it is necessary to choose relations to base the system upon, that is the Guiding Factors.

Therefore again fix upon relations between the Switch and the Crossing Number at the Width (G – H) for a Single Turnout.

The Factors F_1 are thus chosen as $1\frac{1}{2}$, $1\frac{3}{4}$, and 2.

From the F_1's take the corresponding F_5's from Table 46.

The Guiding Factors F_1 and F_5 are placed at the head of the Factor columns.

Now obtain Working Factors F_6 and F_7, which when the Radius is given, will serve to find N and L and which

* It may be necessary to use $\sqrt{R - \frac{1}{2}W}$ instead of $\sqrt{R}$ as explained in the later remarks.

will be all that is required because these Factors will serve in the reverse cases, thus:—

$$\text{If } N = \sqrt{R^*} \div F_6$$
$$\text{then } \underline{\underline{\sqrt{R} = N \times F_6}}$$

$$\text{and if } L = \sqrt{R^*} \times F_7$$
$$\text{then } \underline{\underline{\sqrt{R} = L \div F_7}}$$

To find F_6 take Rule No. 3e, Table 27. and instead of M and M^2 substitute its value in terms of R.

This will give a rule by which F_6 may be found for any W.

To find F_7 take Rule No. 1e. Table 27. and instead of M substitute its value in terms of $\sqrt{R}$ or for convenience of $\sqrt{R} - \frac{1}{2}W$, and instead of N substitute $\sqrt{R} - \frac{1}{2}W \div F_6$.

This will give a rule by which F_7 may be found for any W.

It will be noticed that $\sqrt{R - \frac{1}{2}W}$ is used with the Factors in this Table and not $\sqrt{R}$. $\sqrt{R}$ may be taken, however, in obtaining Crossing Angles with immaterial error and also for the Leads when W is small, say under 4′ 4″, or when only rough results are required.

R is the Radius of the rail upon which an ordinary turnout crossing would lie. In cases where the crossing is on the other rail, as the N_2 in a junction, the gauge, say 5′, must be deducted from R.

Directions for using Factors for Various Widths (Table 48).

The preceding remarks should be studied prior to the following further explanation.

The first thing to decide upon is either F_5 which is the relation of the Switch length which will be used, to the Radius, or alternatively, the Guiding Factor F_1, which is the relation between the Switch length and the Crossing Number for a Single Turnout. It is important that no other crossing must be taken notice of in obtaining this relation.

By comparing the F_5 or F_1 thus decided upon, with those in the column headings of the Table, it will be seen which column to use. For example, if the Radius is to be

600′ and the Switch 15′:—

$$F_5 = S \div \sqrt{R}$$
$$= 15 \div \sqrt{600}$$
$$= 15 \div 24{\cdot}5$$
$$= {\cdot}612$$

The nearest F_5 to this is ·581; so we may take the Working Factors F_6 in the column with this F_5 at the head for finding the Crossing Numbers, because it makes little difference which column we take for these, and the Table enables us to find them rapidly and as accurately as required by using the column headed with the Guiding Factor nearest to the actual Factor.

With regard to the Lead, however, if this is required fairly accurately, in taking the Working Factor F_7, we must allow a proportion due to the difference between the actual F_1 or F_5 and that nearest to it in the Table. Alternatively, we may first find N by a Factor (F_6) and then use Rule 1c (Table 27) to find L.

The same remark as to using the Factors when the main line is curved apply as in the case of the Single Turnout.

SYSTEM OF FACTORS—EXAMPLES.

These examples will illustrate the use of Factors in practice. It may be repeated that in every case the first step is to determine the Width (W), and the second step is, from the Switch S and either N, $\sqrt{R}$, or L, to obtain one of the Guiding Factors F_1, F_5, or F_4.

For the sake of clearness in explanation we shall usually omit to rectify the Working Factors in proportion to the difference between the Guiding Factors as found and those in the Tables, though it will be obvious that more accuracy is obtained by allowing for this difference.

For comparison, the results which would be arrived at by working with the General Rules are in most cases shown in brackets.

Problems 1 to 9 are solved by the use of Table 46 and the remainder by the use of Table 48.

SINGLE TURNOUT, STRAIGHT MAIN.

Problem 1. Crossing No. given.

EXAMPLE.—Given Crossing No. (N) $7\frac{1}{2}$; required the remaining dimensions; the switch length in feet to be not less than $1\frac{3}{4}$ times the Crossing No.

1st.—The Switch should not be less than :—

$$7\tfrac{1}{2} \times 1\tfrac{3}{4} = 13\tfrac{1}{8}'.$$

Assuming that we have no stock switches between 12′ and 15′, then :—

$$S = \underline{\underline{15'}}.$$

2nd.—Find the Guiding Factor :—

$$\begin{aligned} F_1 &= S \div N \\ &= 15 \div 7\tfrac{1}{2} \\ &= \underline{\underline{2}} \end{aligned}$$

3rd.—Find the Radius :—

$$R = N^2 \times F_4$$

Consulting the Table we find that when F_1 is 2 the F_4 is 8·98, so :—

$$\begin{aligned} R &= 7{\cdot}5^2 \times 8{\cdot}98 \\ &= 56{\cdot}25 \times 8{\cdot}98 \\ &= \underline{\underline{505'}} \qquad (507') \end{aligned}$$

4th.—Find the Lead :—

$$\begin{aligned} L &= N \times F_3 \\ &= 7{\cdot}5 \times 7{\cdot}30 \\ &= 54{\cdot}75' = \underline{\underline{54'\ 9''}} \qquad (54'\ 8'') \end{aligned}$$

Problem 2. Radius given.

EXAMPLE.—Radius $8\frac{1}{2}$ chains (561′).

1st.—Choose the Switch.

$$\begin{aligned} \text{The ideal } S &= \sqrt{R} \times {\cdot}581 \text{ (See Chapter XI.).} \\ &= 23{\cdot}68 \times {\cdot}581 \\ &= \underline{\underline{13{\cdot}77'}} \end{aligned}$$

Assuming 15′ is the available length nearest to this,

$$S = \underline{\underline{15'}}.$$

2nd.—

$$\begin{aligned} F_5 &= S \div \sqrt{R} \\ &= 15 \div 23{\cdot}68 \\ &= \underline{\underline{{\cdot}633}}. \end{aligned}$$

Our nearest F_5 to this is ·627, therefore:

$$\begin{aligned} L &= \sqrt{R} \times F_7 \\ &= 23{\cdot}68 \times 2{\cdot}40 \\ &= \underline{\underline{56{\cdot}83'}} \end{aligned}$$

$$\begin{aligned} N &= \sqrt{R} \div F_8 \\ &= 23{\cdot}68 \div 3{\cdot}01 \\ &= \underline{\underline{7{\cdot}87}} \end{aligned}$$

Thus a 1 in $7\frac{3}{4}$ crossing may be used if the curve continues through the crossing or a 1 in 8 if the crossing leg is to be straight.

Problem 3. Lead Given.

EXAMPLE.—Lead 50′ 4″ = 50·33′. Switches 12′.

1st.—

$$\begin{aligned} F_8 &= L \div S \\ &= 50{\cdot}33 \div 12 \\ &= \underline{\underline{4{\cdot}20}} \end{aligned}$$

This lies between the F_8's of 4·33 and 4·08 in the Table, therefore:—

$$\begin{aligned} R &= L^2 \div F_9 \\ &= 50{\cdot}33^2 \div 5{\cdot}51 \\ &= \underline{\underline{460'}} \end{aligned}$$

$$\begin{aligned} N &= L \div F_{10} \\ &= 50{\cdot}33 \div 7{\cdot}09 \\ &= \underline{\underline{7{\cdot}10}} \text{ or say 1 in 7.} \end{aligned}$$

Problem 4. Required the Versed Sine.

EXAMPLE.—Crossing 1 in 8. Switch 15′.

1st.—Find the Guiding Factor.

$$\begin{aligned} F_1 &= S \div N \\ &= 15 \div 8 \\ &= \underline{\underline{1\tfrac{7}{8}}} \end{aligned}$$

In the last column of Table 46 and opposite this F_1 we find:—

$$V = \underline{\underline{8\tfrac{5}{8}''}}$$

If the R or the L are given we must find an F_6 or an F_8 and take the V opposite that Factor.

Problem 5. To shew the results when the turnout curve is assumed as tangential to main line.

If the turnout curve produced is actually tangent to the main line and yet tangential to the switch at its heel, the relation between the turnout dimensions will be as shown by the Factors marked ‡ in Tables 46 and 47.

EXAMPLE.—Crossing 1 in 8.

$$S \div N = 1{\cdot}33$$
$$\therefore S = N \times 1{\cdot}33$$
$$= 10{\cdot}67'$$

$$L = 8 \times 6{\cdot}75$$
$$= 54{\cdot}00'$$

$$R = 8^2 \times 9{\cdot}42$$
$$= 603'$$

Cole gives	L = 53′ 9″	R = 609·7′
Whitelaw gives	L = 54′ 2″	R = 603
Frere gives	L = 54′ 3″	

Slight differences being due to methods of measuring crossing angles, etc.

It is obvious that even if a switch 10·67′ long were available, it would be too short and that 15′ would be a suitable length. In this case:—

$$F_1 = 1\tfrac{7}{8}$$

$$L = 8 \times 7{\cdot}22$$
$$= 57{\cdot}76' \qquad (57'\ 9'')$$

$$R = 8^2 \times 9{\cdot}03$$
$$= 578' \qquad (580')$$

Problem 6. To shew the results for various heel divergences.

EXAMPLE.—What difference will be made in the turnout dimensions by increasing the heel divergence from $4\frac{1}{2}''$ to $4\frac{3}{4}''$. Crossing 1 in 8. Switch 15′. ($F_1 = 1\frac{7}{8}$.)

With $4\frac{1}{2}''$ H (see Problem 5):—

$$R = 578' \qquad (580')$$
$$L = 57{\cdot}76' \qquad (57'\ 9'')$$
$$V = 8\tfrac{3}{4}''$$

With $4\frac{3}{4}''$ H:—

$$R = 8^2 \times 9{\cdot}03 = 578' \qquad (580')$$
$$L = 8 \times 7{\cdot}115 = 56{\cdot}92' \qquad (56{\cdot}93')$$
$$V = 8\frac{3}{8}''$$

The difference in the Radius is so small as only to be shown by carrying the results to decimals, but the difference in Lead and Versed sine should be allowed for.

SINGLE TURNOUT. CURVED MAIN

The notes upon the Principle of Equivalent Radius in Chapter XIII should first be consulted.

Problem 7. Given Radius of Main (R_m) and N.

EXAMPLE.—What is the easiest Radius of turnout curve (R_t) we can obtain from the inside of a 10 chains (= 660') main when the flattest crossing we may use is 1 in 14.

1st $S = N \times F_1$

In such a case as this we may use an F_1 of $1\frac{1}{2}$ so as not to cause the lengths of Switch and Lead to be excessive

$$S = 14 \times 1\frac{1}{2}$$
$$= 21'$$

2nd. Find Equivalent Radius (R_e).

$$R_e = N^2 \times F_1$$
$$= 196 \times 9{\cdot}25$$
$$= 1813'$$

3rd.—From Equivalent Radius Rules:

$$R_t = \frac{R_e \times R_m}{R_m + R_e} \qquad \text{(Rule XIII.—5.)}$$
$$= \frac{1813 \times 660}{660 + 1813}$$
$$= 484'$$

Problem 8. Given R_m and R_t.

EXAMPLE.—What is the shortest Lead and least Crossing No. for a Turnout on a railway where curves must not be less than 400' Radius and the shortest Switches are 10'

The shortest Lead will occur when the main and turnout are both of 400′ Radius and of contra flexure, so:—

$$R_e = \frac{400 \times 400}{400 + 400} \qquad \text{(Rule XIII.—1.)}$$
$$= \underline{\underline{200'}}$$

$$F_5 = S \div \sqrt{R_e}$$
$$= 10 \div 14 \cdot 14$$
$$= \underline{\cdot 707}$$

$$L = \sqrt{R_e} \times 2 \cdot 46$$
$$= \underline{\underline{34 \cdot 78'}}$$

$$N = \sqrt{R_e} \div 3 \cdot 00$$
$$= \underline{\underline{4 \cdot 71}}$$

For the Versed sines:—

V for the factor (V_c) $= \underline{\underline{9''}}$

V for the main on the lead (V_m) $= L^2 \div 8\,R_m$
(From Rule IX.—1.)

$$= 34 \cdot 78^2 \div (8 \times 400)$$
$$= 1209 \cdot 65 \div 3200$$
$$= \cdot 378' = \underline{\underline{4\tfrac{1}{2}''}}$$

With contra flex. V for the turnout (V_t) $= V_c - V_m$
(Rule XIII.—10.)

$$= 9'' - 4\tfrac{1}{2}''$$
$$= \underline{\underline{4\tfrac{1}{2}''}}$$

Prob. 9. Given L and R_m.

EXAMPLE.—Required particulars of a Turnout 75′ in length including switches, in a main of 20 chains ($= 1320'$) Radius, similar flexure.

1st.— Assume $S = 15'$.

Then $L = 75' - 15' = \underline{\underline{60'}}$.

$$F_4 = L \div S$$
$$= 60 \div 15 = \underline{\underline{4 \cdot 00}}.$$

This factor is intermediate to the 4·08 and 3·85 in the Table and so the 15′ switch is suitable.

$$R_e = L^2 \div F_9$$
$$= 3600 \div 5 \cdot 67$$
$$= \underline{\underline{635'}}$$

$$R_t = \frac{1320 \times 635}{1320 + 635} = \underline{\underline{429'}}$$ (Rule XIII.—5.)

$$N = L \div 7 \cdot 17 \quad = \underline{\underline{8 \cdot 37}}$$

For the Versed sines:—

V_c for the Factor $= \underline{\underline{8\frac{1}{2}''}}$

V_m for the main $= L^2 \div 8\,R_m$ (From Rule IX.—1.)
$$= 3600 \div 10560$$
$$= \cdot 341' = 4\frac{1}{8}''$$

V_t for the turnout $= V_c + V_m$ (Rule XIII.—11.)
(with similar flexure)
$$= 8\frac{1}{2}'' + 4\frac{1}{8}''$$
$$= \underline{\underline{1'\ 0\frac{5}{8}''}}$$

TURNOUT WITH STRAIGHT IN FRONT OF CROSSING (Type Ie) Fig. XV.—4.

Prob. 10. Given R and Gap D.

EXAMPLE.—$D = 9''$. $R = 477'$. $S = 15'$.

1st.— $W_1 = G - H - D$
$$= 4'\ 8\frac{1}{2}'' - 4\frac{1}{2}'' - 9'' = \underline{\underline{3'\ 7''}}$$

2nd.— $F_1 = S \div \sqrt{R}$
$$= 15 \div 21{\cdot}84.$$
$$= \underline{\underline{\cdot 686.}}$$

So we may use factors in columns 7 and 8, Table 48, and opposite the W of 3′ 7″, but altered slightly to allow for the difference between ·686 and ·667.

$$N = \sqrt{R} \div F_6$$
$$= 21{\cdot}84 \div 2{\cdot}73$$
$$= \underline{\underline{8{\cdot}00}}$$

$$L_1 = \sqrt{R} \times F_7$$
$$= 21{\cdot}84 \times 2{\cdot}184$$
$$= \underline{\underline{47{\cdot}70'}}.$$

$$L = L_1 + (D \times N)$$
$$= 47{\cdot}70 + (9'' \times 8)$$
$$= \underline{\underline{53{\cdot}70'}}.$$

Prob. 11. Given N and Gap D.

EXAMPLE.—N = 8. D = 9″, S = 15′.

1st.—W_1 = 3′ 7″ as in Problem 10.

2nd.—Make a trial with factor F_6 in column 5.

$$\text{Because } N = \sqrt{R} \div F_6$$
$$\sqrt{R} = N \times F_6$$
$$= 8 \times 2{\cdot}754$$
$$= 22{\cdot}03$$
$$\text{and } R = 22{\cdot}03^2 = \underline{\underline{485'}}$$

Now try which column of factors really should be taken:—

$$F_5 = S \div \sqrt{R}$$
$$= 15 \div 22{\cdot}03$$
$$= \underline{\underline{{\cdot}681}}$$

So it will be nearer to take columns 7 and 8, then:—

$$\sqrt{R} = 8 \times 2{\cdot}735$$
$$= 21{\cdot}88$$
$$R = 21{\cdot}88^2 = \underline{\underline{479'}}$$

$$L_1 = 21{\cdot}88 \times 2{\cdot}170$$
$$= \underline{\underline{47{\cdot}48'}}$$

DOUBLE LINE JUNCTIONS.

(Fig. XVI.-1.)

Prob. 12. Given, Radius of main (R_m) and of Turnout (R_t).

EXAMPLE.—R_m 1079′ (outer rail). R_t 470′ (outer rail). Similar flexure. Clear space between tracks 6′ 0″. Width of rail heads $2\frac{3}{4}''$.

1st.—Obtain Equivalent Radius (R_e).

$$R_e = \frac{1079 \times 470}{1079 - 470} \qquad \text{(Rule XLII.—4.)}$$

$$= \underline{\underline{833'}} \qquad \text{(On outer rail of } 12\tfrac{1}{2} \text{ Chs. on centre line of junction.)}$$

2nd.—

$$S = \sqrt{R \times \cdot 581}$$
$$= \sqrt{833 \times \cdot 581}$$
$$= \underline{\underline{16 \cdot 78'}}$$

To suit switches in stock we will take,

$$S = \underline{\underline{18'}}$$

3rd.—Find the guiding factor F_5

$$F_5 = S : \sqrt{R}$$
$$= 18 : 28 \cdot 86$$
$$= \underline{\underline{\cdot 624}}$$

This is a mean between ·581 and ·667 so we may use factors half way between those in columns 5 and 6 and those in columns 7 and 8

4th.—Form a table with headings as below to receive the dimensions

1	2	3	4	5		6	7	
Crossing.	W	F_8	$\sqrt{R} - \frac{1}{2}W$	Crossing No. $\sqrt{R} - \frac{1}{2}W - E$		F_7	Lead $= \sqrt{R} - \frac{1}{2}W \times F_7$	
	ft. ins.							
N_1	4 4	3·005	28·83	9·59	($9\frac{1}{2}$)	2·103	69·27	(69·23)
N_2	6 1	3·540	28·72	8·11	($8\frac{1}{4}$)	2·936	84·32	(84·30)
N_3	10 $9\frac{1}{2}$	4·685	28·77	6·14	(6)	4·084	117·50	(117·50)
N_4	15 6	5·600	28·72	5·13	(5)	5·000	143·60	(143·49)

5th.—Obtain the W's:—

$$W_1 = 4'\ 8\tfrac{1}{2}'' - 4\tfrac{1}{2}'' = 4'4''$$
$$W_2 = 6'\ 0'' + (2 \times 2\tfrac{3}{4}) - 4\tfrac{1}{2}'' = 6'\ 1''$$
$$W_3 = W_1 + 6'\ 0'' + (2 \times 2\tfrac{3}{4}'') = 10'\ 9\tfrac{1}{2}''$$
$$W_4 = W_3 + 4'\ 8\tfrac{1}{2}'' = 15'\ 6''$$

From Table 48 take the factors applying to these W's and place in 3rd and 6th columns. From R deduct the $\frac{1}{2}$ W's and take square root, placing the results in the 4th column.

Divide the last by the F_6's and multiply them by the F_7's placing the resulting crossing numbers and the leads into their columns. In the case of N_2 marked * the Gauge as well as $\frac{1}{2}$W must be deducted from R.

Prob. 13. Given Angle of Diamond and Space; required the Radius obtainable.

EXAMPLE.—Space (clear) 8' 0"; Diamond 1 in 8.

1st.—W for the diamond (N_1):—

$$4'\ 1'' + 8'\ 0'' + (2 \times 2\tfrac{1}{4}'') = \underline{\underline{12'\ 9\tfrac{1}{2}''}}.$$

2nd. — $$N = \sqrt{R - \tfrac{1}{2}W} \div F_6$$
$$\therefore \quad \sqrt{R - \tfrac{1}{2}W} = N \times F_6$$
$$\therefore \quad R = \underline{\underline{(N \times F_6)^2 + \tfrac{1}{2}W}}$$

3rd.—From Table 48 for a W of 12' 9½" we find:—

(a) When $S = N_1 \times 1\frac{1}{2}$, $F_6 = 5{\cdot}115$
(b) ,, ,, = ,, $\times 1\frac{3}{4}$, ,, = 5·099
(c) ,, ,, = ,, $\times 2$, ,, = 5·089

4th.—Multiply these F_6's by 8, square the results, and add $\frac{1}{2}$ W = say 6', and we obtain radii:—

(a) R = 1680'
(b) ,, = 1670'
(c) ,, = $\underline{\underline{1663'}}$

If the main is curved these Radii will be the Equivalent Radii obtainable.

Prob. 14. Given length of Junction.

EXAMPLE.—Length or Lead from heel of switch to last crossing 127·30′. Space (gauge lines) 6′ 4½′ Switch length 15′

1st.— $W_4 = 15'\ 6''$ as in Prob. 12 and,

$$L_4 = \sqrt{R - \tfrac{1}{2}W} \times F_7$$

$$\therefore \quad \sqrt{R - \tfrac{1}{2}W} = L_4 \div F_7$$

$$\therefore \quad R = (L_4 \div F_7)^2 + \tfrac{1}{2}W$$

2nd.—Assuming we may take F_7 from column 6:—

$$R = (127{\cdot}30 \div 4{\cdot}958)^2 + 7{\cdot}75$$
$$= 25{\cdot}676^2 + 7{\cdot}75$$
$$= \underline{\underline{667}}.$$

3rd.—Obtain an F_5:—

$$F_5 = S \div \sqrt{R}$$
$$= 15 \div 25{\cdot}83$$
$$= \underline{\underline{{\cdot}581}}.$$

So we may use Factors in columns 5 and 6.

The procedure will then be as in Problem 12

If the F_5 we obtained had not corresponded to the F_7 we first assumed, we should have had to revise the F_7 and work out the radius afresh. This in turn would slightly alter F_5.

Prob. 15. Given Radius of Junction Curves and Angle of Diamonds; required the Space.

EXAMPLE.—Radius of Main (R_m), 60 chains. Radius of Branch (R_b), 20 chains. Similar flexure. Diamond Angle (N_3) 1 in 8. Required the necessary Space (S_p) to give these Radii.

1st.—Find the Equivalent Radius (R_e) of the two curves.

$$R_e = \frac{60 \times 20}{60 - 20} = 30 \text{ Chs.} = \underline{\underline{1980'}} \qquad \text{(Rule XIII.—4.)}$$

2nd. $\sqrt{R} \div F_6 = N_3 = 8$

$$\therefore \quad F_6 = \sqrt{R} \div 8$$

$$i.e. \quad F_6 = \sqrt{1980} \div 8 = \underline{\underline{5{\cdot}562}}$$

3rd.—Assuming that $S \div N_1$ will be about $1\frac{3}{4}$, look down column 5 in Table 48 and we find:—

When $F_6 = 5{\cdot}605$ $W = 15'\ 6''$
and ,, ,, $= \underline{5{\cdot}477}$,, $= \underline{14'\ 9\frac{1}{2}''}$

Deducting:—$\cdot 128$ is diff. for $8\frac{1}{2}''$.

$$\begin{array}{r} 5\cdot562 \\ -5\cdot477 \\ \hline \end{array}$$

Diff. for $\cdot 085$ will be $\frac{\cdot 085}{\cdot 128}$ of $8\frac{1}{2}'' = 5\frac{1}{2}''$

$14'\ 9\frac{1}{2}'' + 5\frac{1}{2}'' = \underline{15'\ 3''} = W_3$ for $5\cdot562$

For the space (clear):—

$W_3 = 15'\ 3''$

Less $W_1 = 4'\ 4''$
and rail heads $= 5\frac{1}{2}'' = 4'\ 9\frac{1}{2}''$

$\underline{10'\ 5\frac{1}{2}''}$ Space (clear). (10′ 5″ by General Rules).

DOUBLE TURNOUTS

Prob. 16. To find distance "P." (Fig. XVII. 9.)

EXAMPLE.—$S = 12'$ $R = 150'$ $Y = 9''$.

1st. $w = Y - H$
$= 9'' - 4\frac{1}{2}'' = \underline{4\frac{1}{2}''}$.

2nd.— $F_5 = S \div \sqrt{R}$
$= 12 \div 21{\cdot}21 = \underline{\cdot 566}$.

So we may take the factor F_7 in column 6 and opposite the W of $4\frac{1}{2}''$.

3rd.— $l = \sqrt{R} \times F_7$
$= 21{\cdot}21 \times {\cdot}43 = \underline{9{\cdot}12'}$

4th.— $P = S + l$
$= 12' + 9{\cdot}12' = \underline{21'\ 1\frac{1}{2}''}$. (21′ 0″)

Prob. 17. Type 1, Fig. XVII.–1, Given the particulars of the two turnouts and the Width (w + H) between the third crossing (N_3) and the main.

EXAMPLE.—$S = 12'$ $N_1 = 8$ $N_2 = 7$ $w = 1'\ 0''$ $H = 4\frac{1}{2}''$

1st.— $R_1 = 594'$ $R_2 = 448'$ } By rules for Single
$L_1 = 55'\ 5''$ $L_2 = 49'\ 9''$ } turnouts.

2nd.— $w_1 = G - 2H - w$
$= 4' \; 8\frac{1}{2}'' - 9'' - 1' \; 0'' = \underline{\underline{2' \; 11\frac{1}{2}''}}$

3rd.— Find n_1
$F_1 = S \div N_1$
$= 12 \div 8 = \underline{\underline{1\frac{1}{2}}}$

So for n_1 we may take the factors in column 8 and opposite the W of 2′ 11½″.

$n_1 = \sqrt{R} \div F_6$
$= 24{\cdot}37 \div 2{\cdot}55$
$= \underline{\underline{9{\cdot}56}}$

4th. Find n_2
$F_1 = S \div N_2$
$F_1 = 12 \div 7 = \underline{\underline{1\frac{5}{7}}}$

So we may use column 5 with W of 1′ 0″.

$n_2 = \sqrt{R_2} \div F_6$
$= 21{\cdot}17 \div 1{\cdot}56$
$= \underline{\underline{13{\cdot}57}}$

5th. $N_3 = \dfrac{n_1 \times n_2}{n_1 + n_2}$ (Rule XII.—24.)
$= \dfrac{9{\cdot}56 \times 13{\cdot}57}{9{\cdot}56 + 13{\cdot}57}$
$= \underline{\underline{5{\cdot}66}}$

6th. Find L_3 for $w = 1' \; 0''$
$L_3 = \sqrt{R_2} \times F_7$ (when $F_1 = 1\frac{3}{4}$)
$= 21{\cdot}17 \times {\cdot}90$
$= \underline{\underline{19{\cdot}05'}} = \underline{\underline{19' \; 1''}}$ (say).

7th. Find L_4 for $w_1 = 2' \; 11\frac{1}{2}''$
$L_4 = \sqrt{R_1} \times F_7$ (when $F_1 = 1\frac{1}{2}$)
$= 24{\cdot}37 \times 1{\cdot}787$
$= 43{\cdot}55' = \underline{\underline{43' \; 6\frac{1}{2}''}}$

8th.—As the switches are equal
$P = L_4 - L_3$
$= \underline{\underline{24' \; 5\frac{3}{4}''}}$

Prob. 18. Type 2, Fig. XVII.-6. Given particulars of first turnout. To find L_4, *i.e.*, the situation of crossings N_2 and N_3 so that they lie opposite each other.

EXAMPLE.—$R_1 = 462'$. $S = 12'$.

1st.—N_2 and N_3 will lie at the place where the turnout has diverged from the main to the extent of the gauge, therefore:—

$$W_1 = 2\,G - H$$
$$= 9'\ 0\tfrac{1}{2}''$$

2nd.—
$$F_1 = S \div \sqrt{R}$$
$$= 12 \div 21{\cdot}49$$
$$= {\cdot}558.$$

We may therefore take F_7 a little less than that in column headed $F_5 = {\cdot}581$ and with $W = 9'\ 0\tfrac{1}{2}''$.

3rd.—
$$L_4 = \sqrt{R - \tfrac{1}{2}W} \times 3{\cdot}64$$
$$= \sqrt{457{\cdot}5} \times 3{\cdot}64$$
$$= 21{\cdot}39 \times 3{\cdot}64$$
$$= 77{\cdot}86'$$

THREE THROWS.

Prob. 19. Type 1, Fig. XVIII.-2. Given the particulars of turnouts, which are alike; required the 3rd crossing (N_3).

EXAMPLE.—Longer switch $= 15'$. N_1 and $N_2 = 8$.

1st.—The 3rd crossing will lie in centre of four foot and its W will therefore be:—

$$W_3 = 4'\ 8\tfrac{1}{2}'' \div 2 - 4\tfrac{1}{2}''$$
$$= 1'\ 11\tfrac{3}{4}''.$$

2nd.—By rules for single turnout:—

$$R = 580'.$$

3rd.—
$$F_1 = 15 \div 8$$
$$= 1\tfrac{7}{8}.$$

We may therefore use factors which are a mean between those headed $1\frac{3}{4}$ and 2 and opposite W of $11\frac{3}{4}''$.

4th.— $n = \sqrt{R} \div F_3$
$= 24\cdot08 \div 2\cdot08$
$= \underline{\underline{11\cdot58}}.$

5th.— $N_3 = n \div 2$
$= \underline{\underline{5\cdot79}}$ or $\underline{\underline{1 \text{ in } 5\frac{3}{4}}}.$

6th.— $L_3 = \sqrt{R} \times F_7$
$= 24\cdot08 \times 1\cdot476$
$= \underline{\underline{35\cdot54}}$ or $\underline{\underline{35'\ 6\frac{1}{2}''}}.$

CROSSOVER ROADS.

The case in which factors may be applied is where the curve of the first turnout extends to the crossing of the second turnout (Fig. XIX. 6). This case can be dealt with as in obtaining the second crossing in a double line junction.

The W will be Sp – H, and instead of $\sqrt{R}$

$\sqrt{R - G - \frac{1}{2} W}$ must be taken.

SCISSORS CROSSOVERS.

(Fig. XXI. 1, 2 and 3.)

Considering the diagrams and with the aid of the preceding examples it will be seen that much information may be obtained by the use of factors.

It will be necessary, however, to have the W as a predetermined dimension, a case which usually obtains in practice.

CHAPTER XXV.

SOLUTION OF PROBLEMS BY DRAWING TO SCALE.

The method of solving problems in points and crossings by drawing to scale in many cases presents advantages over that of calculation.

The directions for drawing the work on paper apply largely to the **Method of Lining Out** on the ground if we allow for the fact that instead of lines being drawn with a pencil, they are represented by strings held in place by iron pins resembling surveyors' "arrows"; and that in place of the use of curved rulers, the strings have to be conformed to the curves by employing the method of "quartering" (see Chapter VI.).

Instruments, etc.

The essential requirements will be:—

A fairly large flat table.

Drawing paper, this may be one of the cheaper kinds such as that known as cartridge paper.

Tracing paper, this is useful when it is desired to fit some given arrangement to fall in with certain fixed lines on the drawing.

A well-sharpened, rather hard pencil.

One or two set squares, preferably of celluloid.

A straight-edge, preferably of steel and not less than 3 feet long.

Scale rules:—Firstly, for taking dimensions from the small scale "scheme plans." Secondly, for preparing the large scale detail drawings.

A useful scale rule applying to scheme plans in ordinary use, may contain scales of 40, 66, 41·66, and 208·33 feet to an inch.

An ordinary "universal" scale containing scales of $\frac{1}{8}$ inch, $\frac{1}{4}$ inch, $\frac{1}{2}$ inch, etc., to 1 foot, may serve for the detail plans. A specially made scale rule, however, will

prove more useful, and it is suggested that this be made at least a little over the usual 12ins. in length, so that it contains 100 feet at 8 feet to an inch. It should also contain a 10 feet to the inch scale, as this is often a convenient scale for the purpose.

It is an advantage for the 8 and 10 feet scales to be fully divided at every 2 inches, and the larger scales at every inch.

A set of curved rulers.

Each ruler is marked with a number representing the actual radius in inches with which the curve has been struck.

To find the Number of the Ruler to use for a given Radius of curve on the plan —

Ruler No. = Radius of curve in feet ÷ feet per inch of the scale of the plan.

Conversely Radius in feet = Ruler No. × Scale of plan.

Table 13 gives the Ruler Nos. for various scales and radii.

It will be found that curved rulers of sheet metal are the most satisfactory, because the rulers of flatter curve seem to be cut more exactly, and to retain their accuracy better, than rulers of other materials. The boxes usually supplied have a shortage of the very flat rulers necessary for large scale plans.

There are other instruments, which though not absolutely necessary, will prove useful, such as a large pair of compasses, a small pair of "spring bow" compasses, a parallel ruler, a ruling pen, etc. Indian ink should be used for "inking in" the plan.

The question as to which edge of the rail should be used in drawing and calculation.

The lines on a drawing may alternatively be made to represent :—

1. The inside edges or gauge lines of the rails.

2. The outside edges of the rails.

3. Lines outside of the gauge lines by one-half the thickness of crossing nose.

4. The centre of the rails.

No. 1 is the method strongly recommended, its advantages being that :—

(*a*) Practically all published tables apply to gauge lines.

(*b*) The gauge lines are the critical edges of the rails and are the lines in contact with the wheel flanges.

(*c*) The gauge is constant, regardless of the width of the rail head.

(*d*) The position of the ends of crossing legs is easily defined.

(*e*) In many cases, such as in that of a crossover road, it is desirable that the turnout curve should cease at the crossing nose, and the legs be laid straight. This will be quite in order when the inside edges of the rails are used in calculation by the General Rules. In the case of the outside edges being used, however, the curve must be assumed to extend to the intersection of outside edges and a wider angle of crossing will be arrived at, unless special rules are employed.

No. 2 may be claimed to have the advantages that :—

(*a*) Clearances to structures and spaces between roads are usually measured from outside edge of rails.

(*b*) The intersection of the outside edges at the crossing is a point which may actually be seen on the ground.

(*c*) The outside edge does not become worn by wheel flanges.

No. 3 is occasionally used, the object being that the intersection of the lines will show the actual noses of the crossings.

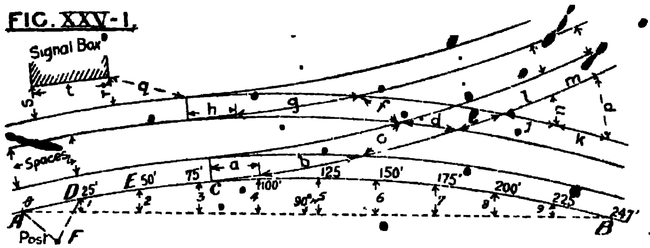
No. 4 is common for small scale general plans and surveys, the lines being plotted 5 feet apart ; the crossing points occur where the spliced part is the same width as the rail.

THE MEASURING AND DRAWING OF EXISTING TRACKWORK.

Those not trained in Surveying may find a first difficulty in laying down on paper the position of existing work which will affect the new. Whilst not proposing to deal with the subject of Surveying, a few notes will show how by the use of nothing more than a string and tape, a not

very extensive or complicated lay-out, may be measured up so that it may be plotted on paper.

FIG. XXV-1.



Example 1.—To survey two curved Main Lines containing Points and Crossings. (Fig XXV.—1.)

1 Stretch a string to act as a straight survey line AB alongside the rail and long enough to cover the new work.

2. Starting at point A make marks, D, E, etc at regular intervals along the rail (in the example at every 25 feet). At the same time note the position of some origin for measuring up the points and crossings later, such as the toe of points at C

3. Measure the square offsets 1, 2, 3, etc., from the survey line to the rail, noting these on a sketch which is best made as the work proceeds The least dimension between a mark and the survey line will be the correct offset in each case

4 If the two mains are parallel, note the Space between them. if not, measure the Space opposite to each offset mark. We shall then have sufficient information to lay down the Main Lines on paper.

5. To "pick up" the Points and Crossings, start at the point C and measure the switch (a), the skew lead (b), the skew distance from crossing to crossing, (c) the diamond sides (d and e), and the dimensions f, g, and h. If working to gauge lines, the blunt nose of the crossings may be measured to, and the fact noted on the plan.

The dimensions to the obtuse crossing must be taken at the centre of the elbow bend.

6. To obtain the direction of the branch line beyond the crossings, the dimensions j, k, l, m, n, and p may be taken, along with a few spaces between the branch lines.

7. Objects adjacent to the track may be "tied" by taking dimensions from two known points or lines, for instance a post F is shown tied to the marks at A and D. The Signal Box may be fixed by the meaurements q, r, s, and t.

8. Measure the Versed sines of the turnouts.

9. Take the position of certain existing rail joints so that unnecessary rail-cutting may be avoided.

To plot the work on paper:—

1. Draw the straight line A B.

2. Mark two small lines one at 25 feet from A, the other at the length of the first offset from the survey line; their intersection will be the point D.

Mark off E in the same manner at its distance from D, and proceed thus until all the points on the rail have been plotted.

3. Join up the points with curved rulers

4. Draw the other rail to gauge.

5. Draw the second road at the measured spaces apart from the first.

6. The positions of the points and crossings may then be marked off, and the branch lines drawn through them.

7. Locate the points fixed by ties, with a pair of compasses. For instance, the intersection of the arcs struck from centres A and D and with radii A F and D F, will give point F.

8. Ink in all the lines, and the plan is ready for receiving the new work.

THE DRAWING OF NEW TRACKWORK.

The drawing of a curve.

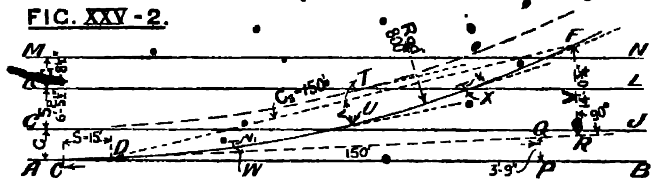
This may be accomplished upon the lines shown in Chapters VI. and VII. After fixing a certain number of points on the curve, they may be joined by the correct curved ruler for the radius.

The drawing of an angle.

This is described in Chapter V. It may be pointed out that the prolongation of short lines should not be depended upon.

Example 2.—To make a drawing of a Double Line Junction, the following dimensions being given:—Main lines straight. Clear space between mains 6ft. Radius of Branch on centre line 12 chains. Switches 15ft. with $4\frac{1}{2}$in. heel divergence. (Fig. XXV.—2.)

FIG. XXV-2.



1st.—Draw the line AB to represent the lower main line rail, next the lines GJ, KL, and MN parallel to AB, at the correct distances apart to represent the other main line rails.

2nd.—Choose a point C for the toe of the switch. From C mark off a distance CP say ten times the switch length, or 150 feet. Mark off PQ—ten times the heel divergence, *i.e.*, $10 \times 4\frac{1}{2}$ins. = 3ft. 9ins. The line drawn from C through Q will be an accurate prolongation of the switch, and therefore a tangent from which the curve may be set, springing from the switch heel D.

3rd.—To lay down the curve of the outer rail, which will be:—792ft. + $\frac{1}{2}$ Sp + G, say 800ft. Radius; calculate a Versed sine (V) for a Sub-chord (C_s), estimating the latter to be long enough to include the whole junction, say 150ft.

$$V = C_s^2 \div 2\,R \qquad \text{(Rule C 9.)}$$
$$= 22500 \div 1600$$
$$= \underline{\underline{14{\cdot}06'}} \text{ or } \underline{\underline{14'\ 0\tfrac{3}{4}''}}$$

Making $DF = C_s = 150$ft., and $RF = V = 14$ft. $0\frac{3}{4}$ins., measured square to CQ, we may by two or three trials find point F.

Draw the chord line DF, bisect it at T and make $TU = v = \frac{1}{4}\,V$.

Obtain the points W and X by joining U to D and F, and making $v_1 = \frac{1}{4}$ v. Join the points D, W, U, X, and F by the correct curve ruler for the scale. The other rail of the branch may then be put in to gauge as shown by the dotted line.

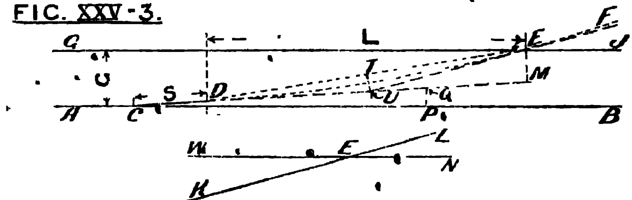
4th.—Mark the intersections of the branch line with the mains, these will be the fine points of the crossings if working to gauge lines.

The leads and any distances desired, may then be scaled.

5th.—To find the crossing angles, draw tangents (see Chapter V.) to the branch line curve at each crossing as shown in fine dotted lines. Scale the crossing angles between these tangents and the main, using a scale twice or four times the scale of the plan. If the mains were curved, it would be necessary to also draw tangents to them at the crossings. The crossing angle would then be the angle between the two tangents.

Example 2.—To draw a turnout with a given Lead and Switch. (Fig. XXV.—3.)

FIG. XXV-3.



1st.—Draw the two lines $A\ B$ and $G\ J$ at 4ft. 8½ins. apart. Choose a point for the switch toe C and mark off the crossing point E at the distance, Lead plus Switch, from C.

2nd.—From C mark off a distance $C\ P$, say four times the switch length. Mark off $P\ Q$ equal to four times the heel divergence, and draw the switch line $C\ Q$.

3rd.—Join $D\ E$ and at its bisection T, draw $T\ U$ square to $D\ E$ and cutting $C\ Q$ at U. Draw the line $U\ E\ F$.

The angle $F\ E\ J$ will be the crossing angle.

The Versed sine will be half $T\ U$. The radius will be, $D\ E$ squared divided by twice $M\ E$.

Example 3.—To draw a turnout with a given Angle of Crossing and Switch. (Fig. XXV.—3.)

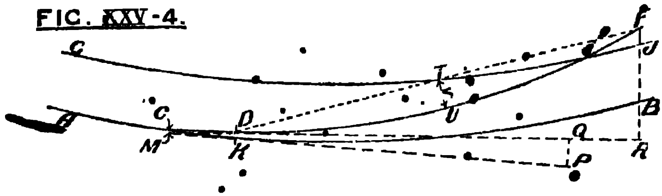
1st.—Draw the main lines and the switch line $C\ Q$ as before.

2nd.—On a piece of tracing paper draw two lines $W\ N$ and $K\ L$ crossing each other at the crossing angle.

3rd.—Place the tracing paper over the drawing, keeping line $W\ N$ on line $G\ J$, and move it lengthwise until the line $K\ E\ L$ cuts the line $C\ Q$ at a point U, so that $D\ U$ is equal to $U\ E$. The point of the crossing will then be at E.

Example 4.—To draw a turnout from a curved main line, with a given switch. (Fig. XXV.—4.)

FIG. XXV-4.



Draw the rails *A B* and *G J* of the curved main.

This may be done by calculating a Versed sine on a chord long enough to cover the turnout, and "quartering"; see Chapter VI.

Mark the toe *C* and the heel *D* of the switch.

Draw a tangent *M P* to the main touching it at point *K* opposite to the switch heel. From point *M* opposite the switch toe, mark off *M P*, say six times the switch length and make *P Q* six times the heel divergence. A line *M Q* will be a tangent to the switch at its heel. The turnout curve may then be drawn as shown previously, springing from this tangent at the switch heel *D*.

The last example is of a turnout of similar flexure; the procedure will be the same with a turnout of contra flexure in the case when it is determined that the side or stock rail must follow the main line curve, a standard straight switch being curved to fit against it. This is objectionable, but it will not be difficult to make the drawing in keeping with one of the methods of avoiding the reverse curve mentioned in Chapter XI.

REFERENCE LIST.

BOOKS.

The list of books includes the works of recent publication dealing with or bearing upon Permanent Way Engineering. Some of the older works are included and also a number of the leading American works, these latter being marked with an asterisk.

Space does not allow of the list being exhaustive, and it is possible that works of value have been unintentionally omitted.

RAILWAYS AND CIVIL ENGINEERING IN GENERAL.

AUTHOR.		PUBLISHER.
Droege, J. A.	Freight Terminals and Yards	Railway Gazette.
,,	Passenger Terminals and Trains	,,
Hadley, E. S.	Railway Working and Appliances	Longmans.
Jenkinson, Lamb, and Travis	Practical Railway Working	Railway Gazette.
	Kempe's Engineer's Year Book	Crosby, Lockwood.
Mills, W. H.	Railway Construction	Longmans.
Molesworth	Pocket Book for Civil Engineers.	Spon.
Merriman	American Civil Engineer's Pocket Book	Chapman Hall.
	Manual of Recommended Practice.	American Rly. Engineering Association
	Maintenance of Way Encyclopaedia	Simmonds-Boardman.
---	Modern Railway Working .. .	Gresham
Rankine	A Manual of Civil Engineering	Griffin.
Raymond, W. G.	*Elements of Railroad Engineering	Chapman, Hall
--	Railway Year Book .. .	Railway Publishing Co.
---	Roorkee Treatise on Civil Engineering, Vol. 2. Section 9 Railways.	Thomason, C. E., College Press, Roorkee.
Sellew, W. H. ..	*Railway Maintenance Engineering.	Constable.
Trautwine .. .	*Civil Engineer's Pocket Book	Chapman, Hall.
Vernon-Harcourt.	Civil Engineering Applied to Construction.	Longmans
Webb, W. L.	*Railroad Construction	Chapman, Hall.
Wellington	*The Economic Theory of the Location of Railways.	,,

BOOKS (*continued*)—PERMANENT WAY.

AUTHOR.		PUBLISHER.
Allen, C. J.	Modern British Permanent Way	Railway Gazette.
Arnall, T.	Permanent Way for Tramways and Street Railways	Railway Engineer
Atkinson, F. W.	Points and Crossings and Junction Diagrams	
Camp, W. M.	*Notes on Track	The Author, Auburn Park, Chicago.
Clayton, S. E.	The Permanent Way Handbook.	Spon.
Cole, W. H.	Permanent Way Material, Platelaying and Points and Crossings	"
Couche, C.	Technical Working of Railways. Vol. I. Permanent Way.	Dulau.
Cuenot	Deformations of Railroad Track	Railway Gazette
Holt, R. B.	Tramway Track Construction and Maintenance	Tramway World
Jones, T. W.	The Permanent Way Pocket Book	Thacker
Perrott & Badger	The Practice of Railway Surveying and Permanent Way Work	Arnold
Rench, W. F.	*Roadway and Track	Simmonds Boardman
Roberts, S. S.	*Track Formulæ and Tables	Chapman, Hall.
Tratman	*Railway Track and Track Work	Constable.
Willard	*Maintenance of Way and Structures	McGraw Hill.
Wilson	*Elements of Railroad Track Construction	Chapman, Hall.
—	The Railway Signal and Permanent Engineer's Pocket Book	Loco. Publishing Co.
—	Questions and Answers on Railway Permanent Way	

BOOKS (*continued*)—SURVEYING AND SETTING OUT.

AUTHOR.		PUBLISHER.
Breed and Hosmer	*The Principles and Practice of Surveying.	Chapman, Hall.
Halden	Setting Out of Tube Railways*.	Spon
Haiste	Data, Railway Curves, &c.* ..	,,
Hearn and Watson	Railway Engineer's Field Book.	,,
Higgins, A. L. ..	Field Manual	Pitman.
,, ..	Transition Spiral and Its Introduction to Railway Curves.	Constable.
Johnson, J. B. ..	*Theory and Practice of Surveying	Chapman, Hall.
Kennedy and Hackwood	Setting Out Curves for Railways, 5 Chains to 3 Miles radius.	Spon.
Middleton and Chadwick	A Treatise on Surveying (2 Vols)	,,
Raymond ..	*Railroad Field Geometry	Chapman, Hall
,, ...	Railroad Field Manual	,,
Searles and Ives ..	*Field Engineering	,,
Shunk	The Field Engineer	Constable.
Stewart, R.	Handbook on Railway Surveying	Spon.
Trautwine	Field Practice of Laying Out Circular Curves.	Chapman Hall
Whitelaw	Surveying as Practised by Civil Engineers and Surveyors.	Crosby, Lockwood.
Williamson	Tables for Setting Out Curves 200 to 4,000 Metres Radius	Spon.

BOOKS (*continued*)—MATHEMATICAL TABLES.

	PUBLISHER.
Barlow's Tables of Squares, Cubes Roots, and Reciprocals of Numbers 1 to 10,000	Spon.
Chambers's Mathematical Tables. Logarithms, Trigonometrical, etc	Chambers.

PERIODICALS.

(In which frequent articles upon Permanent Way appear.)

The Railway Engineer Monthly 33, Fothill St. London, S.W.1

The Railway Gazette Weekly Ditto

*Railway Maintenance Engineer (Chicago). European Office, Simmonds-Boardman Publishing Co., 34, Victoria St. London S.W.1.

INSTITUTIONS, ASSOCIATIONS, ETC.

(Whose Reports, Journals, etc., include matters appertaining to Permanent Way.)

Board of Trade, Ministry of Transport.	Reports printed by H.M. Stationery Office. See their catalogue
International Railway Association.	Reports and Monthly Bulletins published by P. S. King, Gt. Smith St., London, S.W.1.
British Engineering Standards Association.	Reports, etc., obtainable from Secretary, 28, Victoria St. London, S.W.1.
Institution of Civil Engineers .	Minutes of Proceedings issued to Members only
Permanent Way Institution ..	Journal issued thrice annually. This is an educational institution for Inspectors and all employed in Engineering Depts. Secretary, W. E. McLewin, 6, Edith Road, Faversham, Kent.
American Railway Engineering Association.	Manual and Reports obtainable from Secretary, Chicago, Ill., U.S.A.
Indian Railway Board	Technical Papers obtainable from Superintendent of Government Printing, Calcutta, or P. S. King, London.
Tramways and Light Railways Association.	Journal published monthly by the Association. Caxton House, Westminster. Price 1/-.

TABLE I.

WEIGHTS AND MEASURES.

LINEAR AND SURVEYING MEASURE.

		Inches	Links	Feet	Yards	Poles	Chns.	Fur's.	Cms.	Metres.
1 Inch ...	=	...	...	...	...	...	...	...	2·54	...
1 Link ...	=	7·92	...	...	...	...	...	...	20·12	...
1 Foot ...	=	12	1·515	...	...	...	...	...	...	·3048
1 Yard ...	=	36	...	3	...	...	...	...	...	·9144
1 Pole ...	=	...	...	$16\frac{1}{2}$	$5\frac{1}{2}$	...	...	...	...	5·029
1 Chain...	=	792	100	66	22	4	...	...	...	20·116
1 Furlong	=	...	...	660	220	40	10	...	...	201·16
1 Mile ...	=	...	...	5280	1760	320	80	8	...	1609·31

One Fathom = 6 Feet.

SQUARE MEASURE.

		Sq. ins.	Sq. ft.	Sq. yards.	Sq. pols.	Roods.	Acres.
1 Sq. foot ...	=	144	...	...	...	...	...
1 „ yard ...	=	...	9	...	...	...	...
1 „ pole ...	=	...	$272\frac{1}{4}$	$30\frac{1}{4}$	...	...	...
1 Rood ...	=	...	...	1210	40	...	...
1 Acre ...	=	...	...	4840	160	4	...
1 Sq. mile ...	=	...	...	...	...	...	640

1 Sq. inch = 6·451 Sq. centimetres.

1 „ foot = ·0929 „ metres.

1 „ yard = ·8361 „ metres.

10 „ chains = 1 Acre.

TABLE 1 (*continued*)—CUBIC MEASURE.

	Cubic ins.	Cubic feet.	Cubic metres.
1 Cubic foot =	1728	...	·0283
1 „ yard =	...	27	·7645

1 Cubic ft. = 6·2355 Gallons.

ANGULAR MEASURE.

60 Seconds (60″) = 1 Minute.
60 Minutes (60′) = 1 Degree.
90 Degrees (90°) = 1 Right Angle.

By Circular or Radian Measure: A Radian is the angle at the centre of a circle, subtended by an arc equal to the radius. One Radian = 57°2958°, 3·1416 Radians = 180°

AVOIRDUPOIS WEIGHT.

	Ounces.	Pounds.	Qrs.	Cwts.	Kilogram's.
1 Pound ...	16	...	...	...	·4536
1 Quarter ...	...	28	...	...	...
1 Cwt.	...	112	4	...	50·802
1 Ton	...	2240	80	20	1016·048

10 Lbs. of water = 1 Gallon.

METRIC MEASURES.

LINEAR.

	Mms.	Cms.	Metrs.	Inches.	Feet	Yards.
1 Centimetre (cm.) =	10	...	...	·394	...	...
1 Metre (m.) =	1000	100	...	39·371	3·281	1·0936
1 Kilometre (km.) =	...	...	1000	...	...	1093·63

1 Kilometre = ·6214 Mile (⅝ nearly). 60 cm. = 1′ 11⅝″.
1·45 Metres (French Gauge) = 4′ 9·09″.

TABLE 1 (*continued*)—SQUARE.

	Sq. centi-metres.	Sq. metres		Sq. inches	Sq. ft.	Sq. yards.
1 Sq. centimetre (cm^2) =	...	...		·1550	...	...
1 „ metre (m^2) =	10,000	...			10·764	1·196

CUBIC.

	Cubic centimetres.		Cubic feet.	Cubic yards.
1 Cubic metre (m^3) =	1,000,000		35·3166	1·308

WEIGHT

	Grammes.		Pounds.
1 Kilogramme ...... =	1000		2·2046

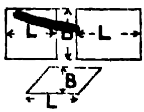
COMPOUND EQUIVALENTS.

1 Mile per hour .... = 1·6093 Kilometres per hour.
1 Foot per second .... = ·3048 Metres per second.
1 Pound per yard .... = ·496 Kilogs. per metre.
1 Kilometre per hour ...... = ·6214 Miles per hour.
1 Metre per Second .... = 3·2809 Feet per second.
1 Kilog. per metre .... = 2·016 Lbs. per yard.
1 Kilog. per sq. millimetre = ·635 Tons per sq. inch.

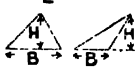
Note.—The Equivalents given throughout Table 1 enable conversions to be made: thus—because 1 foot = ·3048 metres; to convert feet to metres multiply the number of feet by ·3048.

TABLE 2.

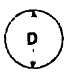
MENSURATION.



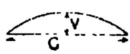
Parallelogram—
Area = Length × Breadth.



Triangle—
Area = Base × $\frac{1}{2}$ Height.



Circle—
Circumference = Diameter × 3·1416.
or „ × $3\frac{1}{7}$ (approx.).
Area = Square of diameter × ·7854.
or — „ „ × $\frac{11}{14}$ (approx.).

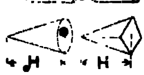


Segment—
Area = Chord × $\frac{2}{3}$ of Versed sine (approx.).

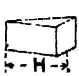
Irregular Figure—
To find Area: Divide the figure into triangles, parallelograms, and segments, and add their areas together;
Or: Divide into parallel strips: Area of each strip = Width × Length on centre line of strip: Add areas of strips together for total.

Parallelopiped*—
Solid Content = Length × Breadth × Depth.

Prism or Cylinder—
Solid Content = Area of end × Length.



Pyramid or Cone—
Solid Content = $\frac{1}{3}$ Area of end × Height.



Wedge—
Solid Content = $\frac{1}{2}$ Area of end × Height.

Sphere—
Solid Content = Cube of diameter × ·5236.

* Such as a rectangular wagon body, box or tank.

TABLE 2—MENSURATION (*continued*).

Irregular Solid Figures (such as Earthwork)—

To find Solid Content: Method 1—Take cross areas at every marked change in section and multiply the mean or average of each two adjacent areas by the distance between them. Add these volumes for the total.

Method 2—Decide upon an equal distance apart "D," at which cross areas shall be taken. Starting at $\frac{1}{2}$ D from one end, take cross areas at every distance D. The last area will occur either more or less than $\frac{1}{2}$ D from the end. Call the distance from the end "E." Multiply the sum of the cross areas, except the last, by D. Add the last area multiplied by $\frac{1}{2}$ D plus E.

These two methods are usually followed, but what is known as "The Prismoidal Rule" is more accurate.

If an end area becomes Nil by dying out to a point, the end portion should be taken as a pyramid.

TABLE 3.

WEIGHTS OF MATERIALS.

Note.—The weights of stone, timber, etc., even of the same description, vary considerably, and the figures may be taken only as a general guide.

BALLAST.

	Lbs. per Cubic foot.	Cubic feet per Ton.
Engine Cinder	56	40
Ironworks Slag	83	27
Limestone	95	24
Granite	95	24
Gravel with sand	100	22
Gravel, clean	90	25

TIMBER.

	Lbs. per Cubic feet.
Baltic Fir, clean	38
,, ,, creosoted	48
Red Pine	34
Pitch Pine	45
Larch	35
Oak	50

	Lbs.
Baltic Fir Sleeper, clean, 9′ × 10″ × 5″ ($3\frac{1}{8}$ c. ft.) =	118
,, ,, creosoted ,, ,, ,, =	148

TABLE 3 (*continued*)—METALS.

	Lbs. per cubic inch.	Lbs. per cubic foot.	Lbs. per square foot. 1in. thick.
Cast Iron	·2604	450	37·5
Wrought Iron	·2777	480	40
Mild Steel	·2833	489·6	40·8

Cast Iron. Sectional Area in sq. ins. × $3\frac{1}{8}$ = lbs. per ft. run.
Wrot. „ „ „ „ × $3\frac{1}{3}$ = „ „
Mild Steel. „ „ „ × 3·4 = „ „
„ „ „ „ × 10·2 = lbs. per yd. run.

Round Iron. Number of eighths of an inch in diameter squared × ·041 = lbs. per foot run.

Weight of Rails.—

Pounds per yard × $1\frac{3}{4}$ = Tons per mile of single rail.

To find weight per yard in lbs., when an odd length of rail has been weighed. Multiply weight of length in lbs. by 36, and divide by number of inches in the length.

TABLE 4.

SQUARES, SQUARE ROOTS, AND RECIPROCALS. OF NUMBERS 1 TO 1000.

Note.—To apply the Table to numbers containing decimals, and to numbers over 1000, see Chapter IV. and Table 4a. For the actual Reciprocal move the decimal point 3 places to the left.

SQUARES, SQUARE ROOTS, AND RECIPROCALS—*continued.*

No.	Square.	Square Root.	Recip. × 1000.	No.	Square.	Square Root.	Recip. × 1000.
1	1	1·0000	1000·000	51	2601	7·1414	19·6078
2	4	1·4142	500·000	52	2704	7·2111	19·2308
3	9	1·7321	333·333	53	2809	7·2801	18·8679
4	16	2·0000	250·000	54	2916	7·3485	18·5185
5	25	2·2361	200·000	55	3025	7·4162	18·1818
6	36	2·4495	166·667	56	3136	7·4833	17·8571
7	49	2·6458	142·857	57	3249	7·5498	17·5439
8	64	2·8284	125·000	58	3364	7·6158	17·2414
9	81	3·0000	111·111	59	3481	7·6811	16·9492
10	100	3·1623	100·000	60	3600	7·7460	16·6667
11	121	3·3166	90·9091	61	3721	7·8102	16·3934
12	144	3·4641	83·3333	62	3844	7·8740	16·1290
13	169	3·6056	76·9231	63	3969	7·9373	15·8730
14	196	3·7417	71·4286	64	4096	8·0000	15·6250
15	225	3·8730	66·6667	65	4225	8·0623	15·3846
16	256	4·0000	62·5000	66	4356	8·1240	15·1515
17	289	4·1231	58·8235	67	4489	8·1854	14·9254
18	324	4·2426	55·5556	68	4624	8·2462	14·7059
19	361	4·3589	52·6316	69	4761	8·3066	14·4928
20	400	4·4721	50·0000	70	4900	8·3666	14·2857
21	441	4·5826	47·6190	71	5041	8·4261	14·0845
22	484	4·6904	45·4545	72	5184	8·4853	13·8889
23	529	4·7958	43·4783	73	5329	8·5440	13·6986
24	576	4·8990	41·6667	74	5476	8·6023	13·5135
25	625	5·0000	40·0000	75	5625	8·6603	13·3333
26	676	5·0990	38·4615	76	5776	8·7178	13·1579
27	729	5·1962	37·0370	77	5929	8·7750	12·9870
28	784	5·2915	35·7143	78	6084	8·8318	12·8205
29	841	5·3852	34·4828	79	6241	8·8882	12·6582
30	900	5·4772	33·3333	80	6400	8·9443	12·5000
31	961	5·5678	32·2581	81	6561	9·0000	12·3457
32	1024	5·6569	31·2500	82	6724	9·0554	12·1951
33	1089	5·7446	30·3030	83	6889	9·1104	12·0482
34	1156	5·8310	29·4118	84	7056	9·1652	11·9048
35	1225	5·9161	28·5714	85	7225	9·2195	11·7647
36	1296	6·0000	27·7778	86	7396	9·2736	11·6279
37	1369	6·0828	27·0270	87	7569	9·3274	11·4943
38	1444	6·1644	26·3158	88	7744	9·3808	11·3636
39	1521	6·2450	25·6410	89	7921	9·4340	11·2360
40	1600	6·3246	25·0000	90	8100	9·4868	11·1111
41	1681	6·4031	24·3902	91	8281	9·5394	10·9890
42	1764	6·4807	23·8095	92	8464	9·5917	10·8696
43	1849	6·5574	23·2558	93	8649	9·6437	10·7527
44	1936	6·6332	22·7273	94	8836	9·6954	10·6383
45	2025	6·7082	22·2222	95	9025	9·7468	10·5263
46	2116	6·7823	21·7391	96	9216	9·7980	10·4167
47	2209	6·8557	21·2766	97	9409	9·8489	10·3093
48	2304	6·9282	20·8333	98	9604	9·8995	10·2041
49	2401	7·0000	20·4082	99	9801	9·9499	10·1010
50	2500	7·0711	20·0000	100	10000	10·0000	10·0000

SQUARES, SQUARE ROOTS, AND RECIPROCALS—*continued.*

No.	Square.	Square Root.	Recip. × 1000.	No.	Square.	Square Root.	Recip. × 1000.
101	10201	10·0499	9·90099	**151**	22801	12·2882	6·62252
102	10404	10·0995	9·80392	**152**	23104	12·3288	6·57895
103	10609	10·1489	9·70874	**153**	23409	12·3693	6·53595
104	10816	10·1980	9·61538	**154**	23716	12·4097	6·49351
105	11025	10·2470	9·52381	**155**	24025	12·4499	6·45161
106	11236	10·2956	9·43396	**156**	24336	12·4900	6·41026
107	11449	10·3441	9·34579	**157**	24649	12·5300	6·36943
108	11664	10·3923	9·25926	**158**	24964	12·5698	6·32911
109	11881	10·4403	9·17431	**159**	25281	12·6095	6·28931
110	12100	10·4881	9·09091	**160**	25600	12·6491	6·25000
111	12321	10·5357	9·00901	**161**	25921	12·6886	6·21118
112	12544	10·5830	8·92857	**162**	26244	12·7279	6·17284
113	12769	10·6301	8·84956	**163**	26569	12·7671	6·13497
114	12996	10·6771	8·77193	**164**	26896	12·8062	6·09756
115	13225	10·7238	8·69565	**165**	27225	12·8452	6·06061
116	13456	10·7703	8·62069	**166**	27556	12·8841	6·02410
117	13689	10·8167	8·54701	**167**	27889	12·9228	5·98802
118	13924	10·8628	8·47458	**168**	28224	12·9615	5·95238
119	14161	10·9087	8·40336	**169**	28561	13·0000	5·91716
120	14400	10·9545	8·33333	**170**	28900	13·0384	5·88235
121	14641	11·0000	8·26446	**171**	29241	13·0767	5·84795
122	14884	11·0454	8·19672	**172**	29584	13·1149	5·81395
123	15129	11·0905	8·13008	**173**	29929	13·1529	5·78035
124	15376	11·1355	8·06452	**174**	30276	13·1909	5·74713
125	15625	11·1803	8·00000	**175**	30625	13·2288	5·71429
126	15876	11·2250	7·93651	**176**	30976	13·2665	5·68182
127	16129	11·2694	7·87402	**177**	31329	13·3041	5·64972
128	16384	11·3137	7·81250	**178**	31684	13·3417	5·61798
129	16641	11·3578	7·75194	**179**	32041	13·3791	5·58659
130	16900	11·4018	7·69231	**180**	32400	13·4164	5·55556
131	17161	11·4455	7·63359	**181**	32761	13·4536	5·52486
132	17424	11·4891	7·57576	**182**	33124	13·4907	5·49451
133	17689	11·5326	7·51880	**183**	33489	13·5277	5·46448
134	17956	11·5758	7·46269	**184**	33856	13·5647	5·43478
135	18225	11·6190	7·40741	**185**	34225	13·6015	5·40541
136	18496	11·6619	7·35294	**186**	34596	13·6382	5·37634
137	18769	11·7047	7·29927	**187**	34969	13·6748	5·34759
138	19044	11·7473	7·24638	**188**	35344	13·7113	5·31915
139	19321	11·7898	7·19424	**189**	35721	13·7477	5·29101
140	19600	11·8322	7·14286	**190**	36100	13·7840	5·26316
141	19881	11·8743	7·09220	**191**	36481	13·8203	5·23560
142	20164	11·9164	7·04225	**192**	36864	13·8564	5·20833
143	20449	11·9583	6·99301	**193**	37249	13·8924	5·18135
144	20736	12·0000	6·94444	**194**	37636	13·9284	5·15464
145	21025	12·0416	6·89655	**195**	38025	13·9642	5·12821
146	21316	12·0830	6·84932	**196**	38416	14·0000	5·10204
147	21609	12·1244	6·80272	**197**	38809	14·0357	5·07614
148	21904	12·1655	6·75676	**198**	39204	14·0712	5·05051
149	22201	12·2066	6·71141	**199**	39601	14·1067	5·02513
150	22500	12·2474	6·66667	**200**	40000	14·1421	5·00000

SQUARES, SQUARE ROOTS, AND RECIPROCALS—*continued.*

No.	Square.	Square Root.	Recip. × 1000	No.	Square.	Square Root.	Recip. ÷ 1000.
201	40401	14·1774	4·97512	**251**	63001	15·8430	3·98406
202	40804	14·2127	4·95050	**252**	63504	15·8745	3·96825
203	41209	14·2478	4·92611	**253**	64009	15·9060	3·95257
204	41616	14·2829	4·90196	**254**	64516	15·9374	3·93701
205	42025	14·3178	4·87805	**255**	65025	15·9687	3·92157
206	42436	14·3527	4·85437	**256**	65536	16·0000	3·90625
207	42849	14·3875	4·83092	**257**	66049	16·0312	3·89105
208	43264	14·4222	4·80769	**258**	66564	16·0624	3·87597
209	43681	14·4568	4·78469	**259**	67081	16·0935	3·86100
210	44100	14·4914	4·76190	**260**	67600	16·1245	3·84615
211	44521	14·5258	4·73934	**261**	68121	16·1555	3·83142
212	44944	14·5602	4·71698	**262**	68644	16·1864	3·81679
213	45369	14·5945	4·69484	**263**	69169	16·2173	3·80228
214	45796	14·6287	4·67290	**264**	69696	16·2481	3·78788
215	46225	14·6629	4·65116	**265**	70225	16·2788	3·77358
216	46656	14·6969	4·62963	**266**	70756	16·3095	3·75940
217	47089	14·7309	4·60829	**267**	71289	16·3401	3·74532
218	47524	14·7648	4·58716	**268**	71824	16·3707	3·73134
219	47961	14·7986	4·56621	**269**	72361	16·4012	3·71747
220	48400	14·8324	4·54545	**270**	72900	16·4317	3·70370
221	48841	14·8661	4·52489	**271**	73441	16·4621	3·69004
222	49284	14·8997	4·50450	**272**	73984	16·4924	3·67647
223	49729	14·9332	4·48431	**273**	74529	16·5227	3·66300
224	50176	14·9666	4·46429	**274**	75076	16·5529	3·64964
225	50625	15·0000	4·44444	**275**	75625	16·5831	3·63636
226	51076	15·0333	4·42478	**276**	76176	16·6132	3·62319
227	51529	15·0665	4·40529	**277**	76729	16·6433	3·61011
228	51984	15·0997	4·38596	**278**	77284	16·6733	3·59712
229	52441	15·1327	4·36681	**279**	77841	16·7033	3·58423
230	52900	15·1658	4·34783	**280**	78400	16·7332	3·57143
231	53361	15·1987	4·32900	**281**	78961	16·7631	3·55872
232	53824	15·2315	4·31034	**282**	79524	16·7929	3·54610
233	54289	15·2643	4·29185	**283**	80089	16·8226	3·53357
234	54756	15·2971	4·27350	**284**	80656	16·8523	3·52113
235	55225	15·3297	4·25532	**285**	81225	16·8819	3·50877
236	55696	15·3623	4·23729	**286**	81796	16·9115	3·49650
237	56169	15·3948	4·21941	**287**	82369	16·9411	3·48432
238	56644	15·4272	4·20168	**288**	82944	16·9706	3·47222
239	57121	15·4596	4·18410	**289**	83521	17·0000	3·46021
240	57600	15·4919	4·16667	**290**	84100	17·0294	3·44828
241	58081	15·5242	4·14938	**291**	84681	17·0587	3·43643
242	58564	15·5563	4·13223	**292**	85264	17·0880	3·42466
243	59049	15·5885	4·11523	**293**	85849	17·1172	3·41297
244	59536	15·6205	4·09836	**294**	86436	17·1464	3·40136
245	60025	15·6525	4·08163	**295**	87025	17·1756	3·38983
246	60516	15·6844	4·06504	**296**	87616	17·2047	3·37838
247	61009	15·7162	4·04858	**297**	88209	17·2337	3·36700
248	61504	15·7480	4·03226	**298**	88804	17·2627	3·35570
249	62001	15·7797	4·01606	**299**	89401	17·2916	3·34448
250	62500	15·8114	4·00000	**300**	90000	17·3205	3·33333

SQUARES, SQUARE ROOTS, AND RECIPROCALS—*continued.*

No.	Square.	Square Root.	Recip. × 1000.	No.	Square.	Square Root.	Recip. × 1000.
301	90601	17·3494	3·32226	**351**	123201	18·7350	2·84900
302	91204	17·3781	3·31126	**352**	123904	18·7617	2·84091
303	91809	17·4069	3·30033	**353**	124609	18·7883	2·83286
304	92416	17·4356	3·28947	**354**	125316	18·8149	2·82486
305	93025	17·4642	3·27869	**355**	126025	18·8414	2·81690
306	93636	17·4929	3·26797	**356**	126736	18·8680	2·80899
307	94249	17·5214	3·25733	**357**	127449	18·8944	2·80112
308	94864	17·5499	3·24675	**358**	128164	18·9209	2·79330
309	95481	17·5784	3·23625	**359**	128881	18·9473	2·78552
310	96100	17·6068	3·22581	**360**	129600	18·9737	2·77778
311	96721	17·6352	3·21543	**361**	130321	19·0000	2·77008
312	97344	17·6635	3·20513	**362**	131044	19·0263	2·76243
313	97969	17·6918	3·19489	**363**	131769	19·0526	2·75482
314	98596	17·7200	3·18471	**364**	132496	19·0788	2·74725
315	99225	17·7482	3·17460	**365**	133225	19·1050	2·73973
316	99856	17·7764	3·16456	**366**	133956	19·1311	2·73224
317	100489	17·8045	3·15457	**367**	134689	19·1572	2·72480
318	101124	17·8326	3·14465	**368**	135424	19·1833	2·71739
319	101761	17·8606	3·13480	**369**	136161	19·2094	2·71003
320	102400	17·8885	3·12500	**370**	136900	19·2354	2·70270
321	103041	17·9165	3·11527	**371**	137641	19·2614	2·69542
322	103684	17·9444	3·10559	**372**	138384	19·2873	2·68817
323	104329	17·9722	3·09598	**373**	139129	19·3132	2·68097
324	104976	18·0000	3·08642	**374**	139876	19·3391	2·67380
325	105625	18·0278	3·07692	**375**	140625	19·3649	2·66667
326	106276	18·0555	3·06749	**376**	141376	19·3907	2·65957
327	106929	18·0831	3·05810	**377**	142129	19·4165	2·65252
328	107584	18·1108	3·04878	**378**	142884	19·4422	2·64550
329	108241	18·1384	3·03951	**379**	143641	19·4679	2·63852
330	108900	18·1659	3·03030	**380**	144400	19·4936	2·63158
331	109561	18·1934	3·02115	**381**	145161	19·5192	2·62467
332	110224	18·2209	3·01205	**382**	145924	19·5448	2·61780
333	110889	18·2483	3·00300	**383**	146689	19·5704	2·61097
334	111556	18·2757	2·99401	**384**	147456	19·5959	2·60417
335	112225	18·3030	2·98507	**385**	148225	19·6214	2·59740
336	112896	18·3303	2·97619	**386**	148996	19·6469	2·59067
337	113569	18·3576	2·96736	**387**	149769	19·6723	2·58398
338	114244	18·3848	2·95858	**388**	150544	19·6977	2·57732
339	114921	18·4120	2·94985	**389**	151321	19·7231	2·57069
340	115600	18·4391	2·94118	**390**	152100	19·7484	2·56410
341	116281	18·4662	2·93255	**391**	152881	19·7737	2·55755
342	116964	18·4932	2·92398	**392**	153664	19·7990	2·55102
343	117649	18·5203	2·91545	**393**	154449	19·8242	2·54453
344	118336	18·5472	2·90698	**394**	155236	19·8494	2·53807
345	119025	18·5742	2·89855	**395**	156025	19·8746	2·53165
346	119716	18·6011	2·89017	**396**	156816	19·8997	2·52525
347	120409	18·6279	2·88184	**397**	157609	19·9249	2·51889
348	121104	18·6548	2·87356	**398**	158404	19·9499	2·51256
349	121801	18·6815	2·86533	**399**	159201	19·9750	2·50627
350	122500	18·7083	2·85714	**400**	160000	20·0000	2·50000

SQUARES, SQUARE ROOTS, AND RECIPROCALS—*continued.*

No.	Square.	Square Root.	Recip. × 1000.	No.	Square.	Square Root.	Recip. × 1000.
401	160801	20·0250	2·49377	**451**	203401	21·2368	2·21730
402	161604	20·0499	2·48756	**452**	204304	21·2603	2·21239
403	162409	20·0749	2·48139	**453**	205209	21·2838	2·20751
404	163216	20·0998	2·47525	**454**	206116	21·3073	2·20264
405	164025	20·1246	2·46914	**455**	207025	21·3307	2·19780
406	164836	20·1494	2·46305	**456**	207936	21·3542	2·19298
407	165649	20·1742	2·45700	**457**	208849	21·3776	2·18818
408	166464	20·1990	2·45098	**458**	209764	21·4009	2·18341
409	167281	20·2237	2·44499	**459**	210681	21·4243	2·17865
410	168100	20·2485	2·43902	**460**	211600	21·4476	2·17391
411	168921	20·2731	2·43309	**461**	212521	21·4709	2·16920
412	169744	20·2978	2·42718	**462**	213444	21·4942	2·16450
413	170569	20·3224	2·42131	**463**	214369	21·5174	2·15983
414	171396	20·3470	2·41546	**464**	215296	21·5407	2·15517
415	172225	20·3715	2·40964	**465**	216225	21·5639	2·15054
416	173056	20·3961	2·40385	**466**	217156	21·5870	2·14592
417	173889	20·4206	2·39808	**467**	218089	21·6102	2·14133
418	174724	20·4450	2·39234	**468**	219024	21·6333	2·13675
419	175561	20·4695	2·38664	**469**	219961	21·6564	2·13220
420	176400	20·4939	2·38095	**470**	220900	21·6795	2·12766
421	177241	20·5183	2·37530	**471**	221841	21·7025	2·12314
422	178084	20·5426	2·36967	**472**	222784	21·7256	2·11864
423	178929	20·5670	2·36407	**473**	223729	21·7486	2·11417
424	179776	20·5913	2·35849	**474**	224676	21·7715	2·10971
425	180625	20·6155	2·35294	**475**	225625	21·7945	2·10526
426	181476	20·6398	2·34742	**476**	226576	21·8174	2·10084
427	182329	20·6640	2·34192	**477**	227529	21·8403	2·09644
428	183184	20·6882	2·33645	**478**	228484	21·8632	2·09205
429	184041	20·7123	2·33100	**479**	229441	21·8861	2·08768
430	184900	20·7364	2·32558	**480**	230400	21·9089	2·08333
431	185761	20·7605	2·32019	**481**	231361	21·9317	2·07900
432	186624	20·7846	2·31482	**482**	232324	21·9545	2·07469
433	187489	20·8087	2·30947	**483**	233289	21·9773	2·07039
434	188356	20·8327	2·30415	**484**	234256	22·0000	2·06612
435	189225	20·8567	2·29885	**485**	235225	22·0227	2·06186
436	190096	20·8806	2·29358	**486**	236196	22·0454	2·05761
437	190969	20·9045	2·28833	**487**	237169	22·0681	2·05339
438	191844	20·9284	2·28311	**488**	238144	22·0907	2·04918
439	192721	20·9523	2·27790	**489**	239121	22·1133	2·04499
440	193600	20·9762	2·27273	**490**	240100	22·1359	2·04082
441	194481	21·0000	2·26757	**491**	241081	22·1585	2·03666
442	195364	21·0238	2·26244	**492**	242064	22·1811	2·03252
443	196249	21·0476	2·25734	**493**	243049	22·2036	2·02840
444	197136	21·0713	2·25225	**494**	244036	22·2261	2·02429
445	198025	21·0950	2·24719	**495**	245025	22·2486	2·02020
446	198916	21·1187	2·24215	**496**	246016	22·2711	2·01613
447	199809	21·1424	2·23714	**497**	247009	22·2935	2·01207
448	200704	21·1660	2·23214	**498**	248004	22·3159	2·00803
449	201601	21·1896	2·22717	**499**	249001	22·3383	2·00401
450	202500	21·2132	2·22222	**500**	250000	22·3607	2·00000

SQUARES, SQUARE ROOTS, AND RECIPROCALS—*continued.*

No.	Square.	Square Root.	Recip. × 1000.	No.	Square.	Square Root.	Recip. × 1000.
501	251001	22·3830	1·99601	**551**	303601	23·4734	1·81488
502	252004	22·4054	1·99203	**552**	304704	23·4947	1·81159
503	253009	22·4277	1·98807	**553**	305809	23·5160	1·80832
504	254016	22·4499	1·98413	**554**	306916	23·5372	1·80505
505	255025	22·4722	1·98020	**555**	308025	23·5584	1·80180
506	256036	22·4944	1·97629	**556**	309136	23·5797	1·79856
507	257049	22·5167	1·97239	**557**	310249	23·6008	1·79533
508	258064	22·5389	1·96850	**558**	311364	23·6220	1·79211
509	259081	22·5610	1·96464	**559**	312481	23·6432	1·78891
510	260100	22·5832	1·96078	**560**	313600	23·6643	1·78571
511	261121	22·6053	1·95695	**561**	314721	23·6854	1·78253
512	262144	22·6274	1·95312	**562**	315844	23·7065	1·77936
513	263169	22·6495	1·94932	**563**	316969	23·7276	1·77620
514	264196	22·6716	1·94553	**564**	318096	23·7487	1·77305
515	265225	22·6936	1·94175	**565**	319225	23·7697	1·76991
516	266256	22·7156	1·93798	**566**	320356	23·7908	1·76678
517	267289	22·7376	1·93424	**567**	321489	23·8118	1·76367
518	268324	22·7596	1·93050	**568**	322624	23·8328	1·76056
519	269361	22·7816	1·92678	**569**	323761	23·8537	1·75747
520	270400	22·8035	1·92308	**570**	324900	23·8747	1·75439
521	271441	22·8254	1·91939	**571**	326041	23·8956	1·75131
522	272484	22·8473	1·91571	**572**	327184	23·9165	1·74825
523	273529	22·8692	1·91205	**573**	328329	23·9374	1·74520
524	274576	22·8910	1·90840	**574**	329476	23·9583	1·74216
525	275625	22·9129	1·90476	**575**	330625	23·9792	1·73913
526	276676	22·9347	1·90114	**576**	331776	24·0000	1·73611
527	277729	22·9565	1·89753	**577**	332929	24·0208	1·73310
528	278784	22·9783	1·89394	**578**	334084	24·0416	1·73010
529	279841	23·0000	1·89036	**579**	335241	24·0624	1·72712
530	280900	23·0217	1·88679	**580**	336400	24·0832	1·72414
531	281961	23·0434	1·88324	**581**	337561	24·1039	1·72117
532	283024	23·0651	1·87970	**582**	338724	24·1247	1·71821
533	284089	23·0868	1·87617	**583**	339889	24·1454	1·71527
534	285156	23·1084	1·87266	**584**	341056	24·1661	1·71233
535	286225	23·1301	1·86916	**585**	342225	24·1868	1·70940
536	287296	23·1517	1·86567	**586**	343396	24·2074	1·70649
537	288369	23·1733	1·86220	**587**	344569	24·2281	1·70358
538	289444	23·1948	1·85874	**588**	345744	24·2487	1·70068
539	290521	23·2164	1·85529	**589**	346921	24·2693	1·69779
540	291600	23·2379	1·85185	**590**	348100	24·2899	1·69492
541	292681	23·2594	1·84843	**591**	349281	24·3105	1·69205
542	293764	23·2809	1·84502	**592**	350464	24·3311	1·68919
543	294849	23·3024	1·84162	**593**	351649	24·3516	1·68634
544	295936	23·3238	1·83824	**594**	352836	24·3721	1·68350
545	297025	23·3452	1·83486	**595**	354025	24·3926	1·68067
546	298116	23·3666	1·83150	**596**	355216	24·4131	1·67785
547	299209	23·3880	1·82815	**597**	356409	24·4336	1·67504
548	300304	23·4094	1·82482	**598**	357604	24·4540	1·67224
549	301401	23·4307	1·82149	**599**	358801	24·4745	1·66945
550	302500	23·4521	1·81818	**600**	360000	24·4949	1·66667

SQUARES, SQUARE ROOTS, AND RECIPROCALS—*continued.*

No.	Square.	Square Root.	Recip. 1000.	No.	Square.	Square Root.	Recip. 1000.
601	361201	24·5153	1·66389	**651**	423801	25·5147	1·53610
602	362404	24·5357	1·66113	**652**	425104	25·5343	1·53374
603	363609	24·5561	1·65837	**653**	426409	25·5539	1·53139
604	364816	24·5764	1·65563	**654**	427716	25·5734	1·52905
605	366025	24·5967	1·65289	**655**	429025	25·5930	1·52672
606	367236	24·6171	1·65017	**656**	430336	25·6125	1·52439
607	368449	24·6374	1·64745	**657**	431649	25·6320	1·52207
608	369664	24·6577	1·64474	**658**	432964	25·6515	1·51976
609	370881	24·6779	1·64204	**659**	434281	25·6710	1·51745
610	372100	24·6982	1·63934	**660**	435600	25·6905	1·51515
611	373321	24·7184	1·63666	**661**	436921	25·7099	1·51286
612	374544	24·7386	1·63399	**662**	438244	25·7294	1·51057
613	375769	24·7588	1·63132	**663**	439569	25·7488	1·50830
614	376996	24·7790	1·62866	**664**	440896	25·7682	1·50602
615	378225	24·7992	1·62602	**665**	442225	25·7876	1·50376
616	379456	24·8193	1·62338	**666**	443556	25·8070	1·50150
617	380689	24·8395	1·62075	**667**	444889	25·8263	1·49925
618	381924	24·8596	1·61812	**668**	446224	25·8457	1·49701
619	383161	24·8797	1·61551	**669**	447561	25·8650	1·49477
620	384400	24·8998	1·61290	**670**	448900	25·8844	1·49254
621	385641	24·9199	1·61031	**671**	450241	25·9037	1·49031
622	386884	24·9399	1·60772	**672**	451584	25·9230	1·48810
623	388129	24·9600	1·60514	**673**	452929	25·9422	1·48588
624	389376	24·9800	1·60256	**674**	454276	25·9615	1·48368
625	390625	25·0000	1·60000	**675**	455625	25·9808	1·48148
626	391876	25·0200	1·59744	**676**	456976	26·0000	1·47929
627	393129	25·0400	1·59490	**677**	458329	26·0192	1·47711
628	394384	25·0599	1·59236	**678**	459684	26·0384	1·47493
629	395641	25·0799	1·58983	**679**	461041	26·0576	1·47275
630	396900	25·0998	1·58730	**680**	462400	26·0768	1·47059
631	398161	25·1197	1·58479	**681**	463761	26·0960	1·46843
632	399424	25·1396	1·58228	**682**	465124	26·1151	1·46628
633	400689	25·1595	1·57978	**683**	466489	26·1343	1·46413
634	401956	25·1794	1·57729	**684**	467856	26·1534	1·46199
635	403225	25·1992	1·57480	**685**	469225	26·1725	1·45985
636	404496	25·2190	1·57233	**686**	470596	26·1916	1·45773
637	405769	25·2389	1·56986	**687**	471969	26·2107	1·45560
638	407044	25·2587	1·56740	**688**	473344	26·2298	1·45349
639	408321	25·2784	1·56495	**689**	474721	26·2488	1·45138
640	409600	25·2982	1·56250	**690**	476100	26·2679	1·44928
641	410881	25·3180	1·56006	**691**	477481	26·2869	1·44718
642	412164	25·3377	1·55763	**692**	478864	26·3059	1·44509
643	413449	25·3574	1·55521	**693**	480249	26·3249	1·44300
644	414736	25·3772	1·55280	**694**	481636	26·3439	1·44092
645	416025	25·3969	1·55039	**695**	483025	26·3629	1·43885
646	417316	25·4165	1·54799	**696**	484416	26·3818	1·43678
647	418609	25·4362	1·54560	**697**	485809	26·4008	1·43472
648	419904	25·4558	1·54321	**698**	487204	26·4197	1·43267
649	421201	25·4755	1·54083	**699**	488601	26·4386	1·43062
650	422500	25·4951	1·53846	**700**	490000	26·4575	1·42857

SQUARES, SQUARE ROOTS, AND RECIPROCALS—*continued.*

No.	Square.	Square Root.	Recip. × 1000.	No.	Square.	Square Root.	Recip. × 1000.
701	491401	26·4764	1·42653	**751**	564001	27·4044	1·33156
702	492804	26·4953	1·42450	**752**	565504	27·4226	1·32979
703	494209	26·5141	1·42248	**753**	567009	27·4408	1·32802
704	495616	26·5330	1·42046	**754**	568516	27·4591	1·32626
705	497025	26·5518	1·41844	**755**	570025	27·4773	1·32450
706	498436	26·5707	1·41643	**756**	571536	27·4955	1·32275
707	499849	26·5895	1·41443	**757**	573049	27·5136	1·32100
708	501264	26·6083	1·41243	**758**	574564	27·5318	1·31926
709	502681	26·6271	1·41044	**759**	576081	27·5500	1·31752
710	504100	26·6458	1·40845	**760**	577600	27·5681	1·31579
711	505521	26·6646	1·40647	**761**	579121	27·5862	1·31406
712	506944	26·6833	1·40449	**762**	580644	27·6043	1·31234
713	508369	26·7021	1·40253	**763**	582169	27·6225	1·31062
714	509796	26·7208	1·40056	**764**	583696	27·6405	1·30890
715	511225	26·7395	1·39860	**765**	585225	27·6586	1·30719
716	512656	26·7582	1·39665	**766**	586756	27·6767	1·30548
717	514089	26·7769	1·39470	**767**	588289	27·6948	1·30378
718	515524	26·7955	1·39276	**768**	589824	27·7128	1·30208
719	516961	26·8142	1·39082	**769**	591361	27·7308	1·30039
720	518400	26·8328	1·38889	**770**	592900	27·7489	1·29870
721	519841	26·8514	1·38696	**771**	594441	27·7669	1·29702
722	521284	26·8701	1·38504	**772**	595984	27·7849	1·29534
723	522729	26·8887	1·38313	**773**	597529	27·8029	1·29366
724	524176	26·9072	1·38122	**774**	599076	27·8209	1·29199
725	525625	26·9258	1·37931	**775**	600625	27·8388	1·29032
726	527076	26·9444	1·37741	**776**	602176	27·8568	1·28866
727	528529	26·9629	1·37552	**777**	603729	27·8747	1·28700
728	529984	26·9815	1·37363	**778**	605284	27·8927	1·28535
729	531441	27·0000	1·37174	**779**	606841	27·9106	1·28370
730	532900	27·0185	1·36986	**780**	608400	27·9285	1·28205
731	534361	27·0370	1·36799	**781**	609961	27·9464	1·28041
732	535824	27·0555	1·36612	**782**	611524	27·9643	1·27877
733	537289	27·0740	1·36426	**783**	613089	27·9821	1·27714
734	538756	27·0924	1·36240	**784**	614656	28·0000	1·27551
735	540225	27·1109	1·36054	**785**	616225	28·0179	1·27389
736	541696	27·1293	1·35870	**786**	617796	28·0357	1·27226
737	543169	27·1477	1·35685	**787**	619369	28·0535	1·27065
738	544644	27·1662	1·35501	**788**	620944	28·0713	1·26904
739	546121	27·1846	1·35318	**789**	622521	28·0891	1·26743
740	547600	27·2029	1·35135	**790**	624100	28·1069	1·26582
741	549081	27·2213	1·34953	**791**	625681	28·1247	1·26422
742	550564	27·2397	1·34771	**792**	627264	28·1425	1·26263
743	552049	27·2580	1·34590	**793**	628849	28·1603	1·26103
744	553536	27·2764	1·34409	**794**	630436	28·1780	1·25945
745	555025	27·2947	1·34228	**795**	632025	28·1957	1·25786
746	556516	27·3130	1·34048	**796**	633616	28·2135	1·25628
747	558009	27·3313	1·33869	**797**	635209	28·2312	1·25471
748	559504	27·3496	1·33690	**798**	636804	28·2489	1·25313
749	561001	27·3679	1·33511	**799**	638401	28·2666	1·25156
750	562500	27·3861	1·33333	**800**	640000	28·2843	1·25000

SQUARES, SQUARE ROOTS, AND RECIPROCALS—*continued.*

No.	Square.	Square Root.	Recip. × 1000.	No.	Square.	Square Root.	Recip. × 1000.
801	641601	28·3019	1·24844	**851**	724201	29·1719	1·17509
802	643204	28·3196	1·24688	**852**	725904	29·1890	1·17371
803	644809	28·3373	1·24533	**853**	727609	29·2062	1·17233
804	646416	28·3549	1·24378	**854**	729316	29·2233	1·17096
805	648025	28·3725	1·24224	**855**	731025	29·2404	1·16959
806	649636	28·3901	1·24069	**856**	732736	29·2575	1·16822
807	651249	28·4077	1·23916	**857**	734449	29·2746	1·16686
808	652864	28·4253	1·23762	**858**	736164	29·2916	1·16550
809	654481	28·4429	1·23609	**859**	737881	29·3087	1·16414
810	656100	28·4605	1·23457	**860**	739600	29·3258	1·16279
811	657721	28·4781	1·23305	**861**	741321	29·3428	1·16144
812	659344	28·4956	1·23153	**862**	743044	29·3598	1·16009
813	660969	28·5132	1·23001	**863**	744769	29·3769	1·15875
814	662596	28·5307	1·22850	**864**	746496	29·3939	1·15741
815	664225	28·5482	1·22699	**865**	748225	29·4109	1·15607
816	665856	28·5657	1·22549	**866**	749956	29·4279	1·15473
817	667489	28·5832	1·22399	**867**	751689	29·4449	1·15340
818	669124	28·6007	1·22249	**868**	753424	29·4618	1·15207
819	670761	28·6182	1·22100	**869**	755161	29·4788	1·15075
820	672400	28·6356	1·21951	**870**	756900	29·4958	1·14943
821	674041	28·6531	1·21803	**871**	758641	29·5127	1·14811
822	675684	28·6705	1·21655	**872**	760384	29·5296	1·14679
823	677329	28·6880	1·21507	**873**	762129	29·5466	1·14548
824	678976	28·7054	1·21359	**874**	763876	29·5635	1·14416
825	680625	28·7228	1·21212	**875**	765625	29·5804	1·14286
826	682276	28·7402	1·21065	**876**	767376	29·5973	1·14155
827	683929	28·7576	1·20919	**877**	769129	29·6142	1·14025
828	685584	28·7750	1·20773	**878**	770884	29·6311	1·13895
829	687241	28·7924	1·20627	**879**	772641	29·6479	1·13766
830	688900	28·8097	1·20482	**880**	774400	29·6648	1·13636
831	690561	28·8271	1·20337	**881**	776161	29·6816	1·13507
832	692224	28·8444	1·20192	**882**	777924	29·6985	1·13379
833	693889	28·8617	1·20048	**883**	779689	29·7153	1·13250
834	695556	28·8791	1·19904	**884**	781456	29·7321	1·13122
835	697225	28·8964	1·19760	**885**	783225	29·7489	1·12994
836	698896	28·9137	1·19617	**886**	784996	29·7658	1·12867
837	700569	28·9310	1·19474	**887**	786769	29·7825	1·12740
838	702244	28·9482	1·19332	**888**	788544	29·7993	1·12613
839	703921	28·9655	1·19190	**889**	790321	29·8161	1·12486
840	705600	28·9828	1·19048	**890**	792100	29·8329	1·12360
841	707281	29·0000	1·18906	**891**	793881	29·8496	1·12233
842	708964	29·0172	1·18765	**892**	795664	29·8664	1·12108
843	710649	29·0345	1·18624	**893**	797449	29·8831	1·11982
844	712336	29·0517	1·18483	**894**	799236	29·8998	1·11857
845	714025	29·0689	1·18343	**895**	801025	29·9166	1·11732
846	715716	29·0861	1·18203	**896**	802816	29·9333	1·11607
847	717409	29·1033	1·18064	**897**	804609	29·9500	1·11483
848	719104	29·1204	1·17925	**898**	806404	29·9666	1·11359
849	720801	29·1376	1·17786	**899**	808201	29·9833	1·11235
850	722500	29·1548	1·17647	**900**	810000	30·0000	1·11111

SQUARES, SQUARE ROOTS, AND RECIPROCALS—*continued.*

No.	Square.	Square Root.	Recip. × 1000.	No.	Square.	Square Root.	Recip. × 1000.
901	811801	30·0167	1·10988	**951**	904401	30·8383	1·05152
902	813604	30·0333	1·10865	**952**	906304	30·8545	1·05042
903	815409	30·0500	1·10742	**953**	908209	30·8707	1·04932
904	817216	30·0666	1·10619	**954**	910116	30·8869	1·04822
905	819025	30·0832	1·10497	**955**	912025	30·9031	1·04712
906	820836	30·0998	1·10375	**956**	913936	30·9192	1·04603
907	822649	30·1164	1·10254	**957**	915849	30·9354	1·04493
908	824464	30·1330	1·10132	**958**	917764	30·9516	1·04384
909	826281	30·1496	1·10011	**959**	919681	30·9677	1·04275
910	828100	30·1662	1·09890	**960**	921600	30·9839	1·04167
911	829921	30·1828	1·09769	**961**	923521	31·0000	1·04058
912	831744	30·1993	1·09649	**962**	925444	31·0161	1·03950
913	833569	30·2159	1·09529	**963**	927369	31·0322	1·03842
914	835396	30·2324	1·09409	**964**	929296	31·0483	1·03734
915	837225	30·2490	1·09290	**965**	931225	31·0644	1·03627
916	839056	30·2655	1·09170	**966**	933156	31·0805	1·03520
917	840889	30·2820	1·09051	**967**	935089	31·0966	1·03413
918	842724	30·2985	1·08932	**968**	937024	31·1127	1·03306
919	844561	30·3150	1·08814	**969**	938961	31·1288	1·03199
920	846400	30·3315	1·08696	**970**	940900	31·1448	1·03093
921	848241	30·3480	1·08578	**971**	942841	31·1609	1·02987
922	850084	30·3645	1·08460	**972**	944784	31·1769	1·02881
923	851929	30·3809	1·08342	**973**	946729	31·1929	1·02775
924	853776	30·3974	1·08225	**974**	948676	31·2090	1·02669
925	855625	30·4138	1·08108	**975**	950625	31·2250	1·02564
926	857476	30·4302	1·07991	**976**	952576	31·2410	1·02459
927	859329	30·4467	1·07875	**977**	954529	31·2570	1·02354
928	861184	30·4631	1·07759	**978**	956484	31·2730	1·02249
929	863041	30·4795	1·07643	**979**	958441	31·2890	1·02145
930	864900	30·4959	1·07527	**980**	960400	31·3050	1·02041
931	866761	30·5123	1·07411	**981**	962361	31·3209	1·01937
932	868624	30·5287	1·07296	**982**	964324	31·3369	1·01833
933	870489	30·5450	1·07181	**983**	966289	31·3528	1·01729
934	872356	30·5614	1·07066	**984**	968256	31·3688	1·01626
935	874225	30·5778	1·06952	**985**	970225	31·3847	1·01523
936	876096	30·5941	1·06838	**986**	972196	31·4006	1·01420
937	877969	30·6105	1·06724	**987**	974169	31·4166	1·01317
938	879844	30·6268	1·06610	**988**	976144	31·4325	1·01215
939	881721	30·6431	1·06496	**989**	978121	31·4484	1·01112
940	883600	30·6594	1·06383	**990**	980100	31·4643	1·01010
941	885481	30·6757	1·06270	**991**	982081	31·4802	1·00908
942	887364	30·6920	1·06157	**992**	984064	31·4960	1·00806
943	889249	30·7083	1·06045	**993**	986049	31·5119	1·00705
944	891136	30·7246	1·05932	**994**	988036	31·5278	1·00604
945	893025	30·7409	1·05820	**995**	990025	31·5436	1·00503
946	894916	30·7571	1·05708	**996**	992016	31·5595	1·00402
947	896809	30·7734	1·05597	**997**	994009	31·5753	1·00301
948	898704	30·7896	1·05485	**998**	996004	31·5911	1·00200
949	900601	30·8058	1·05374	**999**	998001	31·6070	1·00100
950	902500	30·8221	1·05263				

TABLE 4a.

FOR THE APPLICATION OF TABLE 4 TO SQUARES OF NUMBERS WITH FOUR DIGITS.

Diff. for 1 = $(2 \times N) + 1$	Diff. for 6 = $(12 \times N) + 36$
,, ,, 2 = $(4 \times N) + 4$	,, ,, 7 = $(14 \times N) + 49$
,, ,, 3 = $(6 \times N) + 9$	,, ,, 8 = $(16 \times N) + 64$
,, ,, 4 = $(8 \times N) + 16$	,, ,, 9 = $(18 \times N) + 81$
,, ,, 5 = $(10 \times N) + 25$	,, ,, 10 = $(20 \times N) + 100$

N is the number formed of the first 3 digits, followed by 0.

EXAMPLE.—Square of 9716 :—

$$9710^2 = 94284100$$

Add for 6 :—

$$(12 \times 9710) + 36 = 116556$$

$$9716^2 = \underline{\underline{94400656}}$$

Alternatively the square of 9720 may be taken, and the difference for 4 deducted.

TABLE 5.

DECIMAL EQUIVALENTS OF 8ths, 16ths AND 32nds.

Fractions.	Decimals.	Fractions.	Decimals.
$\frac{1}{32}$	·03125	$\frac{1}{2}$	·5
$\frac{1}{16}$	·0625	$\frac{17}{32}$	·53125
$\frac{3}{32}$	·09375	$\frac{9}{16}$	·5625
$\frac{1}{8}$	·125	$\frac{19}{32}$	·59375
$\frac{5}{32}$	·15625	$\frac{5}{8}$	·625
$\frac{3}{16}$	·1875	$\frac{21}{32}$	·65625
$\frac{7}{32}$	·21875	$\frac{11}{16}$	·6875
$\frac{1}{4}$	·25	$\frac{23}{32}$	·71875
$\frac{9}{32}$	·28125	$\frac{3}{4}$	·75
$\frac{5}{16}$	·3125	$\frac{25}{32}$	·78125
$\frac{11}{32}$	·34375	$\frac{13}{16}$	·8125
$\frac{3}{8}$	·375	$\frac{27}{32}$	·84375
$\frac{13}{32}$	·40625	$\frac{7}{8}$	·875
$\frac{7}{16}$	·4375	$\frac{29}{32}$	·90625
$\frac{15}{32}$	·46875	$\frac{15}{16}$	·9375
		$\frac{31}{32}$	·96875

TABLE 6.

INCHES CONVERTED TO DECIMALS OF A FOOT.

Inch.	0″	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″	11″
0	0	·0833	·1667	·2500	·3333	·4167	·5000	·5833	·6667	·7500	·8333	·9167
$\frac{1}{16}$	·0052	·0885	·1719	·2552	·3385	·4219	·5052	·5885	·6719	·7552	·8385	·9219
$\frac{1}{8}$	·0104	·0937	·1771	·2604	·3437	·4271	·5104	·5937	·6771	·7604	·8437	·9271
$\frac{3}{16}$	·0156	·0990	·1823	·2656	·3490	·4323	·5156	·5990	·6823	·7656	·8490	·9323
$\frac{1}{4}$	·0208	·1042	·1875	·2708	·3542	·4375	·5208	·6042	·6875	·7708	·8542	·9375
$\frac{5}{16}$	·0260	·1094	·1927	·2760	·3594	·4427	·5260	·6094	·6927	·7760	·8594	·9427
$\frac{3}{8}$	·0312	·1146	·1979	·2812	·3646	·4479	·5312	·6146	·6979	·7812	·8646	·9479
$\frac{7}{16}$	·0365	·1198	·2031	·2865	·3698	·4531	·5365	·6198	·7031	·7865	·8698	·9531
$\frac{1}{2}$	·0417	·1250	·2083	·2917	·3750	·4583	·5417	·6250	·7083	·7917	·8750	·9583
$\frac{9}{16}$	·0469	·1302	·2135	·2969	·3802	·4635	·5469	·6302	·7135	·7969	·8802	·9635
$\frac{5}{8}$	·0521	·1354	·2188	·3021	·3854	·4688	·5521	·6354	·7188	·8021	·8854	·9688
$\frac{11}{16}$	·0573	·1406	·2240	·3073	·3906	·4740	·5573	·6406	·7240	·8073	·8906	·9740
$\frac{3}{4}$	·0625	·1458	·2292	·3125	·3958	·4792	·5625	·6458	·7292	·8125	·8958	·9792
$\frac{13}{16}$	·0677	·1510	·2344	·3177	·4010	·4844	·5677	·6510	·7344	·8177	·9010	·9844
$\frac{7}{8}$	·0729	·1562	·2396	·3229	·4062	·4896	·5729	·6562	·7396	·8229	·9062	·9896
$\frac{15}{16}$	·0781	·1615	·2448	·3281	·4115	·4948	·5781	·6615	·7448	·8281	·9115	·9948

EXAMPLES.—$2\frac{9}{16}$ = ·2135 ft. ·662 ft. = $7\frac{15}{16}$ ins.

TABLE 7.

FEET AND INCHES CONVERTED TO DECIMALS OF A CHAIN (66 ft.)

Ft.	Chains.	Ft.	Chains	Ft.	Chains	Ft.	Chains.	Ft.	Chains.
1	·0152	14	·2121	27	·4091	40	·6061	53	·8030
2	·0303	15	·2273	28	·4242	41	·6212	54	·8182
3	·0455	16	·2424	29	·4394	42	·6364	55	·8333
4	·0606	17	·2576	30	·4545	43	·6515	56	·8485
5	·0758	18	·2727	31	·4697	44	·6667	57	·8636
6	·0909	19	·2879	32	·4848	45	·6818	58	·8788
7	·1061	20	·3030	33	·5000	46	·6970	59	·8939
8	·1212	21	·3182	34	·5152	47	·7121	60	·9091
9	·1364	22	·3333	35	·5303	48	·7273	61	·9242
10	·1515	23	·3485	36	·5455	49	·7424	62	·9394
11	·1667	24	·3636	37	·5606	50	·7576	63	·9545
12	·1818	25	·3788	38	·5758	51	·7727	64	·9697
13	·1970	26	·3939	39	·5909	52	·7879	65	·9848

Ins.	Chains.	Ins.	Chains.	Ins.	Chains.	Ins.	Chains.
1	·0013	4	·0051	7	·0088	10	·0126
2	·0025	5	·0063	8	·0101	11	·0139
3	·0038	6	·0076	9	·0114	12	·0152

EXAMPLE.—9′ 7″ as decimal of a chain.—

9 ft. — ·1364
\+ 7 ins. — ·0088
·1452 chains.

This Table may be used to convert Feet to Links by moving the decimal point two places; thus:—

5 feet = 7·58 links.

TABLE 8.

CHAINS CONVERTED TO FEET.

Chains.	Feet.	Chains.	Feet.	Chains.	Feet.	Chains.	Feet
1	66	17	1122	45	2970	73	4818
1½	99	18	1188	46	3036	74	4884
2	132	19	1254	47	3102	75	4950
2½	165	20	1320	48	3168	76	5016
3	198	21	1386	49	3234	77	5082
3½	231	22	1452	50	3300	78	5148
4	264	23	1518	51	3366	79	5214
4½	297	24	1584	52	3432	80	5280
5	330	25	1650	53	3498	81	5346
5½	363	26	1716	54	3564	82	5412
6	396	27	1782	55	3630	83	5478
6½	429	28	1848	56	3696	84	5544
7	462	29	1914	57	3762	85	5610
7½	495	30	1980	58	3828	86	5676
8	528	31	2046	59	3894	87	5742
8½	561	32	2112	60	3960	88	5808
9	594	33	2178	61	4026	89	5874
9½	627	34	2244	62	4092	90	5940
10	660	35	2310	63	4158	91	6006
10½	693	36	2376	64	4224	92	6072
11	726	37	2442	65	4290	93	6138
11½	759	38	2508	66	4356	94	6204
12	792	39	2574	67	4422	95	6270
12½	825	40	2640	68	4488	96	6336
13	858	41	2706	69	4554	97	6402
14	924	42	2772	70	4620	98	6468
15	990	43	2838	71	4686	99	6534
16	1056	44	2904	72	4752	100	6600

This Table also serves to convert Links to Feet, thus :—
5 links = 3·30 feet.

TABLE 9.

Minutes converted to Decimals of a Degree.			
Mins.	Degs.	Mins.	Degs.
1	·0167	31	·5167
2	·0333	32	·5333
3	·0500	33	·5500
4	·0667	34	·5667
5	·0833	35	·5833
6	·1000	36	·6000
7	·1167	37	·6167
8	·1333	38	·6333
9	·1500	39	·6500
10	·1667	40	·6667
11	·1833	41	·6833
12	·2000	42	·7000
13	·2167	43	·7167
14	·2333	44	·7333
15	·2500	45	·7500
16	·2667	46	·7667
17	·2833	47	·7833
18	·3000	48	·8000
19	·3167	49	·8167
20	·3333	50	·8333
21	·3500	51	·8500
22	·3667	52	·8667
23	·3833	53	·8833
24	·4000	54	·9000
25	·4167	55	·9167
26	·4333	56	·9333
27	·4500	57	·9500
28	·4667	58	·9667
29	·4833	59	·9833
30	·5000	60	1·0000

TABLE 10.

Seconds converted to Decimals of a Degree.			
Secs.	Degs.	Secs.	Degs.
1	·0003	31	·0086
2	·0006	32	·0089
3	·0008	33	·0092
4	·0011	34	·0094
5	·0014	35	·0097
6	·0017	36	·0100
7	·0020	37	·0103
8	·0022	38	·0106
9	·0025	39	·0108
10	·0028	40	·0111
11	·0031	41	·0114
12	·0033	42	·0117
13	·0036	43	·0120
14	·0039	44	·0122
15	·0042	45	·0125
16	·0044	46	·0128
17	·0047	47	·0131
18	·0050	48	·0133
19	·0053	49	·0136
20	·0056	50	·0139
21	·0058	51	·0142
22	·0061	52	·0144
23	·0064	53	·0147
24	·0067	54	·0150
25	·0070	55	·0153
26	·0072	56	·0156
27	·0075	57	·0158
28	·0078	58	·0161
29	·0081	59	·0164
30	·0083	60	·0167

EXAMPLE.—13 Minutes 47 Seconds is ·2167 + ·0131 = ·2298 of a Degree.

·6759 Degree = ·6667 + ·0092 or 40 Minutes 33 Seconds.

Table 9 also serves to convert Seconds to decimals of a Minute.

TABLE 11.

CURVES IN DEGREES AND MINUTES CONVERTED TO RADIUS IN FEET AND CHAINS.

Degrees.	Minutes.	Radius.			Degrees.	Minutes.	Radius.		
		Feet.	Chs.	Ft.			Feet.	Chs.	Ft.
1		5730	86	54	11	30	499	7	37
1	30	3820	57	58	12		478	7	16
2		2865	43	27	12	30	459	6	63
2	30	2292	34	48	13		442	6	46
3		1910	28	62	13	30	425	6	29
3	30	1637	24	53	14		410	6	14
4		1433	21	47	14	30	396	6	0
4	30	1274	19	20	15		383	5	53
5		1146	17	24	15	30	371	5	41
5	30	1042	15	52	16		359	5	29
6		955	14	31	16	30	348	5	18
6	30	882	13	24	17		338	5	8
7		819	12	27	17	30	329	4	65
7	30	764	11	38	18		320	4	56
8		717	10	57	19		303	4	39
8	30	675	10	15	20		288	4	24
9		637	9	43	22		262	3	64
9	30	604	9	10	24		240	3	42
10		574	8	46	26		222	3	24
10	30	546	8	18	28		207	3	9
11		522	7	60	30		193	2	61

Note.—The Degree is the Measure of the Angle subtended by a 100 feet chord of the curve.

RULE.—Rad. = 50 ÷ Sine $\frac{1}{2}$ D°.

TABLE 12.

MILES PER HOUR EQUIVALENT TO FEET PER SECOND AND SECONDS PER QUARTER MILE.

Feet per second.	Seconds per ¼ mile.	Miles per hour.	Feet per second.	Seconds per ¼ mile.	Miles per hour.
11	120	7½	73	18	50
15	90	10	77	17·1	52½
18	72	12½	81	16·4	55
22	60	15	84	15·7	57½
26	51·4	17½	88	15	60
29	45	20	92	14·4	62½
33	40	22½	95	13·8	65
37	36	25	99	13·3	67½
40	32·7	27½	103	12·9	70
44	30	30	106	12·4	72½
48	27·7	32½	110	12	75
51	25·7	35	114	11·6	77½
55	24	37½	117	11·2	80
59	22·5	40	121	10·9	82½
62	21·2	42½	125	10·6	85
66	20	45	128	10·3	87½
70	19	47½	132	10	90

RULES.—Miles per hour = 900 ÷ seconds per ¼ mile

Do. = Feet per second × $\frac{15}{22}$

EXAMPLE.—If a train travels 66 ft. in 1 second; or, alternatively, if it takes 20 seconds to travel ¼ mile, then the speed is 45 miles per hour.

TABLE 13.

CURVED RULERS FOR VARIOUS SCALES.

EXAMPLE.—A curve of 12 chains radius will need the curved ruler No. 19 when the scale of the plan is 41·66 ft. to an inch.

Rad. Chs.	Rad. Feet.	Scales. Feet to an inch. 4	8	20	30	40	41·66	60	208·33
1	66	16½	8¼	3¼	—	—	—	—	—
2	132	33	16½	6½	4½	3¼	—	—	—
3	198	50	24¾	10	6½	5	4¾	3¼	1
4	264	66	33	13¼	8¾	6½	6¼	4½	1¼
5	330	82	41	16½	11	8¼	8	5½	1½
6	396	99	50	19¾	13¼	10	9½	6½	2
7	462	115	58	23	15½	11½	11	7¾	2¼
8	528	132	66	26½	17½	13¼	12¾	8¾	2½
9	594	148	74	29¾	19¾	14¾	14¼	10	2¾
10	660	165	82	33	22	16½	15¾	11	3¼
11	726	181	91	36	24¼	18¼	17½	12	3½
12	792	198	99	39½	26½	19¾	19	13¼	3¾
13	858	214	107	43	28½	21½	20½	14¼	4
14	924	231	115	46	31	23	22¼	15½	4½
15	990	247	124	50	33	24¾	23¾	16½	4¾
16	1056	264	132	53	35	26½	25¼	17½	5
17	1122	280	140	56	38	28	27	18¾	5½
18	1188	297	148	60	40	30	28½	19¾	5¾
19	1254	313	157	63	42	31½	30	21	6
20	1320	330	165	66	44	33	31½	22	6¼
21½	1419	355	177	71	47	35½	34	23¾	6¾
23	1518	379	190	76	50	38	36½	25¼	7¼
25	1650	412	206	82	55	41	39½	27½	8
27	1782	445	223	89	59	44	43	30	8½
30	1980	495	247	99	66	50	47½	33	9½
35	2310	577	289	115	77	58	55	38½	11
40	2640	660	330	132	88	66	63	44	12¾
45	2970	742	371	148	99	74	71	50	14¼
50	3300	825	412	165	110	82	79	55	16
60	3960	990	495	198	132	99	95	66	19
70	4620	1155	577	231	154	115	111	77	22¼
80	5280	1320	660	264	176	132	127	88	25¼
100	6600	1650	825	330	220	165	158	110	32

No. of curved ruler = Radius in feet ÷ Feet to an inch of scale.
For 66′ to 1″ scale; No. of curved ruler = No of chains radius.
For 33′ to 1″ scale; ,, ,, ,, = Twice ,, ,,
For 10′ to 1″ scale; ,, ,, ,, = No. of feet radius ÷ 10.

This Table applies to rulers which are numbered with their actual radius in inches.

TABLE 14.

RADIUS OF CURVES FROM VERSED SINES MEASURED ON 66 FEET CHORDS.

Rule.—If V_1 is the Versed sine measured on a 33ft. chord, the radius will be that given below for 4 times V_1.

Vers. sine.	Radius.	Radius.		Vers. sine.	Radius	Radius.		Vers sine.	Rad	Radius.	
ins.	feet.	chns.	ft.	ins.	feet.	chns.	ft.	ins.	ft.	chs.	ft.
1/4	26136	396	...	5	1307	19	53	10	654	9	60
1/2	13068	198	...	1/8	1275	19	21	1/4	638	9	41
3/4	8712	132	...	1/4	1245	18	57	1/2	623	9	29
1	6534	99	...	3/8	1216	18	28	3/4	608	9	14
1/8	5808	88	...	1/2	1188	18	...	11	594	9	...
1/4	5227	79	13	5/8	1162	17	40	1/4	581	8	53
3/8	4752	72	...	3/4	1137	17	15	1/2	569	8	41
1/2	4356	66	...	7/8	1112	16	56	3/4	557	8	29
5/8	4021	60	61	6	1089	16	33	12	545	8	17
3/4	3734	56	38	1/8	1067	16	11	1/4	534	8	6
7/8	3485	52	53	1/4	1046	15	56	3/8	528	8	...
2	3267	49	33	3/8	1025	15	35	1/2	523	7	61
1/8	3075	46	39	1/2	1006	15	16	3/4	513	7	51
1/4	2904	44	...	5/8	987	14	63	13	503	7	41
3/8	2751	41	45	3/4	968	14	44	1/4	491	7	32
1/2	2613	39	39	7/8	950	14	26	1/2	485	7	23
5/8	2489	37	47	7	933	14	9	3/4	476	7	14
3/4	2376	36	...	1/8	917	13	59	14	467	7	5
7/8	2273	34	29	1/4	901	13	43	1/8	463	7	1
3	2178	33	...	3/8	886	13	28	1/4	459	6	63
1/8	2091	31	45	1/2	871	13	13	1/2	451	6	55
1/4	2011	30	31	5/8	857	12	65	3/4	444	6	48
3/8	1936	29	22	3/4	843	12	51	15	436	6	40
1/2	1867	28	19	7/8	830	12	38	1/4	429	6	33
5/8	1803	27	21	8	817	12	25	1/2	422	6	26
3/4	1742	26	26	1/4	792	12	...	3/4	416	6	20
7/8	1686	25	36	1/2	769	11	43	16	409	6	13
4	1633	24	49	3/4	747	11	21	1/2	397	6	1
1/8	1584	24	...	9	726	11	...	17	385	5	55
1/4	1538	23	20	1/4	707	10	47	1/2	374	5	44
3/8	1494	22	42	1/2	688	10	28	18	364	5	34
1/2	1452	22	...	3/4	671	10	11	1/2	354	5	24
5/8	1413	21	27					19	345	5	15
3/4	1376	20	56					1/2	336	5	6
7/8	1341	20	21					20	328	4	64

Rule.—Radius in feet $= (6534 \div V'') + \frac{V}{2}$ feet.

TABLE 15.

RADIUS OF CURVES FROM VERSED SINES MEASURED ON 33 FEET CHORDS.

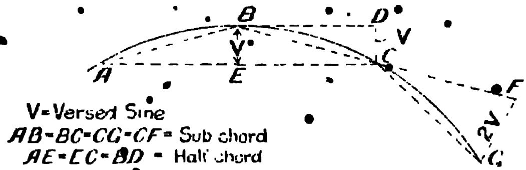
Vers. sine.	Rad.	Rad.		Vers. sine.	Rad.	Rad.		Vers. sine.	Rad.	Rad.	
ins.	feet.	chs.	ft.	ins.	feet.	chs.	ft.	ins.	feet.	chs.	ft.
3	545	8	17	$5\frac{1}{2}$	297	4	33	9	182	2	50
$\frac{1}{8}$	523	7	61	$\frac{5}{8}$	290	4	26	$\frac{1}{2}$	172	2	40
$\frac{1}{4}$	503	7	41	$\frac{3}{4}$	284	4	20	10	164	2	32
$\frac{3}{8}$	484	7	22	$\frac{7}{8}$	279	4	15	$\frac{1}{2}$	156	2	24
$\frac{1}{2}$	467	7	5	6	$272\frac{1}{2}$	4	$8\frac{1}{2}$	11	149	2	17
$\frac{5}{8}$	451	6	55	$\frac{1}{8}$	267	4	3	$\frac{1}{2}$	143	2	11
$\frac{3}{4}$	436	6	40	$\frac{1}{4}$	262	3	64	12	137	2	5
$\frac{7}{8}$	422	6	26	$\frac{3}{8}$	$256\frac{1}{2}$	3	$58\frac{1}{2}$	$\frac{1}{2}$	131	1	65
4	408	6	12	$\frac{1}{2}$	252	3	54	13	126	1	60
$\frac{1}{8}$	396	6	...	$\frac{5}{8}$	247	3	49	$\frac{1}{2}$	122	1	56
$\frac{1}{4}$	384	5	51	$\frac{3}{4}$	243	3	45	14	117	1	51
$\frac{3}{8}$	373	5	43	$\frac{7}{8}$	238	3	40	$\frac{1}{2}$	113	1	47
$\frac{1}{2}$	363	5	33	7	234	3	36	15	110	1	44
$\frac{5}{8}$	353	5	23	$\frac{1}{4}$	226	3	28	16	103	1	37
$\frac{3}{4}$	344	5	14	$\frac{1}{2}$	218	3	20	17	97	1	31
$\frac{7}{8}$	335	5	5	$\frac{3}{4}$	211	3	13	18	92	1	26
5	327	4	63	8	205	3	7	19	87	1	21
$\frac{1}{8}$	319	4	55	$\frac{1}{4}$	198	3	...	20	$82\frac{1}{2}$	1	$16\frac{1}{2}$
$\frac{1}{4}$	311	4	47	$\frac{1}{2}$	193	2	61	22	75	1	9
$\frac{3}{8}$	304	4	40	$\frac{3}{4}$	187	2	55	24	69	1	3
								26	64	...	...
								28	$59\frac{1}{2}$	...	...
								30	57	...	...

RULE.—Radius in feet = $(1633{\cdot}5 \div V'') + \frac{V}{2}$ feet.

TABLE 16.

VERSED SINES & OFFSETS ON VARIOUS CHORDS FOR CURVES OF GIVEN RADII.

(For setting out curves, bending rails, measuring overhang of vehicles, etc.)



Notes.—The Versed sines are to the nearest ⅛″.

Below the dark lines it makes no difference to the V. whether AC or ABC, is taken as the fixed length.

The Sub chords in the last column are to the nearest ¼″.

The Half Chord (EC) is ⅛″ less than BC at place marked *, ¼″ less at †, and ⅜″ less at ‡.

The Versed sines vary as the square of the Sub-chord. Therefore Versed sines for various other lengths ABC may easily be obtained, for example:—

V for 66′ = ¼ of that for 132′; V for 50′ = $\frac{1}{16}$ of that for 200′; etc.

Radius.			Vers. sines (BE) or offsets (DC) in feet and inches for lengths ABC of:—														Sub-Chord (BC) when V = 1 ft.	
Ch.	ft.	feet.	200′		132′		60′		45′		40′		36′		30′			
			ft.	ins.	ft.	ins.	ft.	ins.	ft.	ins.	ft.	ins.	ft.	ins.	ft.	ins.	ft.	ins.
--		60	83	4	36	3⅝	7	6	4	2⅝	3	4	2	8⅜	1	10½	10	11½
1	14	80	62	6	27	2¾	5	7½	3	2	2	6	2	0¼	1	4⅞	*12	7¾
1	34	100	50	0	21	9⅜	4	6	2	6⅜	2	0	1	7½	1	1½	14	1¾
1	59	125	40	0	17	5⅛	3	7¼	2	0¼	1	7¼	1	3½		10¾	15	9¾
2	18	150	33	4	14	6¼	3	0	1	8¼	1	4	1	1		9	17	3⅞
2	43	175	28	7	12	5⅜	2	6⅞	1	5⅜	1	1¾		11⅛		7¾	18	8½
3	2	200	25	0	10	10¾	2	3	1	3¼	1	0		9¾		6¾	20	0
3	42	240	20	10	9	0⅞	1	10½	1	0⅝		10		8⅛		5⅝	21	10⅞
4	0	264	18	11¼	8	3	1	8½		11½		9⅛		7⅜		5⅛	22	11¾
4	36	300	16	8	7	3⅛	1	6		10⅛		8		6½		4½	24	6
5	0	330	15	1¾	6	7¼	1	4⅜		9¼		7¼		5⅞		4⅛	‡25	8¼
5	20	350	14	3⅜	6	2¾	1	3⅜		8⅝		6⅞		5½		3⅞	26	5½
6	4	400	12	6	5	5⅜	1	1½		7⅝		6		4⅞		3⅜	28	3½
6	54	450	11	1⅜	4	10¼	1	0		6¾		5⅜		4⅜		3	30	0
7	0	462	10	9⅞	4	8⅝		11¾		6⅝		5¼		4¼		2⅞	30	4¾

(continued.)

TABLE 16.—*continued.*

Radius.			Vers. sines (BE) or offsets (DC) in feet and inches for lengths ABC of :—									Sub-Chord (BC) when V = 1 ft.	
Ch.	ft.	feet	200′		132′		60′	45′	40′	36′	30′		
			ft.	ins.	ft.	ins.	ins.	ins.	ins.	ins.	ins.	ft.	ins.
7	38	500	10	0	4	4¼	10¾	6⅛	4¾	4	2¾	31	7½
8	0	528	9	5⅝	4	1½	10¼	5¾	4½	3⅝	2½	32	6
8	22	550	9	1⅛	3	11½	9⅞	5½	4⅜	3½	2½	33	2
9	0	594	8	5	3	8	9⅛	5⅛	4	3¼	2¼	34	5¾
9	6	600	8	4	3	7½	9	5	4	3¼	2¼	34	7¾
10	0	660	7	7	3	3⅝	8⅛	4⅝	3⅝	3	2	36	4
10	40	700	7	1¾	3	1⅜	7¾	4⅜	3⅜	2¾	1⅞	37	5
11	0	726	6	10⅝	3	0	7½	4⅛	3¼	2⅝	1⅞	38	1¼
12	0	792	6	3¾	2	9	6⅞	3⅞	3	2½	1¾	39	9½
12	8	800	6	3	2	8¾	6¾	3¾	3	2⅜	1¾	40	0
13	0	858	5	10	2	6½	6¼	3½	2¾	2¼	1⅝	41	5
13	42	900	5	6⅝	2	5	6	3⅜	2⅝	2⅛	1½	42	5
14	0	921	5	4⅞	2	4¼	5⅞	3¼	2⅝	2⅛	1½	43	0
15	0	990	5	0⅝	2	2½	5½	3	2⅜	2	1⅜	44	6
15	10	1000	5	0	2	2⅛	5⅜	3	2⅜	2	1⅜	44	8½
16	0	1056	4	8¾	2	0¾	5⅛	2⅞	2¼	1⅞	1¼	45	11½
16	41	1100	4	6⅝	1	11¾	5	2¾	2⅛	1¾	1¼	46	10¾
17	0	1122	4	5½	1	11⅜	4⅞	2¾	2⅛	1¾	1¼	47	4½
18	0	1188	4	2½	1	10	4½	2½	2	1⅝	1⅛	48	9
18	12	1200	4	2	1	9¾	4½	2½	2	1⅝	1⅛	49	0
19	0	1254	3	11⅞	1	8⅞	4¼	2⅜	1⅞	1½	1⅛	50	1
19	46	1300	3	10⅛	1	8⅛	4⅛	2⅜	1⅞	1½	1	51	0
20	0	1320	3	9½	1	7¾	4⅛	2¼	1⅞	1½	1	51	4½
21	0	1386	3	7¼	1	6⅞	3⅞	2¼	1¾	1½	1	52	8
21	33	1419	3	6¼	1	6⅜	3¾	2⅛	1¾	1⅜	1	53	3
22	48	1500	3	4	1	5½	3⅝	2	1⅝	1¼	⅞	54	9
25	0	1650	3	0⅜	1	3⅞	3¼	1⅞	1½	1⅛	⅞	57	5¼
30	0	1980	2	6¼	1	1¼	2¾	1½	1¼	1	⅝	62	11
30	20	2000	2	6	1	1	2¾	1½	1¼	1	⅝	63	3
35	0	2310	2	2		11⅜	2⅜	1⅜	1	⅞	⅝	67	11¾
37	58	2500	2	0		10½	2⅛	1¼	1	¾	½	70	8½
40	0	2640	1	10¾		9⅞	2	1⅛	⅞	¾	½	72	8
45	30	3000	1	8		8¾	1¾	1	¾	⅝	½	77	5½
50	0	3300	1	6⅛		8	1⅝	⅞	¾	⅝	⅜	81	3
60	0	3960	1	3⅛		6⅝	1⅜	¾	⅝	½	⅜	89	0
70	0	4620	1	1		5⅝	1⅛	⅝	½	⅜	¼	96	1½
75	50	5000	1	0		5¼	1⅛	⅝	½	⅜	¼	100	0
80	0	5280		11⅜		5	1	⅝	½	⅜	¼	102	9

TABLE 17.

DECREASE IN LENGTH OF INNER RAIL ON CURVES. Gauge 4' 8½"

RULES FOR 30' RAILS :—

Radius in feet = 1695 ÷ Decrease in inches.
Decrease in inches = 1695 ÷ Radius in feet.
" " = 25·68 ÷ Radius in chains.

Notes.—The decreases for 45' and 60' rails are 1½ and 2 times those for 30' rails, because decrease is proportional to rail length.

Decreases for other gauges may be obtained by multiplying the decreases for 4' 8½" by figures in Table 20; thus with same radius, decrease for 5' 3" gauge is 1·11 times that for 4' 8½" gauge. Refer also to Table 17a

Radius.			Outer rail.		Radius.			Outer rail.	
feet	chs.	ft.	30'	45'	feet.	chs.	ft.	30'	45'
			ins.	ins.				ins.	ins.
13560	205	30	⅛	3/16	565	8	37	3	4½
6780	102	48	¼	⅜	542	8	14	⅛	11/16
4520	68	32	⅜	9/16	522	7	60	¼	⅞
3390	51	24	½	¾	502	7	40	⅜	5 1/16
2712	41	6	⅝	15/16	484	7	22	½	¼
2260	34	16	¾	1⅛	468	7	6	⅝	7/16
1937	29	23	⅞	5/16	452	6	56	¾	⅝
1695	25	45	1	½	437	6	41	⅞	13/16
1507	22	55	⅛	11/16	424	6	28	4	6
1356	20	36	¼	⅞	411	6	15	⅛	3/16
1233	18	45	⅜	2 1/16	399	6	3	¼	⅜
1130	17	8	½	¼	387	5	57	⅜	9/16
1043	15	53	⅝	7/16	376	5	46	½	¾
968	14	44	¾	⅝	366	5	36	⅝	15/16
904	13	46	⅞	13/16	357	5	27	¾	7⅛
847	12	55	2	3	348	5	18	⅞	5/16
798	12	6	⅛	3/16	339	5	9	5	½
753	11	27	¼	⅜	323	4	59	¼	⅞
714	10	54	⅜	9/16	308	4	44	½	8¼
678	10	18	½	¾	295	4	31	¾	⅝
646	9	52	⅝	15/16	282	4	18	6	9
616	9	22	¾	4⅛	271	4	7	¼	⅜
590	8	62	⅞	5/16	261	3	63	½	¾

(continued.)

TABLE 17—*continued.*

Radius.			Outer Rail.		Radius.			Outer Rail.	
feet.	chns.	ft.	30′	45′	feet.	chns.	ft.	30′	45′
			ins.	ins.				ins.	ins.
251	3	53	6¾	10⅛	178	2	46	9½	14¼
242	3	44	7	½	174	2	42	¾	⅝
234	3	36	¼	⅞	169	2	37	10	15
226	3	28	½	11¼	165	2	33	¼	⅛
219	3	21	¾	⅝	161	2	29	½	¾
212	3	14	8	12	158	2	26	¾	16⅛
205	3	7	¼	⅞	154	2	22	11	½
199	3	1	½	¾	151	2	19	¼	⅞
194	2	62	¾	13⅛	147	2	15	½	17¼
188	2	56	9	½	144	2	12	¾	⅝
183	2	51	¼	⅞	141	2	9	12	18

TABLE 17a.

DECREASE IN LENGTH OF INNER RAIL ON CURVES, FROM VERSED SINE (V) ON RAIL LENGTH.

Gauge 4′ 8½″.

Outer Rail.	Decrease of Inner Rail.
ft.	
30	V × 1·256 or say V × 1¼
33	V × 1·141 ,, V × 1⅐
36	V × 1·047 ,, V × 1$\frac{1}{20}$
45	V × ·837 ,, V × ⅚
60	V × ·628 ,, V × ⅝

TABLE 18.

THEORETICAL SUPER-ELEVATION.

Gauge 4′ 8½″. Centre to Centre of rails 4′ 11″.

RULE.—$e = \cdot 06 \times \frac{V^2}{R}$

V = Velocity in miles per hour.
R = Radius of Curve in chains.
e = Super-elevation in inches.

Radius. in chs.	Radius. in feet.	Curve in Deg.	Curve in M.	Versine 66′ Chord in inches	15	20	25	30	35	40	45	50	55	60	65	70	Max. Spd.* M. per H.
5	330	17	26	19¾	2¾	4¾	7½										12
6	396	14	30	16½	2¼	4	6¼										15
7	462	12	26	14⅛	2	3⅜	5⅜	7¾									20
8	528	10	52	12⅜	1¾	3	4¾	6¾									21
9	594	9	39	11	1½	2⅝	4⅛	6	8⅛								23
10	660	8	41	9⅞	1⅜	2⅜	3¾	5⅜	7⅜								25
11	726	7	54	9	1¼	2⅛	3⅜	4⅞	6⅝								27
12	792	7	14	8¼	1⅛	2	3⅛	4½	6⅛	8							28
13	858	6	41	7⅝	1	1⅞	2⅞	4⅛	5⅝	7⅜							30
14	924	6	12	7⅛	1	1¾	2⅝	3⅞	5¼	6⅞							32
15	990	5	47	6⅝	⅞	1⅝	2½	3⅝	4⅞	6⅜	8⅛						34
16	1056	5	26	6⅛	⅞	1½	2⅜	3⅜	4½	6	7⅝						[illegible]
17	1122	5	6	5⅞	¾	1⅜	2¼	3⅛	4⅜	5⅝	7⅛						37
18	1188	4	49	5½		1⅜	2⅛	3	4⅛	5⅜	6¾	8⅜					39
20	1320	4	20	5		1¼	1⅞	2¾	3⅝	4¾	6⅛	7½					42
22	1452	3	57	4½		1⅛	1¾	2½	3⅜	4⅜	5½	6¾	8¼				44
25	1650	3	28	4		1	1½	2⅛	3	3⅞	4⅞	6	7¼	8⅝			48
30	1980	2	54	3¼		¾	1¼	1¾	2½	3¼	4	5	6	7¼	8½		52
35	2310	2	29	2¾			1	1½	2⅛	2¾	3½	4¼	5¼	6⅛	7¼	8⅜	55
40	2640	2	10	2½			⅞	1⅜	1⅞	2⅜	3	3¾	4½	5⅜	6⅜	7⅜	60
50	3300	1	44	2			¾	1⅛	1½	1⅞	2⅜	3	3⅝	4⅜	5⅛	5⅞	67
60	3960	1	27	1⅝				⅞	1¼	1⅝	2	2½	3	3⅝	4¼	4⅞	70
70	4620	1	14	1⅜				¾	1	1⅜	1¾	2⅛	2⅝	3⅛	3⅝	4¼	70
80	5280	1	5	1¼					⅞	1¼	1½	1⅞	2¼	2¾	3⅛	3⅝	70
100	6600	0	52	1					¾	1	1¼	1½	1¾	2⅛	2½	3	70

The Super-elevation (e) decreases proportionally to the increase of radius, thus for 120 chns. radius e will be one half that for 60 chns. radius, etc.

For general suggestions as to Super-elevation, see Chapter IX.

* Recommended with first class track in good condition.

TABLE 19.

FOR OBTAINING PRACTICAL SUPER-ELEVATION, AFTER THE THEORETICAL ELEVATION HAS BEEN TAKEN FROM TABLE 18.

(As suggested in Chap. IX.) (Practical = ⅝ of Theoretical.)

	INCHES.															
Theoretical Elevation.	1	1⅛	1¼	1⅜	1½	1⅝	1¾	1⅞	2	2⅛	2¼	2⅜	2½	2⅝	2¾	2⅞
Suggested Elevation.	⅝	¾	¾	⅞	1	1	1⅛	1⅛	1¼	1⅜	1½	1½	1⅝	1⅝	1¾	1¾
	INCHES.															
Theoretical Elevation	3	3⅛	3¼	3⅜	3½	3⅝	3¾	3⅞	4	4⅛	4¼	4⅜	4½	4⅝	4¾	4⅞
Suggested Elevation.	1⅞	2	2	2⅛	2¼	2¼	2⅜	2⅜	2½	2⅝	2⅝	2¾	2⅞	2⅞	3	3
	INCHES.															
Theoretical Elevation	5	5⅛	5¼	5⅜	5½	5⅝	5¾	5⅞	6	6⅛	6¼	6⅜	6½	6⅝	6¾	6⅞
Suggested Elevation.	3⅛	3¼	3¼	3⅜	3½	3½	3⅝	3⅝	3¾	3⅞	3⅞	4	4⅛	4⅛	4¼	4¼
	INCHES.															
Theoretical Elevation.	7	7⅛	7¼	7⅜	7½	7⅝	7¾	7⅞	8	8⅛	8¼	8⅜	8½	8⅝	8¾	9
Suggested Elevation.	4¼	4⅜	4⅜	4½	4⅝	4¾	4¾	4⅞	5	5⅛	5⅛	5¼	5¼	5⅜	5½	5⅝

Note.—To find Super-elevation for other gauges, multiply the elevation for 4′ 8½″ gauge, by figures in Table 20.

TABLE 20.

PROPORTION OF VARIOUS GAUGES TO 4′ 8½″

(For applying Tables 17, 18 and 19 to other gauges.

Gauge.		Multiplier.			
ft.	ins.				
5	6	1·17	or	1⅙	Approximate.
5	3	1·11	,,	1⅑	,,
3	6	·74	,,	¾	,,
3	3⅜	·70	,,	7/10	,,
3	0	·64	,,	⅝	,,
2	6	·53	,,	½	,,
1	11⅝	·42	,,	⅖	,,

Note.—In this table the assumption is made that the width of rail head varies in proportion to the Gauge.

TABLE 21.

LENGTH OF TRANSITION CURVES WITH VARIOUS SHIFTS.

Note.—R = Radius of Circular Curve to which transition runs from a straight. For transitions between Reverse Curves use $\sqrt{\frac{R \times r}{R + r}}$ in place of $\sqrt{R}$, and for Compound Curves use $\sqrt{\frac{R \times r}{R - r}}$

Shift (S). ft.	ins	Length of Transition Curve (L). in chains of 66′.	in feet.
2	9	$\sqrt{R} \times \cdot 1000$	$\sqrt{R} \times 8 \cdot 124$
2	6	,, ·953	,, 7·746
2	3	,, ·905	,, 7·348
2	0	,, ·852	,, 6·928
1	9	,, ·798	,, 6·481
1	6	,, ·738	,, 6·000
1	3	,, ·674	,, 5·477
1	0	,, ·603	,, 4·900
	9	,, ·522	,, 4·243
	6	,, ·426	,, 3·464
	3	,, ·301	,, 2·450

TABLE 22.

OFFSETS FOR TRANSITION CURVES.

For 8 Offsets.			For 10 Offsets.			For 12 Offsets.		
Offset No.	Multiply end offset by.	Offsets with 2′ 9″ Shift. ft. ins.	Offset No.	Multiply end offset by.	Offsets with 2′ 9″ Shift ft. ins.	Offset No.	Multiply end offset by.	Offsets with 2′ 9″ Shift. ft. ins.
1	·002	¼	1	·001	⅛	1	·0006	$\frac{1}{16}$
2	·0156	2	2	·008	1	2	·0046	⅝
3	·053	7	3	·027	3⅝	3	·0156	2
4	·125	1 4½	4	·064	8½	4	·037	4[illegible]
5	·244	2 8¼	5	·125	1 4½	5	·072	9½
6	·422	4 7¾	6	·216	2 4½	6	·125	1 4½
7	·670	7 4½	7	·343	3 9¼	7	·198	2 2¼
8	1·000	11 0	8	·512	5 7⅝	8	·296	3 3⅛
			9	·729	8 0¼	9	·422	4 7¾
			10	1·000	11 0	10	·579	6 4⅜
						11	·770	8 5⅝
						12	1·000	11 0

Note.—If only half the number of offsets are required, omit the odd numbered offsets.

TABLE 23.

CIRCULAR CURVES. RULES RELATING TO THE LENGTHS.

This Table and the next contain all Rules likely to be useful in practical and theoretical work. In Chap. VI. an abridged table is given, containing only the more generally useful Rules, with their statement in words. Other short practical rules are given in Chap. IX.

For explanation of symbols, see Chap. VI. and Figs VI.-1 & 2.

Required.	Given.	Rule No.*	Formula.	Remarks.
R	$\mathcal{C}$ & V	1	$\frac{(\frac{1}{2}C)^2+V^2}{2V}$	If V^2 is neglected R will be too small by $\frac{1}{2}V$.
,,	C_s & V	2	$C_s{}^2 \div 2V$	
,,	O & AG & GC	3	$\frac{AG\times GC}{2\times O}$	
,,	V & O & X	4	$\frac{X^2+V^2+O^2-(2\times V\times O)}{2\times(V-O)}$	
,,	$\mathcal{C}$ & O & X	5	$\frac{(\frac{1}{2}C+X)\times(\frac{1}{2}C-X)}{2\times O}$ †	or, $\frac{(\frac{1}{2}C)^2-X^2}{2\times O}$ †
,,	T & E	6	$\frac{T^2-E^2}{2\times E}$	

* Wherever these Rules are quoted in the book their Nos. are preceded by "C."

† Approximate Rules which are near enough for usual purposes.

TABLE 23—*continued.*

Required.	Given.	Rule No.*	Formula.	Remarks.
V	R & C	7	$R - \sqrt{R^2 - (\frac{1}{2}C)^2}$	or, $R - \sqrt{(R + \frac{1}{2}C) \times (R - \frac{1}{2}C)}$
,,	,,	8	$(\frac{1}{2}C)^2 \div 2R$ †	† will be too small by $\frac{V^2}{2R}$
,,	R & C_s	9	$C_s^2 \div 2R$	
,,	R & T	10	$R - \frac{R^2}{\sqrt{T^2 - R^2}}$	
,,	R & E	11	$(R \times E) \div (R - E)$	
O	R & AG & GC	12	$(AG \times GC) \div 2R$	
,,	R & AF & FC	12a	$(AF \times FC) \div 2R$ †	
,,	R & V & X	13	$\sqrt{R^2 - X^2} - (R - V)$	
,,	R & C & X	13a	$\frac{(\frac{1}{2}C + X) \times (\frac{1}{2}C - X)}{2 \times R}$	or $\frac{(\frac{1}{2}C)^2 - X^2}{2 \times R}$

* Wherever these Rules are quoted in the book their Nos. are preceded by "C."
† Approximate Rules which are near enough for usual purposes.

(*continued.*)

TABLE 23—*continued.*

Required.	Given.	Rule No.*	Formula.	Remarks.
X	R, V & O	14	$\sqrt{R^2 - (O + R - V)^2}$	
C	R & V	15	$2 \times \sqrt{(2 \times R \times V) - V^2}$	If V^2 is neglected C will be too great by $\frac{2V^2}{C}$
"	C_s & V	16	$2 \times \sqrt{C_s^2 - V^2}$	
C_s	R & V	17	$\sqrt{2 \times R \times V}$	
"	C & V	18	$\sqrt{(\frac{1}{2}C)^2 + V^2}$	
T	R & C	19	$\frac{C \times R}{\sqrt{(2R)^2 - C^2}}$	or $\frac{C \times R}{\sqrt{(2R + C) \times (2R - C)}}$
"	R & V	20	$\frac{\sqrt{V \times R^2 \times (2R - V)}}{R - V}$	
"	R & E	21	$\sqrt{(2R + E) \times E}$	This arises from Euclid III.–36.

* Wherever these Rules are quoted in the book their Nos. are preceded by "C."

TABLE 23—*continued.*

Required.	Given.	Rule No.*	Formula.	Remarks.
E	R & T	22	$\sqrt{T^2 + R^2} - R$	
"	R & V	23	$(R \times V) \div (R - V)$	
"	R & C_s	24	$\dfrac{C_s^2 \times R}{2R^2 - C_s^2}$	
"	R & C	25	$\dfrac{2R^2}{\sqrt{(2R)^2 - C^2}} - R$	or $\dfrac{2R^2}{\sqrt{(2R + C) \times (2R - C)}} - R$
V_s	V	26	$V \div 4$ †	

* Wherever these Rules are quoted in the book their Nos. are preceded by "C."

† Approximate Rules which are near enough for usual purposes.

TABLE 24.

CIRCULAR CURVES. RULES RELATING TO THE ANGLE AND THE LENGTHS, INCLUDING THE ARC.

Refer to General Notes at head of Table 23.

To find the Radius from the Angle D and a Length.

Given.	Rule No.*	Formula.
$\frac{1}{2}C$	31	$\frac{1}{2}C \div$ Sine D
C_s	32	$C_s \div 2$ Sine $\frac{1}{2}D$
	33	or $C_s \div$ Chord † D
V	34	$V \div$ Versine D
T	35	$T \div$ Tangent D
E	36	$E \div$ (Secant D − 1)
R − V	37	(R − V) ÷ Cosine D
Arc	38	$\frac{\text{Arc}}{2 \times 3{\cdot}1416} \times \frac{360^\circ}{2 \times D^\circ}$
	39	or $\frac{\text{Arc}}{{\cdot}01745 \times 2D^\circ}$

* Wherever these Rules are quoted in the book their numbers are preceded by "C."

‡ See Mathematical Tables, such as Chambers'.

To find the Lengths from the Angle D and the Radius.

Re-quir'd	Rule No.*	Formula.
$\frac{1}{2}C$	40	R × Sine D
C_s	41	R × 2 Sine $\frac{1}{2}D$
	42	R × Chord † D
V	43	R × Versine D
T	44	R × Tangent D
E	45	R × (Secant D − 1)
R − V	46	R × Cosine D
Arc	47	$R \times 2 \times 3{\cdot}1416 \times \frac{2D^\circ}{360^\circ}$
	48	or ·01745 × R × 2D°
	49	or ·0002909 × R × 2 D mins.
	50	or Radius × the number in a table of "Arcs to radius one," which is opposite to twice the angle D. ‡

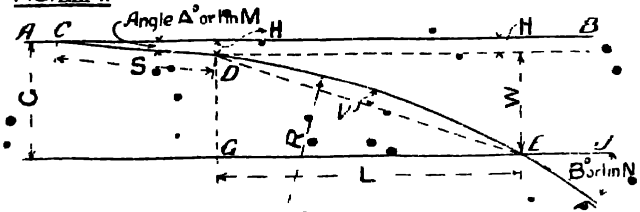
To find the Angle D in degrees, or its Trigonometrical Ratios from the Radius and a Length.

Given.	Rule No.*	Formula.
$\frac{1}{2}C$	51	Sine D = $\frac{1}{2}C$ ÷ R
R − V	52	Cosine D = (R − V) ÷ R
E	53	Secant D = (R + E) ÷ R
$\frac{1}{2}C$	54	Cosec. D = R ÷ $\frac{1}{2}C$
T	55	Tan. D = T ÷ R
T	56	Cot D = R ÷ T
C_s	57	Sine $\frac{1}{2}D$ = C_s ÷ 2R
	58	Chord † D = C_s ÷ R
V	59	Versine D = V ÷ R
Arc	60	$D^\circ = 360^\circ \times \frac{\frac{1}{2}A}{2 \times R \times 3{\cdot}1416}$
,,	61	A ÷ R = No. opposite 2D° in a table of "Arcs to radius one." ‡

† This does not refer to a chord length, but to the number headed "Chord" in Mathematical Tables.

TABLE. 25.

FIG. XIII-1.



SWITCHES AND CROSSINGS.

General Trigonometrical Rules (a).

Note.—In addition to the aid of Tables 28 and 30. trigonometrical tables will be required except for Rules 2a and 7a.

Given in addition to S and H, and switch angle (A°).	Rule No.	To find	Formula.	Rule in words. ("W" means Width from switch heel line to rail on which the crossing lies.)
Crossing angle (B°)	1a	Lead (L)	$W \times \text{Cot}\left(\frac{A+B}{2}\right)$	Add the switch and crossing angles in degrees and minutes and divide by 2. Multiply the Cot of this angle by W
"	2a	Radius (R)	$W \div (\text{Cos A} - \text{Cos B})$	From the Cos of the switch angle deduct the Cos of the crossing angle and divide the result into W*.

* Or take the reciprocal of the result and multiply it by W.

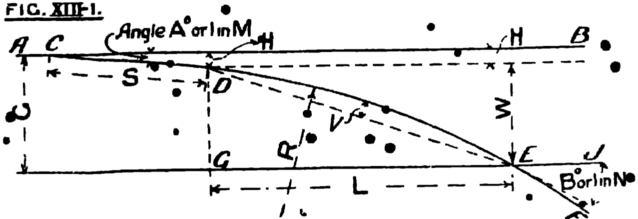
(continued.)

TABLE 25—*continued.*

Given in addition to S and H, and switch angle (A°).	Rule No.	To find	Formula.	Rule in words. ("W" means Width from switch heel line to rail on which the crossing lies.)
Radius (R)	3a	Crossing angle (B)	$\text{Cos } B = \text{Cos } A - \frac{W}{R}$	Divide W by the radius and deduct the result from Cos. of switch angle. The remainder will be Cos. of crossing angle.
"	4a	Lead (L)	First find B by Rule 3a and then use Rule 1a.	
Lead (L)	5a	Crossing angle (B)	$\text{Cot}\left(\frac{A + B}{2}\right) = \frac{L}{W}$	Divide Lead by W. Find the angle which the result is the Cot. of. From that angle deduct half the switch angle, the remainder will be half the crossing angle.
"	6a	Radius (R)	First find B by Rule 5a and then use Rule 2a.	
Radius and Crossing angle	7a	Width (W)	$R \times (\text{Cos } A - \text{Cos } B)$	From Cos of switch angle deduct Cos. of crossing angle and multiply by the radius.
"	8a	Versed sine (V)	$R \times \text{Vers.}\left(\frac{B - A}{2}\right)$	From the crossing angle deduct the switch angle and divide by 2. Multiply the Versed sine of this angle by the radius.

TABLE 26.

FIG. XIII-1.



SWITCHES AND CROSSINGS.

GENERAL C.L. MEASURE RULES (b).

(Accurate.)

Note.—See Table 27 for simplified approximate rules.

Given in addition to S and H, and switch angle number (M)*	Rule No	To find	Formula.	Rule in words. ("W" means Width from switch heel line to rail on which the crossing lies.)
Crossing angle (N)	1b	Lead (L)	$2 \times W \times \frac{(M \times N) - \frac{1}{4}\dagger}{M + N}$	From the product of M and N deduct ·25, then divide by their sum and multiply by twice W.
"	2b	Radius (R)	$W \times \frac{(4 \times M^2 \times N^2) + M^2 + N^2 + \frac{1}{4}\dagger}{2 \times (M^2 - N^2)}$	To four times the product of the squares of M and N add the square of M and the square of N, also add ·25. Divide this sum by twice the difference of the squares of M and N. Then multiply by W (page 206).

* $M = S \div H$.

† The $\frac{1}{4}$ may usually be neglected.

(continued.)

L

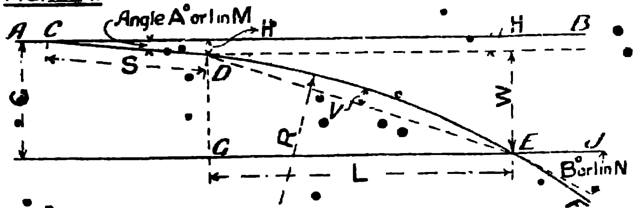
TABLE 26—*continued.*

Given in addition to S and H, and switch angle number (M)*	Rule No.	To find	Formula.	Rule in words. ("W" means Width from switch heel line to rail on which the crossing lies.)
Radius (R)	3b	Crossing number (N)	$\sqrt{\dfrac{M^2 \times (2R - W) - \frac{W}{4}\dagger}{(2 \times R) + (4 \times W \times M^2) + W}}$	1st.—From twice R deduct W, then multiply by the square of M and deduct ¼ of W. 2nd.—Multiply 4 times W by the square of M and add twice R and W. Divide the 1st result by the 2nd and extract the square root of quotient (page 239).
”	4b	Lead (L)	First find N by Rule 3b and then use Rule 1b.	
Lead (L)	5b	Crossing No. (N)	$\dfrac{(L \times M) + \frac{W}{2}}{(2 \times M \times W) - L}$	1st.—Multiply Lead by M and add ½ W. 2nd.—Multiply twice M by W and deduct Lead. Divide the 1st result by the 2nd.
”	6b	Radius (R)	First find N by Rule 5b and then use Rule 2b.	
Radius and Crossing number	7b	Width (W)	$\dfrac{2 \times R \times (M^2 - N^2)}{(4 \times M^2 \times N^2) + M^2 + N^2 + \frac{1}{4}\dagger}$	Multiply twice the Radius by the difference of the squares of M and N. Divide the result by 4 times the product of the squares of M and N plus the squares of M and N plus ·25.
Crossing No. or Lead	8b	Versed sine (V)	Use Rules "c"	

* $M = S \div H$. † The ¼ may usually be neglected.

TABLE 27.

FIG. XIII-1.



SWITCHES AND CROSSINGS.

GENERAL C.L. MEASURE RULES (c).

(Approx.)

Note.—These rules are sufficiently accurate for Turnout Problems and many other purposes.

Given in addition to S and H, and switch angle number (M)*	Rule No.	To find	Formula.	Rule in words. ("W" means Width from switch heel line to rail on which the crossing lies.)
Crossing number (N)	1c	Lead (L)	$2 \times W \times \frac{M \times N}{M + N}$	Divide the product of M and N by their sum and multiply by twice W§ (see page 201).
"	1d	"	$W \div \left(\frac{1}{2 \times N} + \frac{1}{2 \times M}\right)$ †	Divide W by the sum of the reciprocals of twice M and twice N.
"	2c	Radius (R)	$2 \times W \times \frac{M^2 \times N^2}{M^2 - N^2}$	Divide the product of the squares of M and N by the difference of those squares. Then multiply by twice W (page 228).

* $M = S \div H$. † These rules are similar to those preceding them, but adapted for use with reciprocals.
§ Usually it is better to multiply twice W, M and N together and then divide by the sum of M and N. The rule is given as it is to meet other requirements.

(continued.)

TABLE 27—*continued.*

Given in addition to S and H, and switch angle number (M)*	Rule No.	To find	Formula.	Rule in words. ("W" means Width from switch heel line to rail on which the crossing lies.)
Crossing number (N)	2d	Radius (R)	$W \div \left(\frac{1}{2 \times N^2} - \frac{1}{2 \times M^2}\right)$ †	Deduct the reciprocal of twice the square of M from that of twice the square of N and divide the result into W.
Radius (R)	3c	Crossing number (N)	$M \times \sqrt{\frac{R - \frac{1}{2}W}{R + (2 \times W \times M^2)}}$	1st.—From R deduct ½ W. 2nd.—Multiply twice W by M squared and add R. Divide the first result by the second, and extract square root. Then multiply by M (page 203).
"	4c	Lead (L)	First find N by Rule 3c and then use Rule 1c.	
"	4d	"	$\sqrt{W \times (2R - W) + \frac{R^2}{M_2}} - \left(\frac{R}{M}\right)$ ‡	
Radius and Crossing number	4e	"	$\frac{R \times (M - N)}{N \times M}$	Multiply radius by difference between M and N, then divide by the product of those numbers (page 210).

* M = S ÷ H. † These rules are similar to those preceding them, but adapted for use with reciprocals.
‡ These rules are only for use in special cases.

TABLE 27—*continued.*

Given in addition to S and H, and switch angle number (M)*	Rule No.	To find	Formula.	Rule in words. ("W" means Width from switch heel line to rail on which the crossing lies.)
Lead (L)	5c	Crossing number (N)	$\frac{L \times M}{(2 \times M \times W) - L}$	Multiply twice M by W and deduct Lead. Divide result into product of Lead and M (see page 204)
"	6c	Radius (R)	First find N by Rule 5c and then use Rule 2c.	
"	6d	Radius	$\frac{M \times (W^2 + L^2)}{2 \times (W \times M - L)}$ ‡	1st.—Add the square of W to the square of Lead and multiply by M. 2nd.—Multiply W by M, deduct Lead, and multiply by 2. 3rd.—Divide first result by the second (page 219).
Lead and crossing number	6e	"	$\frac{L \times N \times M}{M - N}$	Multiply together the Lead and N and M, then divide by the difference of M and N (page 202).
Radius and crossing number	7c	Width (W)	$\frac{R \times (M^2 - N^2)}{2 \times M^2 \times N^2}$	Multiply radius by difference of squares of M and N. Divide result by twice product of squares of M and N.

* $M = S \div H$. ‡ These rules are only for use in special cases

(continued.)

TABLE 27—*continued.*

Given in addition to S and H, and switch angle number (M)*	Rule No.	To find	Formula.	Rule in words. ("W" means Width from switch heel line to rail on which the crossing lies.)
Crossing number	8c	Versed sine (V)	$\frac{W}{4} \times \frac{M-N}{M+N}$	Divide the difference of M and N by their sum, then multiply by $\frac{1}{4}$ of W.
Crossing number and Lead	9c	"	$\frac{(L \div N) - W}{4}$	Divide Lead by N and deduct W, then divide by 4 (page 202).
Lead and M	10c	"	$\frac{W - (L \div M)}{4}$	Divide Lead by M and deduct from W, then divide by 4 (page 210).
Radius and Lead	11c	Width (W)	$R - \sqrt{R^2 - L^2 - \frac{2 \times L \times R}{M}}$ ‡	
"	11d	"	$\frac{L^2}{2 \times R} + \frac{L}{M}$ ‡ (Further approximation of Rule 11c.)	Divide square of Lead by twice radius and add the result of dividing Lead by M (page 266).

* M = S ÷ H. ‡ These Rules are only for use in special cases.

TABLE 28.

ANGLES OF SWITCHES IN DEGREES WITH COSINES AND COTANGENTS.

Switch length (S). Ft.	ins.	Heel Divergence H 3ins. Degs.	m.	s.	Cosine.	M & Cotan	H 3½ ins. Degs	m	s	Cosine.	M & Cotan	H 4ins. Degs.	m.	s	Cosine.	M & Cotan
2	6	5	42	38	·9950374	10		......		......	......		......		......	......
4	0	3	34	35	·9980526	16		......		......	......		......		......	......
5	0	2	51	45	·9987522	20		......		......	......		......		......	......
6	0	2	23	9	·9991332	24	2	46	59	·9988205	20·57	3	10	47	·9984605	18
7	6	1	54	33	·9994449	30	2	13	37	·9992448	25·71	2	32	41	·9990139	22·5
9	0	1	35	28	·9996144	36	1	51	22	·9994753	30·86	2	7	16	·9993148	27
10	0		......		......	...	1	40	14	·9995750	34·29	1	54	33	·9994449	30
12	0		......		......	...	1	23	32	·9997048	41·14	1	35	28	·9996144	36
		Heel Divergence H 4½ ins.					H 4⅝ in.					H 4¾ ins.				
6	0	3	34	35	·9980526	16	3	40	31	·9979434	15·568	3	46	28	·9978310	15·158
9	0	2	23	9	·9991332	24	2	27	8	·9990843	23·351	2	31	6	·9990342	22·737
10	0	2	8	51	·9992977	26⅔	2	12	16	·9992600	25·946	2	16	0	·9992176	25·263
12	0	1	47	24	·9995121	32	1	50	23	·9994845	31·135	1	53	21	·9994565	30·316
14	0	1	32	3	·9996415	37⅓	1	34	37	·9996213	36·324	1	37	10	·9996067	35·368
15	0	1	25	56	·9996876	40	1	28	19	·9996701	38·919	1	30	42	·9996580	37·895
16	0	1	20	33	·9997255	42⅔	1	22	48	·9997102	41·514	1	25	2	·9996941	40·421
18	0	1	11	36	·9997831	48	1	13	36	·9997708	46·703	1	15	35	·9997583	45·473
20	0	1	4	27	·9998243	53⅓	1	6	14	·9998144	51·892	1	8	2	·9998042	50·526
21	0	1	1	23	·9998406	56	1	3	5	·9998317	54·487	1	4	47	·9998225	53·052
24	0		53	42	·9998781	64		55	12	·9998711	62·270		56	41	·9998640	60·631
		Heel Divergence H 5½ ins.														
30	0		52	31	·9998833	65·455		......		......	......		......		......	......

Note.—The value "M" used in rules in Tables 26 and 27 and the Cotangent are both taken as equal to S ÷ H.

TABLE 29.

LENGTHS OF SWITCHES,

In relation to the Curve, the Crossing Angle, and the Lead.

Gauge 4ft. 8½ins.* Heel divergence (H) 4½ins.†

Switch length.	MAXIMUM					
	Equiv. Radius (R). $=S^2 \div \cdot581^2$			Crossing Angle (N). $=S \div 1\frac{3}{4}$ ‡	Lead (L). $=S \times 4\cdot08$	
Ft. (S).	Ft.	Chs.	ft.	1 in —	Ft.	ins.
6	107	1	41	3·43 say 3½	24	6
9	240	3	42	5·14 „ 5	36	9
10	296	4	32	5·71 „ 5¾	40	10
12	427	6	31	6·86 „ 6¾	49	0
14	580	8	52	8·00 „ 8	57	2
15	666	10	6	8·57 „ 8½	61	3
16	758	11	32	9·14 „ 9¼	65	3
18	960	14	36	10·30 „ 10¼	73	9
20	1,185	17	63	11·43 „ 11½	81	7
21	1,306	19	52	12·00 „ 12	85	8
24	1,706	25	56	13·71 „ 14	98	0

EXAMPLE.—According to the above table, a 15ft. switch should not be used with a Radius of more than 666ft., nor with a Crossing more than 8½, nor with a Lead more than 61ft. 3ins.

Notes.—*The R, but not the N and L, apply to any gauge.

†With 4in. H, the R may be 12% more; with 4¾in. H, the R should be 6% less.

‡The table is based on C.L.M., but the use of R.A. or Isosceles M., will not appreciably alter the dimensions.

TABLE 30.

CROSSING ANGLES BY CENTRE LINE MEASURE.

Converted to Degrees, and to Right Angle and Isosceles Measures; also their Trigonometrical Ratios.

Angle of Crossing C.L.M.	Deg	M.	S.	Sine.	Cosine.	Tangent	Cotang. which = Rt.A.M.	Secant.	Cosec.	Isosceles Measure
1	53	7	48	·8000	6000000	1·3333	·750	1·6667	1·250	1·118
1⅛	47	55	30	·7423	6701030	1·1077	·903	1·4923	1·347	1·231
1¼	43	36	10	·6896	7241370	·9524	1·050	1·3810	1·450	1·346
1⅜	39	57	58	·6423	7661234	·8381	1·193	1·3048	1·557	1·463
1½	36	52	11	·6000	8000000	·7500	1·333	1·2500	1·667	1·581
1⅝	34	12	19	·5621	8270270	·6797	1·471	1·2092	1·779	1·700
1¾	31	53	26	·5282	8490564	·6222	1·607	1·1778	1·893	1·820
1⅞	29	51	16	·4980	8672200	·5742	1·742	1·1531	2·008	1·941
2	28	4	20	·4706	8823530	·5333	1·875	1·1334	2·125	2·061
2¼	25	3	26	·4235	9058822	·4675	2·139	1·1039	2·361	2·305
2½	22	37	11	·3846	9230770	·4167	2·400	1·0833	2·600	2·549
2¾	20	36	34	·3520	9360000	·3761	2·659	1·0684	2·841	2·796
3	18	55	28	·3243	9459460	·3429	2·917	1·0571	3·083	3·041
3¼	17	29	32	·3006	9537572	·3152	3·173	1·0485	3·327	3·288
3½	16	15	36	·2800	9600000	·2917	3·429	1·0417	3·571	3·535
3¾	15	11	20	·2620	9650655	·2715	3·683	1·0362	[illegible]	3·783
4	14	15	—	·2461	9692308	·2540	3·937	1·0318	4·062	4·031
4¼	13	25	10	·2321	9726962	·2386	4·191	1·0281	4·3[illegible]9	4·279
4½	12	40	48	·2195	9756098	·2250	4·444	1·0250	4·555	4·528
4¾	12	1	4	·2082	9780822	·2129	4·697	1·0224	4·803	4·776
5	11	25	16	·1980	9801980	·2020	4·950	1·0202	5·050	5·025
5¼	10	52	50	·1888	9820224	·1922	5·202	1·0183	5·298	5·274
5½	10	23	19	·1803	9836065	·1833	5·455	1·0167	5·545	5·523
5¾	9	56	22	·1726	9849906	·1752	5·706	1·0152	5·793	5·772
6	9	31	38	·1655	9862070	·1678	5·958	1·0140	6·042	6·021
6¼	9	8	52	·1590	9872814	·1610	6·210	1·0129	6·290	6·270
6½	8	47	50	·1529	9882353	·1548	6·461	1·0119	6·538	6·519
6¾	8	28	22	·1473	9890860	·1490	6·713	1·0110	6·787	6·768
7	8	10	16	·1421	98984[illegible]	·1436	6·964	1·0103	7·036	7·018
7¼	7	53	25	·1373	9905370	·1386	7·215	1·0096	7·284	7·268
7½	7	37	40	·1327	9911504	·1339	7·467	1·0089	7·533	7·517
7¾	7	22	58	·1285	9917098	·1296	7·718	1·0084	7·7[illegible]2	7·767
8	7	9	8	·1245	9922178	·1255	7·969	1·0079	8·031	8·[illegible]6
8¼	6	56	10	·1208	9926806	·1217	8·220	1·0074	8·280	8·266
8½	6	43	58	·1172	9931034	·1181	8·471	1·0069	8·529	8·515
8¾	6	32	26	·1139	9934906	·1147	8·721	1·0065	8·779	8·765
9	6	21	34	·1108	9938462	·1115	8·972	1·0062	9·028	9·014
9¼	6	11	16	·1078	9941733	·1084	9·223	1·0058	9·277	9·264
9½	6	1	32	·1050	9944751	·1056	9·474	1·0056	9·526	9·513
9¾	5	52	16	·1023	994[illegible]541	·1028	9·724	1·0053	9·776	9·763

TABLE 80—*continued.*

Angle of Crossing C.L.M.	Deg. M. S.	Sine.	Cosine.	Tangent	Cotang., which = Rt. A. M.	Secant.	Cosec.	Isosceles Measure
10	5 43 28	·0997	·9950125	·1002	9·975	1·0050	10·025	10·012
10½	5 27 9	·0950	·9954751	·0955	10·476	1·0045	10·524	10·512
11	5 12 18	·0907	·9958762	·0911	10·977	1·0041	11·023	11·011
11½	4 58 44	·0868	·9962264	·0871	11·478	1·0038	11·522	11·511
12	4 46 18	·0832	·9965338	·0835	11·979	1·0035	12·021	12·010
12½	4 34 52	·0799	·9968051	·0801	12·480	1·0032	12·520	12·510
13	4 24 18	·0768	·9970458	·0770	12·981	1·0030	13·019	13·009
13½	4 14 31	·0740	·9972603	·0742	13·481	1·0027	13·518	13·509
14	4 5 27	·0713	·9974522	·0715	13·982	1·0026	14·018	14·009
15	3 49 4	·0666	·9977802	·0667	14·983	1·0022	15·017	15·008
16	3 34 46	·0624	·9980488	·0626	15·984	1·0020	16·016	16·008
17	3 22 10	·0588	·9982714	·0589	16·985	1·0017	17·015	17·007
18	3 10 54	·0555	·9984380	·0556	17·986	1·0016	18·014	18·007
19	3 0 54	·0526	·9986159	·0527	18·987	1·0014	19·013	19·006
20	2 51 50	·0500	·9987508	·0500	19·9875	1·0013	20·0125	20·006

TABLE 31.

TURNOUTS FROM A STRAIGHT MAIN LINE,

With a given Angle of Crossing by Centre Line Measure and a Straight Switch.

Gauge 4ft. 8½ins. Switch heel divergence 4¼ins.

(Refer to Fig XIII.—1.)

Lead is distance on straight from switch heel to fine point of crossing.

For the Lead measured along the curved turnout rail, divide 18 inches by the Number of the Crossing, and add the result to the Lead on the straight.

Radius is of curve between heel of switch and crossing point.

Versed sine is on above curve If the string for setting the versed sine is tied to blunt nose of crossing, the slight increase in the versed sine is negligible.

This Table is based on Rules 1a, 2a, and 8a in Table 25. The number in 7th column indicates whether the switch is suitable to the crossing, the ideal recommended being 1·75.

For the application of this Table when the main is curved, see Chapters XIII. and XV.

TABLE 31—*continued.*

For Switch Lengths not included in Table: the Lead, Radius, and Versed sine may be obtained by proportion; *e.g.*, for 18ft. switch take the average of Leads, etc., shown for 16 and 20 feet switches.

4¾ins. Heel divergence.—The last column shows the decrease in the Lead as compared with that for 4½ins. divergence. Example: for 1 in 8 crossing 16ft. switch, 4¾ins. divergence, the Lead will be 58ft. 4ins. less 9ins. = 57ft. 7ins. The Radii will be practically the same. The Versed sines will be ¼in. less.

Irish Gauge.—With a Gauge of 5ft. 3ins. and 4½ins. Switch heel divergence, the Leads, Radii, and Versed sines may be obtained by adding ⅛ to those in this Table. Example—Crossing 1 in 8, Switch 15ft.

	Lead.	Radius.	Versed sine.
With 4′ 8½″ Gauge	57′ 9″	580′	8¾″
Add ⅛	7′ 2⅝″	72½′	1$\frac{1}{16}$″
Giving for 5′ 3″ Gauge...	64′ 11⅝″	652½′	9$\frac{13}{16}$″

This is because with the same heel divergence (H), the Leads, Radii, and Versed sines are proportional to Gauge minus H.

Angle of crossing by C.L.M.	Switch length.	Lead.		Radius.	Radius.		Versed sine.	Switch ÷ by C.L.M. of crossing.	Decrease in Lead for 4¾in. H.	
	Feet.	Ft.	ins.	Feet.	Chs.	ft.	Ins.		Ft.	ins.
1	6	8	0	10·9	...		12	6·00		½
1¼	6	9	11	15·8	...		11⅛	4·80		
1½	6	11	9	21·9	...		10¾	4·00		
1¾	6	13	6	29·1	...		10½	3·43		
2	6	15	3	37·5	...		10¼	3·00		1½
2¼	6	17	0	47·0	...		9⅞	2·67		
2½	6	18	7	57·8	...		9½	2·40		
2¾	6	20	3	69·8	1	4	9¼	2·18		
3	6	21	9	83·2	1	17	9	2·00		3
3¼	6	23	3	97·8	1	32	8⅝	1·85		
3¼	9	24	8	95·3	1	29	9⅞	2·77		
3½	6	24	9	113·9	1	48	8⅜	1·71		
3½	9	26	5	110·7	1	45	9¾	2·57		

TABLE 31—*continued.*

Angle of crossing by C.L.M.	Switch length.	Lead.		Radius.	Radius.		Vers. sine.	Switch ÷ by C.L.M. of crossing.	Decrease in Lead for 4¾in. H.
	Feet.	Ft.	ins.	Feet.	Chs.	ft.	Ins.		Ft. ins.
3¾	6	26	2	131·4	1	65	8⅛	1·60	
3¾	9	28	0	127·2	1	61	9½	2·40	
4	6	27	7	150·3	2	18	7⅞	1·50	5
4	9	29	8	144·9	2	13	9¼	2·25	
4	12	30	9	143·1	2	11	10⅛	3·00	4
4¼	6	29	0	171·0	2	39	7½	1·41	
4¼	9	31	3	164·0	2	32	9⅛	2·12	
4¼	12	32	5	161·6	2	30	10	2·82	
4½	6	30	4	193·0	2	61	7¼	1·33	
4½	9	32	9	185·3	2	52	8⅞	2·00	
4½	12	34	2	181·3	2	49	9¾	2·67	
4¾	9	34	3	206	3	8	8¾	1·89	
4¾	12	35	9	202	3	4	9⅝	2·53	
5	9	35	9	229	3	31	8½	1·80	6
5	12	37	5	224	3	26	9½	2·40	5
5¼	9	37	3	253	3	55	8⅜	1·71	
5¼	12	39	0	248	3	50	9⅜	2·29	
5½	9	38	8	279	4	15	8⅛	1·64	
5½	12	40	7	272	4	8	9¼	2·18	
5¾	9	40	2	306	4	42	8	1·57	
5¾	12	42	2	298	4	34	9	2·09	
6	9	41	6	335	5	5	7⅞	1·50	8
6	12	43	9	326	4	62	8⅝	2·00	7
6¼	9	42	11	365	5	35	7⅜	1·44	
6¼	12	45	3	354	5	24	8¾	1·92	
6½	9	44	3	398	6	2	7½	1·38	
6½	12	46	9	384	5	54	8⅝	1·85	
6½	15	48	5	378	5	48	9⅜	2·31	
6½	16	48	10	377	5	47	9½	2·46	
6¾	9	45	7	431	6	35	7¼	1·33	
6¾	12	48	3	416	6	20	8½	1·78	
6¾	15	50	0	409	6	13	9¼	2·22	
6¾	16	50	6	407	6	11	9½	2·37	
7	9	46	11	467	7	5	7⅛	1·29	10
7	12	49	9	448	6	52	8⅜	1·71	
7	15	51	7	440	6	44	9⅛	2·14	

TABLE 31—*continued.*

Angle of crossing by C.L.M.	Switch length.	Lead.		Radius.	Radius.		Vers. sine.	Switch ÷ by C.L.M. of crossing.	Decrease in Lead for 4¾in. H.	
	Feet.	Ft.	ins.	Feet.	Chs.	ft.	Ins.		Ft.	ins.
7	16	52	1	439	6	43	9⅜	2·29		8
7¼	12	51	2	483	7	21	8¼	1·66		
7¼	15	53	2	473	7	11	9	2·07		
7¼	16	53	8	472	7	10	9¼	2·21		
7½	12	52	7	518	7	56	8⅛	1·60		
7½	15	54	8	507	7	45	8⅞	2·00		
7½	16	55	3	505	7	43	9⅛	2·13		
7¾	12	54	0	555	8	27	7⅞	1·55		
7¾	15	56	3	513	8	15	8¾	1·94		
7¾	16	56	10	540	8	12	9	2·07		
8	12	55	5	594	9	0	7¾	1·50		11
8	15	57	9	580	8	52	8⅜	1·87		
8	16	58	4	577	8	49	8⅞	2·00		9
8¼	12	56	10	634	9	40	7⅜	1·45		
8¼	15	59	3	619	9	25	8½	1·82		
8¼	16	59	10	615	9	21	8¾	1·94		
8½	12	58	2	676	10	16	7¼	1·41		
8½	15	60	9	658	9	61	8⅜	1·76		
8½	16	61	5	654	9	60	8⅝	1·88		
8¾	15	62	2	699	10	39	8⅜	1·71		
8¾	16	62	11	695	10	35	8½	1·83		
9	15	63	7	742	11	16	8¼	1·67		11
9	16	64	4	737	11	11	8½	1·78		
9	20	66	9	725	10	65	9¼	2·22		10
9¼	15	65	0	786	11	60	8⅛	1·62		
9¼	16	65	10	781	11	55	8⅜	1·73		
9¼	20	68	3	767	11	41	9⅛	2·16		
9½	15	66	6	831	12	39	8	1·58		
9½	16	67	3	826	12	34	8¼	1·68		
9½	20	69	10	810	12	18	9⅛	2·11		
9¾	15	67	11	879	13	21	7⅞	1·54		
9¾	16	68	9	872	13	14	8¼	1·64		
9¾	20	71	5	855	12	63	9	2·05		
10	15	69	3	927	14	3	7¾	1·50	1	1
10	16	70	2	919	13	61	8⅛	1·60		
10	20	72	11	901	13	43	8⅞	2·00		11

TABLE 31—*continued.*

Angle of crossing by C.L.M.	Switch length.	Lead.		Radius.	Radius.		Versed sine.	Switch ÷ by C.L.M. of crossing.	Decrease in Lead for 4¾in. H.	
	Feet.	Ft.	ins.	Feet.	Chs.	ft.	Ins.		Ft.	ins.
10	21	73	6	898	13	40	9	2·10		
10½	16	73	0	1020	15	30	7⅞	1·52		
10½	20	76	0	996	15	6	8¾	1·90		
10½	21	76	7	993	15	3	8⅞	2·00		
11	16	75	9	1126	17	4	7⅝	1·45	1	3
11	20	79	0	1098	16	42	8½	1·82		
11	21	79	8	1093	16	37	8¾	1·91	1	1
11½	16	78	6	1238	18	50	7½	1·39		
11½	20	81	11	1205	18	17	8⅜	1·74		
11½	21	82	8	1199	18	11	8⅝	1·83		
12	16	81	2	1358	20	38	7¼	1·33	1	5
12	20	84	10	1318	19	64	8¼	1·67		
12	21	85	7	1311	19	57	8⅜	1·75		
12	24	87	7	1296	19	42	8⅞	2·00	1	2
12½	20	87	9	1437	21	51	8⅛	1·60		
12½	21	88	6	1428	21	42	8¼	1·68		
12½	24	[illegible]	7	1410	21	24	8¾	1·92		
13	20	90	7	1560	23	42	7⅞	1·54	1	6
13	21	91	5	1551	23	33	8⅛	1·61		
13	24	93	7	1530	23	12	8⅝	1·85	1	4
13½	20	93	4	1692	25	42	7¾	1·48		
13½	21	94	3	1680	25	30	8	1·56		
13½	24	96	7	1656	25	6	8½	1·78		
14	20	96	1	1827	27	45	7⅝	1·43	1	7
14	21	97	0	1814	27	32	7¾	1·50		
14	24	99	6	1786	27	4	8⅜	1·71	1	5
15	24	105	4	2066	31	20	8⅛	1·60	1	7
16	24	111	0	2370	35	60	7¾	1·50	1	10
17	24	116	5	2697	40	57	7½	1·41	2	0
18	24	121	9	3052	46	16	7¼	1·33	2	1
19	24	127	0	3433	52	1	7	1·26	2	2
20	24	132	1	3846	58	18	6⅞	1·20	2	4

TABLE 32.—Spare.

TABLE 33.

DOUBLE LINE JUNCTIONS.

Gauge 4′ 8½″. H 4½″. Space between tracks, 6′ 0″ clear, 6′ 5½″ gauge lines. Crossings in C.L. Measure. Leads are from heel of switch to fine point of Crossing.

For explanation of symbols see Chapter XVI. and Fig. XVI.—1.

12ft. Switches.

Radius on centre line.	Radius of outer rail.	Angles of Crossings N_1		N_2		N_3		N_4		Leads (in feet and inches). L_1		L_2		L_3		L_4	
Ch.	Ft									Ft.	in.	Ft.	in.	Ft.	in.	Ft.	in.
3½	239	5·16	5	4·32	4½	3·27	3¼	2·71	2¾	38	5	46	1	63	11	77	6
4	272	5·49	5½	4·60	4¼	3·[illegible]	3½	2·90	3	40	7	48	10	67	10	82	4
4½	305	5·81	5¾	4·88	5	3·70	[illegible]	3·08	3	42	7	51	4	71	5	86	11
5	338	[illegible]·11	6	5·14	5¼	3·89	4	3·24	3¼	44	5	53	8	74	10	91	2
5½	371	6·40	6¼	5·39	5½	4·08	4	3·40	3½	46	0	55	9	78	0	95	[illegible]
6	404	6·66	6½	5·62	5¾	4·26	4¼	3·55	3½	47	8	57	10	80	11	99	0
6½	437	6·91	7	5·83	6	4·43	4½	3·69	3¾	49	2	59	10	83	9	102	6
7	470	7·16	7¼	6·04	6	4·59	4½	3·83	3¾	50	7	61	[illegible]	[illegible]	6	105	11
7½	503	7·39	7¼	6·24	6¼	4·75	4¾	3·96	4	51	11	63	6	89	1	109	3
8	536	7·62	7½	6·44	6½	4·90	5	4·09	4	53	3	65	2	91	7	112	4
8½	569	7·81	7¾	6·63	6¾	5·04	5	4·21	4¼	54	6	66	9	93	11	115	4
9	602	8·05	8	6·82	6	5·18	5¼	4·33	4¼	55	7	68	3	96	2	[illegible]	2

15ft. Switches.

Ch.	Ft.	N_1		N_2		N_3		N_4		Ft.	in.	Ft.	in.	Ft.	in.	Ft.	in.
6	404	6·71	6¾	5·65	5¾	4·27	4¼	3·56	3½	49	8	60	0	83	1	101	1
6½	437	6·97	7	5·87	6	4·44	4½	3·71	3¾	51	5	62	2	86	2	104	10
7	470	7·22	7¼	6·09	6	4·61	4½	3·85	3¾	53	0	64	2	89	1	108	5
7½	503	7·46	7½	6·30	6¼	4·77	4¾	3·98	4	54	6	66	0	91	10	111	11
8	536	7·70	7¾	6·50	6½	4·92	5	4·11	4	55	11	67	10	94	5	115	3
8½	569	7·93	8	6·70	6¾	5·07	[illegible]	4·23	4¼	57	3	69	7	96	11	118	5
9	602	8·14	8	6·89	7	5·21	5¼	4·35	4¼	58	7	71	3	99	5	121	6
9½	635	8·36	8¼	7·07	7	5·35	5¼	4·47	4½	59	10	72	11	101	10	124	5
10	668	8·56	8½	7·24	7¼	5·48	5½	4·59	4½	61	1	74	6	104	2	127	[illegible]
10½	701	8·76	8¾	7·41	7½	5·61	5½	4·70	4¾	62	3	76	0	106	4	130	0
11	734	8·96	9	7·58	7½	5·74	5¾	4·81	4¾	63	4	77	6	108	6	132	9
11½	767	9·15	9¼	7·75	7¾	5·87	5¾	4·91	5	64	5	78	10	110	6	135	5
12	800	9·34	9¼	7·91	8	6·00	6	5·02	5	65	6	80	2	112	6	137	11
12½	833	9·52	9½	8·06	8	6·12	6¼	5·12	5¼	66	7	81	6	114	5	140	5
13	866	9·68	9¾	8·21	8¼	6·24	6¼	5·22	5¼	67	[illegible]	82	10	116	4	142	10

DOUBLE LINE JUNCTIONS—*continued.*

18FT. SWITCHES.

Radius on centre line.	Radius of outer rail.	ANGLES OF CROSSINGS.								LEADS. (In feet and inches.)							
		N_1		N_2		N_3		N_4		L_1		L_2		L_3		L_4	
Chs.	Ft.									F.	ins.	F.	ins.	F.	ins.	F	ins
10	668	8·63	8½	7·29	7¼	5·50	5½	4·60	4½	63	3	76	8	106	6	129	10
10½	701	8·83	8¾	7·46	7½	5·64	5¾	4·71	4¾	64	7	78	4	108	10	132	8
11	734	9·02	9	7·63	7¾	5·76	5¾	4·82	4¾	65	10	79	11	111	1	135	5
11½	767	9·21	9¼	7·80	7¾	5·89	6	4·92	5	67	3	81	6	113	3	138	2
12	800	9·40	9½	7·96	8	6·02	6	5·03	5	68	1	83	0	115	5	140	9
12½	833	9·59	9½	8·12	8¼	6·14	6¼	5·13	5	69	2	84	5	117	6	143	4
13	866	9·78	9¾	8·28	8¼	6·26	6¼	5·23	5¼	70	3	85	9	119	5	145	11
13½	899	9·96	10	8·43	8½	6·38	6½	5·33	5¼	71	4	87	1	121	4	148	5
14	932	10·13	10¼	8·58	8½	6·50	6½	5·43	5½	72	4	88	5	123	2	150	11
15	998	10·47	10½	8·87	9	6·72	6¾	5·61	5½	74	3	90	11	126	11	155	8
16	1064	10·80	10¾	9·14	9¼	6·93	7	5·79	5¾	76	1	93	4	130	6	160	2
17	1130	11·12	11	9·41	9½	7·13	7	5·97	6	77	11	95	8	133	11	164	6
18	1196	11·41	11½	9·67	9¾	7·33	7¼	6·14	6	79	9	97	11	137	2	168	7
19	1262	11·69	11¾	9·93	10	7·53	7½	6·31	6¼	81	6	100	0	140	4	172	7
20	1328	11·97	12	10·19	10¼	7·72	7¾	6·47	6½	83	1	102	0	143	4	176	6
21½	1427	12·40	12½	10·54	10½	8·00	8	6·70	6¾	85	5	104	11	147	10	182	3

21FT. SWITCHES.

Chs.	Ft.									F.	ins.	F.	ins.	F.	ins.	F.	ins.
12	800	9·46	9½	7·98	8	6·03	6	5·03	5	70	0	84	11	117	6	143	2
13	866	9·82	9¾	8·29	8¼	6·27	6¼	5·24	5¼	72	6	88	0	121	9	148	4
14	932	10·18	10¼	8·60	8½	6·51	6½	5·43	5½	74	9	90	9	125	9	153	5
15	998	10·53	10½	8·90	9	6·73	6¾	5·62	5½	76	10	93	5	129	8	158	4
16	1064	10·87	10¾	9·18	9¼	6·95	7	5·80	5¾	78	9	96	0	133	4	163	0
17	1130	11·19	11¼	9·46	9½	7·16	7¼	5·99	6	80	7	98	5	136	10	167	5
18	1196	11·49	11½	9·73	9¾	7·36	7¼	6·16	6¼	82	5	100	9	140	3	171	8
19	1262	11·80	11¾	9·99	10	7·56	7½	6·32	6¼	84	2	103	0	143	7	175	10
20	1328	12·09	12	10·25	10¼	7·75	7¾	6·48	6½	85	11	105	2	146	10	179	10
21½	1427	12·50	12½	10·60	10½	8·01	8	6·71	6¾	88	7	108	4	151	4	185	10

Note—For switch lengths not given in table:—

(i.) The difference in the crossing angles will be negligible.

(ii.) Approximate leads may be obtained by proportion, *e.g.*: for 20ft. switches deduct one-third of difference between leads given for 18ft. and 21ft. switches, from leads given for 21ft. switches.

TABLE 34.

RADII OBTAINED IN A JUNCTION WITH VARIOUS SPACES BETWEEN TRACKS, THE DIAMOND CROSSING BEING 1 IN 8.

Gauge 4′ 8½″ Switch heel divergence 4½″ *

Space between two main tracks				Switch length.	Radius from a straight main †			Increase in radius for 1′ of space.	S ÷ √ R.
Clear		Gauge lines							
Ft.	ins	Ft.	ins	Ft.	Ft.	Chs.	ft.	Ft ‡	
6	0	6	5½	15	1145	21	59	133·9	·394
				16	1437	21	51	133·2	·422
				18	1426	21	40	132·2	·477
				20	1419	21	33	131·5	·531
				21	1416	21	30	131·2	·558
				24	1409	21	23	130·5	·639
6	6	6	11½	16	1504	22	52	—	·428
				24	1474	22	22	—	·625
7	0	7	5½	16	1571	23	53	—	·404
				24	1539	23	21	—	·612
8	0	8	5½	20	1682	25	32	—	·[illegible]
				24	1670	25	20	—	·587
10	0	10	5½	20	1945	29	31	—	·453
				24	1931	29	17	—	·546
12	0	12	5½	20	2208	33	30	—	·426
				24	2192	33	14	—	·513

Notes.—

* With 4¾″ divergence the Radii will be slightly greater, about 4 ft. average.

† If both lines are curved this will be "Equivalent Radius."

‡ Example.—Switches 24′ Space 11′ 6″ *i.e.*, 5½′ more than 6′:—

Radius for 6′ space	=	1409′
Add 130·5 × 5½	=	718′
Radius for 11′ 6″ space	=	2127′

TABLE 35.

DOUBLE TURNOUTS.

Width* between gauge lines at toe of second pair of switches.

To find Width (Y) add the appropriate dimensions "A" and "B" together. Example:—Switches 15ft.; distance between toes of switches 28ft. (therefore l=28′−15′=13′); radius 400ft.; H=4½″. Then A+B=2½″+8⅜″=10⅞″=Y.

DIMENSION A (in inches).

Equiv. Radius of 1st turnout.	Distances in feet from heel of 1st to toe of 2nd switch (l Fig. xvii.–8).										
	6	7	8	9	10	11	12	13	14	15	16
200	1⅛	1½	1⅞	2⅜	3	3⅝	4⅜	...	...	...	...
250	⅞	1¼	1½	2	2⅜	2⅞	3½	...	...	...	...
300	⅝	1	1¼	1⅝	2	2⅜	2⅞	...	...	...	...
350	...	⅞	1⅛	1⅜	1⅞	2⅛	2½	...	...	...	...
400	...	¾	1	1⅛	1⅜	1⅞	2⅛	2½	...	...	...
450	...	⅝	⅞	1⅛	1⅜	1⅝	1⅞	2¼	2¾	...	...
500	...	...	¾	1	1¼	1½	1¾	2	2⅜	...	...
550	...	...	¾	⅞	1⅛	1⅜	1⅝	1⅞	2⅛	2½	...
600	...	...	⅝	¾	1	1¼	1½	1¾	2	2¼	2½
650	...	...	...	¾	⅞	1⅛	1⅜	1½	1¾	2	2⅜
700	...	...	...	¾	⅞	1	1¼	1½	1⅝	1⅞	2¼
800	...	...	...	⅝	¾	⅞	1⅛	1¼	1½	1¾	1⅞

DIMENSION B (in inches).

Switch length.	Divergence (H).	Distances in feet between switch toes (P Fig. xvii.–8).													
		18	19	20	21	22	23	24	25	26	27	28	29	30	31
Feet.	Inches.														
12	4½	6¾	7⅛	7½	7⅞	8¼	8⅝	9	9⅜	...	...	...	...	...	...
15	4½	...	...	...	...	6⅝	6⅞	7¼	7½	7¾	8⅛	8⅜	8¾	9	9¼
16	4½	...	...	...	...	6¼	6½	6¾	7	7¼	7⅝	7⅞	8¼	8½	8¾
12	4¾	7⅛	7½	7⅞	8⅜	8¾	9⅛	9½	9⅞	...	...	...	...	...	...
15	4¾	...	...	...	...	7	7¼	7⅝	7⅞	8¼	8½	8⅞	9⅛	9½	9⅞
16	4¾	...	...	...	...	6½	6⅞	7⅛	7⅜	7¾	8	8¼	8⅝	8⅞	9¼

* Y on Figs. XVI.—1 and 8.

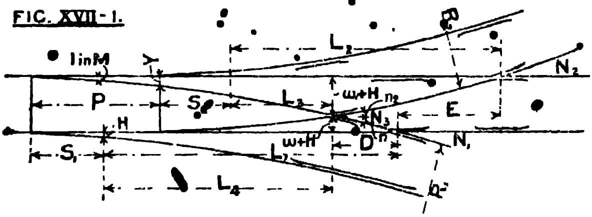
$A = l^2 \div 2R$. $B = P \div M$. $M = S \div H$.

TABLE 36.

DOUBLE TURNOUTS.

(Type 1.)

FIG. XVII-1.



Gauge 4ft. 8½ins. H 4½ins. Middle road straight.

Crossing Angle.						Radii of Turnouts.		Leads.											
N_1	N_2	N_3	S_1 & S_2	P	Y	R_1	R_2	L_1		L_2		L_3		D		E		w + H	
			ft.	ft.	ins.	ft.	ft.	ft.	ins.	ft.	ins.	ft.	ins.	ft.	ins.	ft.	ins.	ft.	ins.
6	5	4¼	12	21	9⅜	326	224	43	9	37	5	14	5	8	4	14	8	1	3¾
6	6	4½	12	21	9⅜	326	326	43	9	43	9	15	2	7	7	21	0	1	2¼
6	7	4¾	12	21	9⅜	326	448	43	9	49	9	15	7½	7	1½	27	0	1	1½
7	6	4¾	12	21	9	448	326	49	9	43	9	17	10	10	11	15	0	1	5¼
7	7	5¼	12	21	9	448	448	49	9	49	9	18	6½	10	2½	21	0	1	4
[illegible]	8	5½	12	21	9	448	594	49	9	55	5	18	11	9	10	26	8	1	3½
8	7	5½	12	21	[illegible]	594	448	55	5	49	9	21	0	13	5	15	4	1	6¼
8	8	5¾	12	21	8¾	594	594	55	5	55	5	21	6	12	11	21	0	1	5½
7	7	5¼	12	24	10⅞	448	448	49	9	49	9	16	8½	9	0½	24	0	1	2½
7	8	5½	12	24	10⅞	448	594	49	9	55	5	17	1	8	8	29	8	1	1¾
8	7	5½	12	24	10½	594	448	55	5	49	[illegible]	19	3½	12	1½	18	4	1	4½
8	8	5¾	12	24	10½	594	594	55	5	55	5	19	9½	11	7½	24	0	1	4
8	8	6	15	27	9½	580	580	57	9	57	9	20	1	10	8	27	0	1	2¾
8	9	6¼	15	27	9½	580	742	57	9	63	7	20	8	10	1	32	10	1	2
9	8	6¼	15	27	9¼	742	580	63	7	57	9	22	8	13	11	21	2	1	4¾
9	9	6½	15	27	9¼	742	742	63	7	63	7	23	5	13	2	27	0	1	4
8	8	6	15	30	11¼	580	580	57	9	57	9	18	0	9	9	30	0	1	[illegible]
8	9	6¼	15	30	11¼	580	742	57	9	63	7	18	7	9	2	35	10	1	1
9	8	6¼	15	30	10⅞	742	580	63	7	57	9	21	4	12	3	24	2	1	3½
9	9	6½	15	30	10⅞	742	742	63	7	63	7	21	10	11	9	30	0	1	3

The dimensions given for 15ft. switches (4½ins. H) will apply practically to 16ft. switches with 4¾ins. H and the same P.

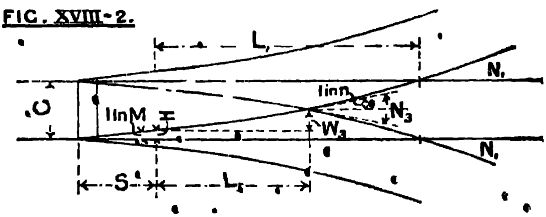
TABLE 37.

THREE THROWS.
(Type 1.)

MAIN LINE CROSSINGS OPPOSITE.

Gauge 4' 8½". Switch heel divergence 4½".

FIG. XVIII-2.



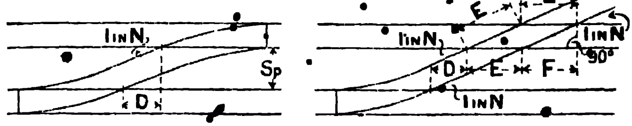
Length of longer Switch.	Angle of main line Crossing.	Radius of Turnouts.	Lead of Turnouts.	Angle of 3rd or middle Crossing.	Lead of 3rd Crossing.
S	N_1 & N_2	R. in feet.	L_1	N_3	L_3
Feet.			Ft. ins.		Ft. ins.
12	5	224	37 5	3·65 3½	23 7
12	6	326	43 9	4·36 4¼	27 2
15	7	440	51 7	5·09 5	32 2
15	8	580	57 9	5·79 5¾	35 6
18	9	730	65 8	6·53 6½	40 8
18	10	908	71 8	7·22 7¼	43 11

TABLE 38.

DISTANCES BETWEEN CROSSINGS.

Main lines parallel, all roads straight.

Gauge 4ft. $8\frac{1}{2}$ins. Rail Heads $2\frac{3}{4}$ins. wide.



RULES.—D = Sp × Cot B − G × Cosec B, E = G × Cosec B, F = G × Cot B. For examples see Chapters XIX. and XX.

Angle of crossing by C.L.M.	Distance apart of fine points with Sp. 6′ $5\frac{1}{2}$″ (6′ 0″ clear). D on sketch	Add for each inch of space.	Add for each foot of space.	Length of sides of diamond E on sketch	Distance from the obtuse to opposite the acute crossing. F on sketch	Fine point to actual nose. $\frac{3}{4}$″ nose.	Fine point to actual nose. $\frac{5}{8}$″ nose.
	Ft. ins.	Ins.	Ft. ins.	Ft. ins.	Ft. ins.	Ins.	Ins.
1	–1 $0\frac{3}{4}$	$\frac{3}{4}$	9	*5 $10\frac{1}{2}$	*3 $6\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{8}$
$1\frac{1}{8}$	– $6\frac{1}{8}$	$\frac{7}{8}$	$10\frac{3}{4}$	6 4	4 3	$\frac{7}{8}$	$\frac{3}{4}$
$1\frac{1}{4}$	– $\frac{1}{2}$	1	1 $0\frac{3}{4}$	6 10	4 $11\frac{1}{2}$	$\frac{7}{8}$	$\frac{3}{4}$
$1\frac{3}{8}$	$4\frac{1}{2}$	$1\frac{1}{8}$	1 $2\frac{1}{4}$	7 4	5 $9\frac{1}{2}$	1	$\frac{7}{8}$
$1\frac{1}{2}$	$9\frac{1}{4}$	$1\frac{3}{8}$	1 4	7 10	6 $3\frac{1}{2}$	$1\frac{1}{8}$	$\frac{7}{8}$
$1\frac{5}{8}$	1 $1\frac{1}{2}$	$1\frac{1}{2}$	1 $5\frac{5}{8}$	8 $4\frac{1}{2}$	6 $11\frac{1}{2}$	$1\frac{1}{4}$	1
$1\frac{3}{4}$	1 $5\frac{5}{8}$	$1\frac{5}{8}$	1 $7\frac{1}{4}$	8 $10\frac{1}{2}$	7 $6\frac{1}{2}$	$1\frac{3}{8}$	$1\frac{1}{8}$
$1\frac{7}{8}$	1 $9\frac{3}{8}$	$1\frac{3}{4}$	1 $8\frac{7}{8}$	9 $5\frac{1}{2}$	8 $2\frac{1}{2}$	$1\frac{3}{8}$	$1\frac{1}{8}$
2	2 $1\frac{1}{4}$	$1\frac{7}{8}$	1 $10\frac{1}{2}$	10 0	8 10	$1\frac{1}{2}$	$1\frac{1}{4}$
$2\frac{1}{4}$	2 $8\frac{3}{8}$	$2\frac{1}{8}$	2 $1\frac{5}{8}$	11 $1\frac{1}{2}$	10 1	$1\frac{5}{8}$	$1\frac{3}{8}$
$2\frac{1}{2}$	*3 3	$2\frac{3}{8}$	2 $4\frac{3}{4}$	12 3	11 $3\frac{1}{2}$	$1\frac{7}{8}$	$1\frac{1}{2}$
$2\frac{3}{4}$	3 $9\frac{1}{2}$	$2\frac{5}{8}$	2 $7\frac{7}{8}$	13 $4\frac{1}{2}$	12 6	2	$1\frac{3}{4}$
3	4 4	$2\frac{7}{8}$	2 11	14 6	13 $8\frac{1}{2}$	$2\frac{1}{4}$	$1\frac{7}{8}$
$3\frac{1}{4}$	4 10	$3\frac{1}{8}$	3 $2\frac{1}{8}$	15 8	14 11	$2\frac{3}{8}$	2
$3\frac{1}{2}$	5 4	$3\frac{3}{8}$	3 $5\frac{1}{8}$	16 10	16 $1\frac{1}{2}$	$2\frac{5}{8}$	$2\frac{1}{8}$
$3\frac{3}{4}$	5 10	$3\frac{5}{8}$	3 $8\frac{1}{4}$	17 $11\frac{1}{2}$	17 4	$2\frac{3}{4}$	$2\frac{3}{8}$
4	6 $3\frac{1}{2}$	$3\frac{7}{8}$	3 $11\frac{1}{4}$	19 $1\frac{1}{2}$	18 $6\frac{1}{2}$	3	$2\frac{1}{2}$
$4\frac{1}{4}$	6 $9\frac{1}{2}$	$4\frac{1}{8}$	4 $2\frac{1}{4}$	20 $3\frac{1}{2}$	19 9	$3\frac{1}{8}$	$2\frac{5}{8}$
$4\frac{1}{2}$	7 3	$4\frac{1}{2}$	4 $5\frac{3}{8}$	21 $5\frac{1}{2}$	20 11	$3\frac{3}{8}$	$2\frac{3}{4}$
$4\frac{3}{4}$	7 $8\frac{1}{2}$	$4\frac{3}{4}$	4 $8\frac{5}{8}$	22 $7\frac{1}{2}$	22 $1\frac{1}{2}$	$3\frac{1}{2}$	3
5	8 $2\frac{1}{2}$	5	4 $11\frac{3}{8}$	23 9	23 $3\frac{1}{2}$	$3\frac{3}{4}$	$3\frac{1}{8}$
$5\frac{1}{4}$	8 8	$5\frac{1}{4}$	5 $2\frac{3}{4}$	24 11	24 6	$3\frac{7}{8}$	$3\frac{1}{4}$

* Distances below these points are to the nearest $\frac{1}{2}$″

TABLE 38—*continued.*

Angle of crossing by C.L.M.	Distance apart of fine points with Sp. 6′ 5½″ (6′ 0″ clear). D on sketch.		Add for each inch of space.	Add for each foot of space		Length of sides of diamond. E on sketch.		Distance from the obtuse to opposite the acute crossing. F on sketch.		Fine point to actual nose. ¾″ nose.	Fine point to actual nose. ⅝″ nose.
	Ft.	ins.	Ins.	Ft.	ins.	Ft.	ins.	Ft.	ins.	Ins.	Ins.
5½	9	2	5½	5	5½	26	1	25	8	4⅛	3⅜
5¾	9	7	5¾	5	8½	27	3	26	10½	4¼	3⅝
6	10	0½	6	5	11½	28	5½	28	0½	4½	3¾
6¼	10	6	6¼	6	2½	29	7½	29	3	4⅝	3⅞
6½	10	11½	6½	6	5½	30	9½	30	5	4⅞	4
6¾	11	5	6¾	6	8½	31	11½	31	7½	5	4¼
7	11	10	7	6	11⅝	33	1½	32	9½	5¼	4⅜
7¼	12	4	7¼	7	2⅝	34	3½	33	11½	5⅜	4½
7½	12	9	7½	7	5⅝	35	5½	35	2	5⅝	4⅝
7¾	13	2½	7¾	7	8⅝	36	7½	36	4	5¾	4⅞
8	13	7½	8	7	11⅝	37	10	37	6	6	5
8¼	14	1	8¼	8	2⅝	39	0	38	8½	6⅛	5⅛
8½	14	6½	8½	8	5⅝	40	2	39	10½	6⅜	5¼
8¾	15	0	8¾	8	8⅝	41	4	41	0½	6½	5½
9	15	5	9	8	11⅝	42	6	42	3	6¾	5⅝
9¼	15	10½	9¼	9	2⅝	43	8	43	5	6⅞	5¾
9½	16	4	9½	9	5⅝	44	10	44	7	7⅛	5⅞
9¾	16	9	9¾	9	8¾	46	0½	45	9	7¼	6⅛
10	17	2½	10	9	11¾	47	2½	46	11½	7½	6¼
10½	18	1½	10½	10	5¾	49	6½	49	4	7⅞	6½
11	19	0	11	10	11¾	51	11	51	8	8¼	6⅞
11½	19	10½	11½	11	5¾	54	3	54	0½	8⅝	7⅛
12	20	9	12	11	11¾	56	7	56	4½	9	7½
12½	21	8	12½	12	5¾	58	11½	58	9	9⅜	7¾
13	22	6½	13	12	11¾	61	3½	61	1½	9¾	8⅛
13½	23	5	13½	13	5¾	63	8	63	5½	10⅛	8⅜
14	24	3½	14	13	11¾	66	0	65	10	10½	8¾
15	26	0½	15	14	11¾	70	8½	70	6½	11¼	9⅜
16	27	10	16	15	11¾	75	5	75	3	12	10
17	29	7	17	16	11⅞	80	1½	79	11½	12¾	10⅝
18	31	4	18	17	11⅞	84	9½	84	8	13½	11¼
19	33	1	19	18	11⅞	89	6	89	4½	14¼	11⅞
20	34	10	20	19	11⅞	94	3	94	1	15	12½

TABLE 39.

CROSSOVER ROADS.

Main Lines straight. Turnout curves reversing with no straight between. (See Fig. XIX.—3.)

Table shewing distance apart of crossings and saving in distance over that for straight between crossings.

Gauge 4ft. 8½ins. Rail width 2¾ins.

Distance apart of crossings. D on sketch, Table 38.	Spaces (between gauge lines).									
	ft.	ins.	ft.	ins.	ft.	ins	ft.	ins.	ft.	ins.
	6	5½	7	5½	8	5½	9	5½	10	5½
	Crossing 1 in 7.				**Radius 438ft.**					
On straight ...	11	10	18	9¾	25	9¼	32	9	39	8½
Reverse curve..	11	3½	17	5¾	23	5	29	1¾	34	7¼
Saving		6½	1	4	2	4¼	3	7¼	5	1¼
	Crossing 1 in 8.				**Radius 578ft.**					
On straight ...	13	7¼	21	7¼	29	7	37	6¼	45	6¼
Reverse curve..	13	0	20	1	26	11	33	5¼	39	9
Saving		7¼	1	6¼	2	8	4	1¼	5	9¼
	Crossing 1 in 9.				**Radius 740ft.**					
On straight ...	15	5¼	24	5	33	4½	42	4¼	51	4
Reverse curve..	14	8½	22	8¾	30	5	37	9½	44	11
Saving		8¾	1	8¼	2	11½	4	6¾	6	5
	Crossing 1 in 10.				**Radius 925ft.**					
On straight ...	17	2½	27	2¼	37	2	47	2	57	1½
Reverse curve..	16	5¼	25	4½	33	11¼	42	2	50	1
Saving		9¼	1	9¾	3	2¾	5	0	7	0½

Notes.—The curve is taken at the same radius on centre line of 4ft. as that of a turnout with the given angle of crossing and 15ft. switch. (Table 31.)

The above figures shew that the saving in distance is so small as practically never to warrant the omission of the straight between crossings, when the main lines are straight.

TABLE 40.

SCISSORS CROSSOVERS.—Type 1.

(See Fig. XXI.-1-page 262.)

Gauge 4ft. 8½ins. H 4¼in. Crossing angles C. L. Measure.

Crossovers straight between straight mains.

(*a*) SPACE BETWEEN MAINS 6ft. clear. 6ft 5¼ins. gauge lines.*

$W_3 + H = 3$ft. 2¾ins.

Angle of crossing on main. N_1	Switch length. S	Radius of turnouts. R			Lead of turnouts L		Angle of obtuse. N_2		Angle of crossings in 6ft. N_3		X		l		D (From Table 38).	
	ft.	ft.	chs.	ft.	ft.	ins.					ft	ins.	ft	ins.	ft.	ins.
7	12	418	6	52	49	9	3·50	3½	4·24	4¼		10¼	38	3	11	10
7½	15	507	7	45	54	8	3·75	3¾	4·56	4½		10¼	42	5	12	9
8	15	580	8	52	57	9	4·00	4	4·86	5		10¼	44	7½	13	7¾
8½	15	658	9	64	60	9	4·25	4¼	5·16	5¼		10¼	46	10	14	6½
9	18	730	11	4	65	8	4·50	4½	5·47	5½		10⅜	50	11	15	5¼
9½	18	816	12	24	68	8	4·75	4¾	5·77	5¾		10⅜	53	0½	16	4
10	18	908	13	50	71	8	5·00	5	6·07	6		10⅜	55	4	17	2½

(*b*) SPACE 6ft. 4ins. clear. 6ft 9½ins. gauge lines.

$W_3 + H = 3$ft. 4⅛ins.

7	12		4·13	4¼	1	0¼	39	8	14	2
7½	15		4·41	4½	1	0¼	43	11	15	3
8	15	R, L and N_2	4·73	4¾	1	0¼	46	2	16	3¾
8½	15	as above	5·02	5	1	0¼	48	6	17	4½
9	18	(independent of the space).	5·33	5½	1	0⅜	52	8	18	5¼
9½	18		5·62	5¾	1	0⅜	55	0	19	6
10	18		5·90	6	1	0⅜	57	3	20	6[illegible]

(*c*) SPACE 6ft. 6in. clear. 6ft. 11½in. gauge lines

$W_3 + H = 3$ft. 5¾ins.

7	12		4·08	4	1	1⅛	40	4	15	4
7½	15		4·38	4½	1	1⅛	44	8	16	5¾
8	15	R, L and N_2	4·68	4¾	1	1¼	47	0	17	7¼
8½	15	as above	4·96	5	1	1¼	49	4	18	9½
9	18	(independent of the space).	5·26	5¼	1	1¼	53	6	19	11
9½	18		5·54	5½	1	1⅜	55	11	21	1
10	18		5·84	5¾	1	1⅜	58	4	22	2½

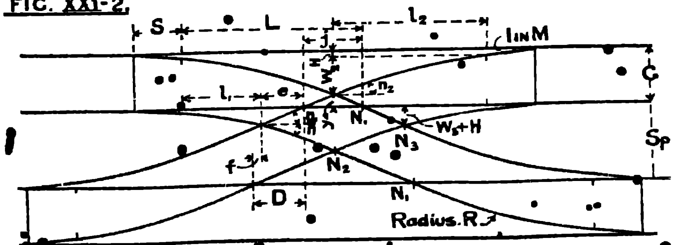
N_2, X, and D will always be the same for the same N_1 and space, whatever the length of switch.

• With 16ft. switches and 4¾in. H and the same crossing N_1, the R L N_2 and l will be practically the same as given for 15ft. switches, 4¼in. H.

* Special crossing chairs will usually be necessary in any scissors crossovers with a 6ft. space.

TABLE 41.

FIC. XXI-2.



SCISSORS CROSSOVERS.

Type 2.

Gauge 4′ 8½″. H 4½″. Crossing angles C.L. Measure. Crossovers straight between straight mains. Crossings N_3 and N_1 lie on the same timber with a 6ft. space.

SPACE BETWEEN TRACKS 6′ 0″ clear, 6′ 5½″ gauge lines.
All dimensions except D and f will apply when space is other than 6′ 0″.

Angle of crossings on main. N_1	Switch length. S	Radius of Turnouts R	Lead of Turnouts L		From Table 38. D		e		f		l_1		Angle of crossings in 6ft. N_3		j		W_2+H		v		Angle of obtuse. N_2	
	Feet.	Feet.	Ft.	ins.			Ft.	ins.	Ft.	ins.	Ft	ins.			Ft.	ins.	Ft.	ins.	Ft.	ins.		
7	12	448	49	9	11	10	10	10½		11¾	21	5	4·49	4½	17	5	3	6¾	1	2⅜	4·06	4
7¼	15	473	53	2	12	3½	11	3¼	1	0¼	23	8	4·68	4¾	18	2¾	3	6¾	1	2¼	4·22	4¼
7½	15	507	54	8	12	9	11	8	1	1	24	2½	4·84	5	18	9½	3	6¾	1	2½	4·37	4½
7¾	15	543	56	3	13	2½	12	2¾		11¾	25	6	4·98	5	19	0¼	3	6⅞	1	1⅞	4·50	4½
8	15	580	57	9	13	7¾	12	7½	1	0¼	25	7	5·14	5¼	19	6½	3	6¾	1	1¾	4·64	4¾
8½	15	658	60	9	14	6½	13	5	1	1½	26	7½	5·45	5½	20	8½	3	6¾	1	1¾	4·92	5
9	18	730	[illegible]	8	15	5¼	14	2½	1	2¾	29	4½	5·79	5¼	22	1	3	6¾	1	1¾	5·23	5¼
9½	18	816	68	6	16	4	15	0	1	4	30	6	6·11	6	23	2	3	6¾	1	1¾	5·50	5½
10	18	908	71	8	17	2½	15	9½	1	5	31	6½	6·42	6½	24	4	3	6¾	1	1¾	5·79	5¼

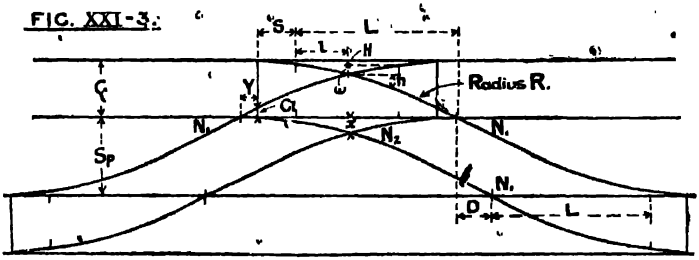
In the above W_3+H is 1′ 6¾″ for N_1 of 7, 7¼, and 7½; and 1′ 7″ for the remainder.
With 16ft. switches and 4½″ H, all dimensions will be practically the same as given for 15ft. switches 4½″ H.

TABLE 42.

SCISSORS CROSSOVERS.

(Type 3.)

FIG. XXI-3.



Gauge 4′ 8½″. Angles C.L. Measure. Crossovers straight between straight mains. Space, 6′ 5½″ gauge lines. All dimensions except D apply when Space is over 6′ 5½″.

Angle of crossings on mains. N_1	Switch length. S	Switch heel divergence. H	Radius of turnouts. R	Lead of turnouts. L		Clearance for points. Cl	Y		l		x		Angle of obtuse. N		D from Table 38.	
	feet	ins.	feet	ft.	in.	in.	ft.	in.	ft.	in	ft.	in.			ft.	in.
7	12	$4\frac{1}{2}$	448	49	9	9	5	3	16	3	1	$2\frac{1}{8}$	7·40	$7\frac{1}{2}$	11	10
7	15	$4\frac{1}{2}$	440	51	7	9	5	3	15	8	1	$0\frac{1}{2}$	8·25	$8\frac{1}{4}$	11	10
7	15* curved	$4\frac{3}{4}$	441	50	2	9	5	3	14	$11\frac{1}{2}$	1	1	7·92	8	11	10
8	12	$4\frac{1}{2}$	594	55	5	9	6	0	18	$8\frac{1}{2}$	1	3	7·96	8	13	$7\frac{3}{4}$
8	15† curved	5	591	54	$2\frac{1}{2}$	8	5	4	16	11	1	$2\frac{3}{4}$	8·07	8	13	$7\frac{3}{4}$
$8\frac{1}{2}$	15†	5	674	56	10	8	5	8	18	1	1	$3\frac{1}{8}$	8·30	$8\frac{1}{4}$	14	$6\frac{1}{2}$

* Switch curved to ⅛in. Versed sine.
† " " ¼in. "

TABLE 43.

OBTUSE CROSSINGS.

Length of Splay and Distance between Point Rails.

Crossing Angle 1 in. (C.L.M.)	Length of Splay (y, Fig. XX.-4). Rail head 2⅝in. ½in. point.	Rail head 2⅝in. ⅝in. point.	Rail head 2⅝in. ¾in. point.	Rail head 2¼in. ½in. point.	Rail head 2¼in. ⅝in. point	Rail head 2¼in. ¾in. point	Distance between diamond points. (Twice e, Fig. xx.-4.) Flangeway, F=1⅜in. ½in. point.	⅝in. point.	¾in. point.
	ins.	ins.	ins.	ins.	ins.	ins.	ft. ins.	ft. ins.	ft. ins.
1	2⅛	2	1⅞	2¼	2⅛	2	4½	4⅞	5
1½	3 3/16	3	2 13/16	3⅜	3 3/16	3	6¾	7⅛	7½
2	4¼	4	3¾	4½	4¼	4	9	9¼	10
2½	5 5/16	5	4 11/16	5⅝	5 5/16	5	11¼	11⅞	1 0½
3	6⅜	6	5⅝	6¾	6⅜	6	1 1½	1 2¼	1 3
3½	7 7/16	7	6 9/16	7⅞	7 7/16	7	1 3¾	1 4⅝	1 5½
4	8½	8	7½	9	8½	8	1 6	1 7	1 8
4½	9 9/16	9	8 7/16	10⅛	9 9/16	9	1 8¼	1 9⅜	1 10½
5	10⅝	10	9⅜	11¼	10⅝	10	1 10½	1 11¾	2 1
5½	11 11/16	11	10 5/16	12⅜	11 11/16	11	2 0¾	2 2⅛	2 3½
6	12¾	12	11¼	13½	12¾	12	2 3	2 4½	2 6
6½	13 13/16	13	12 3/16	14⅝	13 13/16	13	2 5¼	2 6⅞	2 8½
7	14⅞	14	13⅛	15¾	14⅞	14	2 7½	2 9¼	2 11
7½	15 15/16	15	14 1/16	16⅞	15 15/16	15	2 9¾	2 11⅝	3 1½
8	17	16	15	18	17	16	3 0	3 2	3 4
9	19⅛	18	16⅞	20¼	19⅛	18	3 4½	3 6¾	3 9
10	21¼	20	18¾	22½	21¼	20	3 9	3 11½	4 2

Distances y and e for flatter angles will be in proportion, *e.g.* for 1 in 14 take twice the distances given for 1 in 7.

The table will enable the crossing angle to be obtained from the splay when the obtuse point rails are out of position.

TABLE 44.

OBTUSE CROSSING DISTANCES.

d and d_1 on Fig. XX.-1. Gauge 4ft. 8½ins.

Angle. (C.L.M.)	d	d_1	Angle. (C.L.M.)	d	d_1	Angle. (C.L.M.)	d	d_1
	ft. ins.	ft. ins		ft. ins.	ins.		ft. ins.	ins.
1	5 3⅛	2 4¼	4	4 9	7	7	4 8⅝	4
1½	4 11½	1 6⅝	4½	4 8⅞	6¼	7½	4 8⅝	3¾
2	4 10¼	1 2⅛	5	4 8¾	5⅝	8	4 8⅝	3½
2½	4 9⅝	11¼	5½	4 8¾	5⅛	8½	4 8⅝	3⅜
3	4 9¼	9⅜	6	4 8¾	4¾	9	4 8⅝	3⅛
3½	4 9⅛	8	6½	4 8⅝	4⅜	10	4 8⅝	2⅞

d_1 for 1 in 12 is half that for 1 in 6, etc.

d and d_1 for other gauges may be obtained by multiplying the above dimensions by the figures given in Table 20.

TABLE 45.

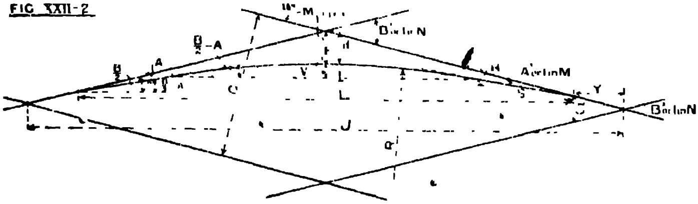
SLIP ROADS IN DIAMONDS.

Gauge 4′ 8½″ Switch heel divergence 4½″

Through lines straight. Slip rail tangential to straight switch.

Dimensions refer to outer rail of curve of slip.

FIG. XXII-2



Angle of crossing C. L. M. N	Distance apart of fine points of V crossing J ft.	J ins.	Distance V crossing to switch toe Y ft	Y ins	Clearance for points Cl ft	Cl ins.	Length of switch S ft.	Slip length including switches L ft	L ins	Radius of slip rail R ft	Versine of slip rail V ins	Diamond to slip rail (gauge lines) d ft	d ins.
8	75	6	5	4		8	15	64	10	461	$3\frac{7}{8}$	1	$1\frac{3}{8}$
8	75	6	6	0		9	15	63	6	447	$3\frac{3}{4}$	1	$1\frac{1}{4}$
8	75	6	6	3		$9\frac{3}{8}$	15	63	0	440	3 [illegible]	1	$1\frac{1}{8}$
8	75	6	7	9		$11\frac{5}{8}$	15	60	0	400	$3\frac{3}{8}$	*1	0 [illegible]
8	75	6	5	4		8	12	64	10	653	$3\frac{7}{8}$	1	4
8	75	6	6	0		9	12	63	6	632	$3\frac{3}{4}$	1	$3\frac{3}{4}$
8	75	6	6	3		$9\frac{3}{8}$	12	63	0	624	$3\frac{5}{8}$	1	$3\frac{1}{2}$
8	75	6	7	9		$11\frac{5}{8}$	12	60	0	576	$3\frac{3}{8}$	1	$2\frac{5}{8}$
8	75	6	9	$2\frac{3}{4}$	1	$1\frac{7}{8}$	12	57	0	528	$3\frac{1}{8}$	1	$1\frac{1}{4}$
8	75	6	10	9	1	$4\frac{1}{8}$	12	54	0	480	$2\frac{3}{4}$	1	1
$7\frac{3}{4}$	73	$1\frac{1}{2}$	5	$0\frac{3}{4}$		$7\frac{7}{8}$	15	63	0	418	$3\frac{7}{8}$	1	$1\frac{3}{8}$
$7\frac{3}{4}$	73	$1\frac{1}{2}$	6	$6\frac{3}{4}$		$10\frac{1}{8}$	15	60	0	380	$3\frac{1}{2}$	*1	$0\frac{1}{2}$
$7\frac{3}{4}$	73	$1\frac{1}{2}$	6	$6\frac{3}{4}$		$10\frac{1}{8}$	12	60	0	541	$3\frac{5}{8}$	1	$2\frac{3}{4}$
$7\frac{3}{4}$	73	$1\frac{1}{2}$	8	$0\frac{3}{4}$	1	$0\frac{1}{2}$	12	57	0	496	$3\frac{1}{4}$	1	2
$7\frac{3}{4}$	73	$1\frac{1}{2}$	9	$6\frac{3}{4}$	1	$2\frac{3}{4}$	12	54	0	451	3	1	$1\frac{1}{8}$
$7\frac{1}{2}$	70	$9\frac{1}{4}$	5	$4\frac{5}{8}$		$8\frac{5}{8}$	12	60	0	508	$3\frac{7}{8}$	1	$3\frac{1}{8}$
$7\frac{1}{2}$	70	$9\frac{1}{4}$	6	$10\frac{5}{8}$		11	12	57	0	466	$3\frac{1}{2}$	1	$2\frac{1}{8}$
$7\frac{1}{2}$	70	$9\frac{1}{4}$	8	$4\frac{3}{8}$	1	$1\frac{3}{8}$	12	54	0	423	$3\frac{1}{8}$	1	$1\frac{3}{8}$
$7\frac{1}{4}$	68	$5\frac{1}{8}$	5	$8\frac{1}{2}$		$9\frac{1}{2}$	12	57	0	437	$3\frac{3}{4}$	1	$2\frac{1}{2}$

TABLE 45—*continued.*

Angle of crossing C L M N	Distance apart of fine points of V crossings. J		Distance V crossing to switch toe. Y		Clearance for points. Cl		Length of switch S	Slip length including switches L		Radius of slip rail. R	Vers. sine of slip rail. V	Diamond to slip rail. (gauge lines) d	
	ft.	ins.	ft.	ins.	ft.	ins.	ft.	ft.	ins.	ft.	ins.	ft.	ins.
7¼	68	5⅛	7	2½	1	0	12	51	0	397	3⅜	1	1½
7	66	1	4	6½		7¾	12	57	0	411	4	1	2⅜
7	66	1	6	0½		10⅜	12	54	0	373	3⅜	1	1¾
7	66	1	7	6½	1	0⅞	12	51	0	336	3¼	*1	0⅞
6¾	63	8¾	4	10⅜		8⅞	12	54	0	350	3⅞	1	2
6¾	63	8¾	6	4⅜		11¼	12	51	0	315	3½	1	1
6½	61	4⅝	5	2¼		9⅜	12	51	0	295	3⅝	1	1¼
6½	61	4⅝	6	8¼	1	0⅜	12	48	0	263	3¼	*1	0¼
6½	61	4⅝	5	8¼		10½	10	50	0	380	3½	1	2¾
6½	61	4⅝	5	2¼		9⅜	9	51	0	468	3½	1	4¼
6½	61	4⅝	6	8¼	1	0⅜	9	48	0	425	3⅛	1	3⅛
6¼	59	0⅜	4	0¼		7¼	12	51	0	277	3⅞	1	1½
6¼	59	0⅜	5	6¼		10⅝	12	48	0	246	3½	*1	0½
6¼	59	0⅜	4	6¼		8⅝	10	50	0	353	3⅞	1	3
6¼	59	0⅜	6	0¼		11½	10	47	0	318	3½	1	2
6¼	59	0⅜	4	0¼		7¾	9	51	0	430	3¼	1	4½
6¼	59	0⅜	5	6¼		10⅝	9	48	0	391	3½	1	3½
6	56	8¼	4	3⅞		8¼	12	48	0	230	3¼	*1	0¾
6	56	8¼	4	10⅛		9⅜	10	47	0	295	3¼	1	2¼
6	56	8¼	4	4⅛		8¾	9	48	0	360	3¾	1	3¼
5¾	51	1⅛	5	2		10¾	10	44	0	243	3½	1	1½
5¾	54	4⅛	4	3⅛		9¾	9	45	0	298	3⅝	1	2⅞
5½	52	0	4	0		8¾	10	44	0	225	3⅞	1	1¾
5½	52	0	5	0		10⅞	9	12	0	244	3½	1	2
5¼	49	7⅞	4	3⅞		9⅞	10	41	0	182	3 [illegible]	*1	0⅞
5¼	49	7⅞	5	4	1	0⅛	9	39	0	196	3 [illegible]	1	1¼
5	47	3¾	4	7⅞		11⅛	10	38	0	144	3¾	*1	0
5	47	3¾	4	1⅞		10	9	39	0	180	3⅝	1	1½

* With chaired track, the distances "d" in these cases, will not usually allow room for check rails to clear chair jaws.

For length of diamond sides see Table 38.

With 16ft. switches and ¾ins H, the Radii will be 29ft. less, the V ⅛in. less, and the "d" ¼in. less than those given for 15ft. switches, therefore 16ft. switches are not usually suitable for slip roads.

TABLE 46.

FACTORS FOR THE SINGLE TURNOUT. (H $4\frac{1}{2}''$.)

Switch heel divergence (H) $4\frac{1}{2}$ins. Gauge 4ft. $8\frac{1}{2}$ins.
M = Angle of switch by C.L.M. $M = S \div H$.
N = Angle of crossing by C.L.M.
R = Radius of outer rail of turnout curve or equiv. radius.
L = Lead from switch heel to fine point of crossing.

S = Switch length.
V = Versed sine on turnout curve, in length of lead when the main is straight.
F_1, F_2 = Factors.
All dimensions must be in feet.

When N is given.				When Radius is given.			When Lead is given.			
When $S \div N$ is F_1	$M \div N$ is F_2	For Lead $\times N$ by F_3	For radius $\times N$ squared by F_4 †	When $S \div \sqrt{R}$ is F_5	For Crossing $\div \sqrt{R}$ by F_6	For Lead $\times \sqrt{R}$ by F_7	When $L \div S$ is F_8	For Rad. $\div L^2$ by F_9 †	For Crossing $\div L$ by F_{10}	V will be inches.
$1\frac{1}{4} = 1{\cdot}25$	$3\frac{1}{3}$	6·67	9·53	·405	3·09	2·16	5·33	4·667	6·67	7
‡$1\frac{1}{3} = 1{\cdot}333$	$3\frac{5}{9}$	6·75	9·42	·433	3·07	2·20	5·09	4·850	6·75	$7\frac{1}{4}$‡
$1\frac{1}{2} = 1{\cdot}5$	4	6·93	9·25	·493	3·04	2·28	4·62	5·200	6·93	$7\frac{3}{4}$
$1\frac{5}{8} = 1{\cdot}625$	$4\frac{1}{3}$	7·04	9·16	·538	3·05	2·33	4·33	5·417	7·04	$8\frac{1}{8}$
*$1\frac{3}{4} = 1{\cdot}75$	$4\frac{2}{3}$	7·14	9·09	·581	3·02	2·37	4·08	5·614	7·14	$8\frac{3}{8}$*
$1\frac{7}{8} = 1{\cdot}875$	5	7·22	9·03	·627	3·01	2·40	3·85	5·777	7·22	$8\frac{5}{8}$
$2 = 2{\cdot}0$	$5\frac{1}{3}$	7·30	8·98	·667	3·00	2·43	3·65	5·933	7·30	$8\frac{7}{8}$
$2\frac{1}{4} = 2{\cdot}25$	6	7·43	8·92	·752	2·99	2·49	3·30	6·191	7·43	$9\frac{1}{4}$

Notes.—The values of the switch whether obtained by F_1 F_5 or F_8 will correspond, *e.g.*, when $S = N \times 1\frac{1}{3}$, $S = \sqrt{R} \times {\cdot}493$, and S also $= L \div 4{\cdot}62$. *This line gives the relations recommended for general use. ‡This line gives the relations when the turnout curve produced is tangential to the main. †And add 2ft. For Examples see Chap. XXIV.

TABLE 47.

FACTORS FOR THE SINGLE TURNOUT. (H $4\frac{3}{4}$ ins.)

Gauge 4ft. $8\frac{1}{2}$ ins. Symbols same as for Table 46.

When N is given.				When Radius is given.			When Lead is given.			
When $S \div N$ is F_1	$M \div N$ is F_2	For Lead $\times N$ by F_3	For radius $\times N$ squared by F_4 †	When $S \div \sqrt{R}$ is F_5	For Crossing $\div \sqrt{R}$ by F_6	For Lead $\sqrt{R}$ by F_7	When $L \div S$ is F_8	For Rad. $\div L^2$ by F_9 †	For Crossing $\div L$ by F_{10}	V will be inches.
$1\frac{1}{4} = 1·25$	3·16	6·55	9·59	·404	3·10	2·12	5·24	4·48	6·55	$6\frac{3}{4}$
† 1·365	3·45	6·68	9·42	·444	3·07	2·18	4·89	4·74	6·68	$7\frac{1}{8}$
$1\frac{1}{2} = 1·5$	3·79	6·82	9·27	·493	3·05	2·24	4·55	5·02	6·82	$7\frac{1}{2}$
$1\frac{5}{8} = 1·625$	4·105	6·94	9·17	·537	3·03	2·29	4·27	5·25	6·94	$7\frac{7}{8}$
* $1\frac{3}{4} = 1·75$	4·42	7·03	9·09	·581	3·02	2·33	4·02	5·44	7·03	$8\frac{1}{8}$
$1\frac{7}{8} = 1·875$	4·74	7·12	9·03	·624	3·00	2·37	3·80	5·62	7·12	$8\frac{3}{8}$
2 = 2·0	5·05	7·20	8·98	·667	2·99	2·40	3·60	5·77	7·20	$8\frac{5}{8}$
$2\frac{1}{4} = 2·25$	5·68	7·33	8·90	·754	2·98	2·46	3·26	6·04	7·33	$9\frac{1}{8}$

Notes.—
As per Table 46.

TABLE 48.

FACTORS FOR VARIOUS WIDTHS.

For use in solution of Junction, Crossover, Double Turnout, and Special Single Turnout Problems, etc.

Gauge 4′ 8½″ Switch heel divergence 4½″ **Rail heads 2¾″** S = Switch length. N = Crossing No. (C.L.M.) N_1 = **Crossing No. at W for the single turnout.** F = **Factor.** R = Radius of outer rail. L = **Lead of crossing in question from switch heel.** W = Width across track from switch heel to rail on which crossing lies.

W	Applying to cases shewn in the various diagrams of the book as below.	$F_1 = S \div N_1 = 1\frac{1}{2}$, $F_5 = S \div \sqrt{R} = \cdot 493$		$F_1 = 1\frac{3}{4}$, $F_5 = \cdot 581$		$F_1 = 2$, $F_5 = \cdot 667$	
Feet ins.		† $N = \sqrt{R} \div$ by F_6	† $L = \sqrt{R} \times$ by F_7	† $N = \sqrt{R} \div$ by F_6	† $L = \sqrt{R} \times$ by F_7	† $N = \sqrt{R} \div$ by F_6	† $L = \sqrt{R} \times$ by F_7
1	2	3	4	5	6	7	8
4½	XVII.-9, w for pts D. Turnout.	1·153	·392	1·080	·434	1·032	·470
6	" " "	1·257	·496	1·190	·545	1·147	·585
9	XVII.-1, w. XX.-2, W_3* XX.-3, w.	1·442	·681	1·384	·739	1·348	·784
1 0	" " "	1·606	·845	1·554	·908	1·522	·960
1 11¾	XVIII.-2, 3 throws, crossing in c. of 4′	2·130	1·371	2·092	1·446	2·067	1·566
2 11½	XVII.-1, w_1 when w is 1′ 0″ XX.-2, W_2	2·548	1·787	2·516	1·870	2·496	1·932
3 2½	" " " " 9″	2·644	1·884	2·614	1·970	2·595	2·030
3 7	XV.-4, W_1 S. Turnout when D = 9″	2·783	2·021	2·754	2·108	2·735	2·170
3 10	" " " " D = 3″	2·871	2·109	2·843	2·199	2·825	2·264
4 4	XIV.-1, W Ordinary Turnout.	3·040	2·280	3·014	2·369	2·997	2·437
†5 4½	XXI.-1, W_2 Scissors, 6′ 4″ space.	3·360	2·601	3·335	2·690	3·320	2·757
†5 5¼	" " " 6′ 6″ space.	3·384	2·620	3·360	2·717	3·345	2·784
6 1	XVI.-1*, W_2, 6′ 0″ space. XIX.-6, W_2*.	3·570	2·808	3·547	2·902	3·533	2·970
6 7	" " 6′ 6″ " "	3·707	2·949	3·685	3·037	3·672	3·107

TABLE 48—*continued.*

W		Applying to cases shewn in the various diagrams of the book as below		$F_1 = S \div N_1 = 1\frac{1}{2}$ $F_5 = S \div \sqrt{R} = \cdot 493$		$F_1 = 1\frac{3}{4}$ $F_5 = \cdot 581$		$F_1 = 2$ $F_5 = \cdot 667$	
				† $N = \sqrt{R}$ ÷ by F_6	† $L = \sqrt{R}$ ÷ by F_7	† $N = \sqrt{R}$ ÷ by F_6	† $L = \sqrt{R}$ × by F_7	† $N = \sqrt{R}$ ÷ by F_6	† $L = \sqrt{R}$ × by F_7
Feet	ins.								
1		2		3	4	5	6	7	8
7	1	XVI.-1*. W_2. 7′ 0″ space	XIX.-6. W_2*	3·840	3·07[illegible]	3·818	3·175	3·806	3·246
8	1	„ „ 8′ 0″ „	„	4·032	3·328	4·072	3·432	4·060	3·508
9	$0\frac{1}{2}$	XVII.-6. $W_{\frac{1}{2}}$. D Turnout.		4·320	3·567	4·301	3·662	4·290	3·726
9	1	XVI.-1*. W_2. 9′ 0″ space	XIX.-6*	4·32[illegible]	3·570	4·311	3·665	4·299	3·729
10	1	„ „ 10′ 0″ „	„	4·555	3·768	4·537	3·884	4·516	3·965
10	$9\frac{1}{2}$	XVI.-1. $W_{\frac{1}{2}}$. 6′ 0″ „	„	4·708	3·942	4·690	4·042	4·680	4·124
11	$3\frac{1}{2}$	„ „ 6′ 6″ „	„	4·813	4·056	4·796	4·159	4·785	4·242
11	$9\frac{1}{2}$	„ „ 7′ 0″ „	„	4·915	4·148	4·899	4·251	4·889	4·336
12	$9\frac{1}{2}$	„ „ 8′ 0″ „	„	5·116	4·364	5·099	4·451	5·089	4·517
13	$9\frac{1}{2}$	„ „ 9′ 0″ „	„	5·307	4·559	5·292	4·646	5·282	4·713
14	$9\frac{1}{2}$	„ „ 10′ 0″ „	„	5·4[illegible]	4·730	5·477	4·832	5·468	4·909
15	6	„ $W_{\frac{1}{4}}$. 6′ 0″ „	„	5·6[illegible]	4·860	5·605	4·958	5·596	5·042
16	0	„ „ 6′ 6″ „	„	5·7[illegible]	4·943	5·695	5·047	5·685	5·125
16	6	„ „ 7′ 0″ „	„	5·7[illegible]	5·046	5·781	5·135	5·772	5·203
17	6	„ „ 8′ 0″ „	„	5·965	5·215	5·951	5·305	5·943	5·373
18	6	„ „ 9′ 0″ „	„	6·130	5·367	6·117	5·472	6·109	5·556
19	6	„ „ 10′ 0″ „	„	6·291	5·532	6·278	5·633	6·270	5·692

* R − G must be used instead of R in these cases

† To give greater accuracy, take $\sqrt{R - \frac{1}{4}W}$ instead of $\sqrt{R}$, especially with W's over 4′

‡ Apply only when scissors are in a curved main and turnout curves extend to N_2. For examples see Chap. XXIV.

TABLE 49.

BULL HEAD RAILWAY RAILS.—BRITISH STANDARD SECTIONS.

Weight per yard.	Depth. A	Width. B	Web. C	D	E	F	G	Area.
lbs.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	Sq. ins.
60	$4\frac{3}{4}$	$2\frac{5}{16}$	$\frac{17}{32}$	$1\frac{27}{64}$	1 [illegible]	$\frac{3}{8}$	$\frac{1}{4}$	5·869
65	$4\frac{7}{8}$	$2\frac{3}{8}$	$\frac{9}{16}$	$1\frac{1}{2}$	$1\frac{3}{32}$	$\frac{3}{8}$	$\frac{1}{4}$	6·338
70	5	$2\frac{7}{16}$	$\frac{19}{32}$	$1\frac{37}{64}$	$1\frac{9}{64}$	$\frac{3}{8}$	$\frac{5}{16}$	6·883
75	$5\frac{1}{8}$	$2\frac{1}{2}$	$\frac{5}{8}$	$1\frac{41}{64}$	$1\frac{13}{64}$	$\frac{3}{8}$	$\frac{5}{16}$	7·318
80	$5\frac{3}{8}$	$2\frac{9}{16}$	$\frac{21}{32}$	$1\frac{11}{16}$	$1\frac{7}{32}$	$\frac{3}{8}$	$\frac{5}{16}$	7·802
85*	$5\frac{15}{32}$	$2\frac{11}{16}$	$\frac{11}{16}$	$1\frac{3}{4}$	$1\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	8·331
85R	$5\frac{15}{32}$	$2\frac{11}{16}$	$\frac{11}{16}$	$1\frac{13}{16}$	$1\frac{3}{16}$	$\frac{1}{2}$	$\frac{1}{4}$	8·331
90*	$5\frac{37}{64}$	$2\frac{3}{4}$	$\frac{3}{4}$	$1\frac{25}{32}$	$1\frac{19}{64}$	$\frac{1}{2}$	$\frac{1}{4}$	8·811
90R	$5\frac{37}{64}$	$2\frac{3}{4}$	$\frac{3}{4}$	$1\frac{53}{64}$	$1\frac{7}{32}$	$\frac{1}{2}$	$\frac{1}{4}$	8·811
95*	$5\frac{23}{32}$	$2\frac{3}{4}$	$\frac{3}{4}$	$1\frac{29}{32}$	$1\frac{11}{32}$	$\frac{1}{2}$	$\frac{1}{4}$	9·284
95R	$5\frac{23}{32}$	$2\frac{3}{4}$	$\frac{3}{4}$	$1\frac{15}{16}$	$1\frac{5}{16}$	$\frac{1}{2}$	$\frac{1}{4}$	9·284
100	$5\frac{29}{32}$	$2\frac{3}{4}$	$\frac{3}{4}$	$2\frac{3}{32}$	$1\frac{11}{32}$	$\frac{1}{2}$	$\frac{1}{4}$	9·800

The dimensions figured on the diagram are the same for all sections.

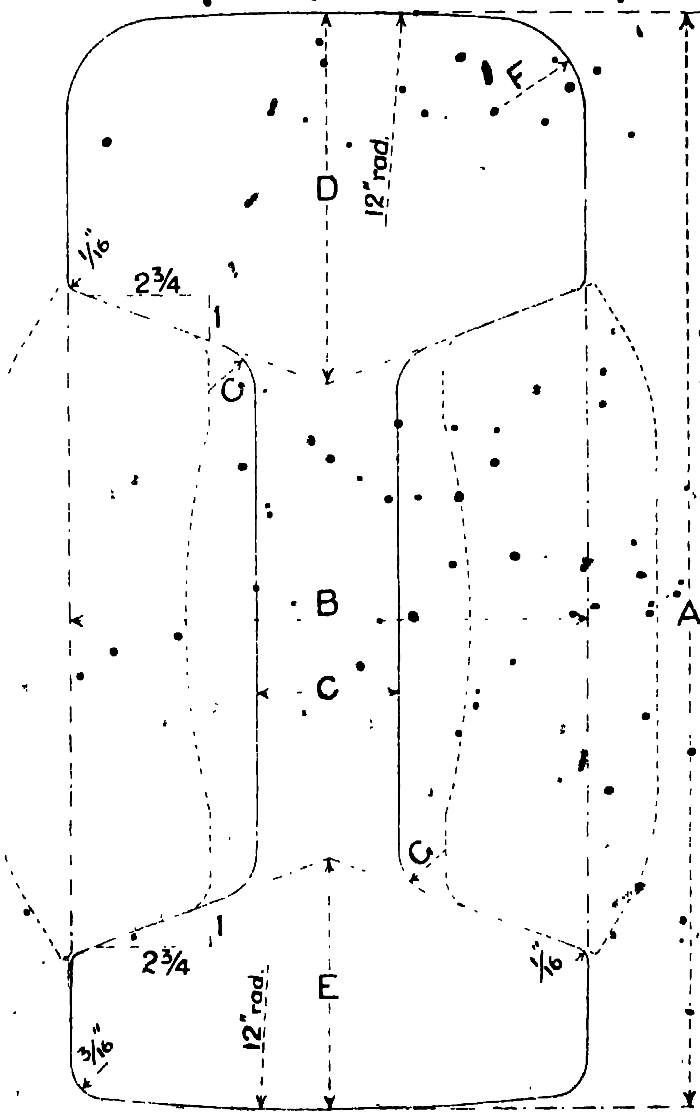
The sections marked * in the table obtained until June, 1922, when they were revised to the sections marked "R."

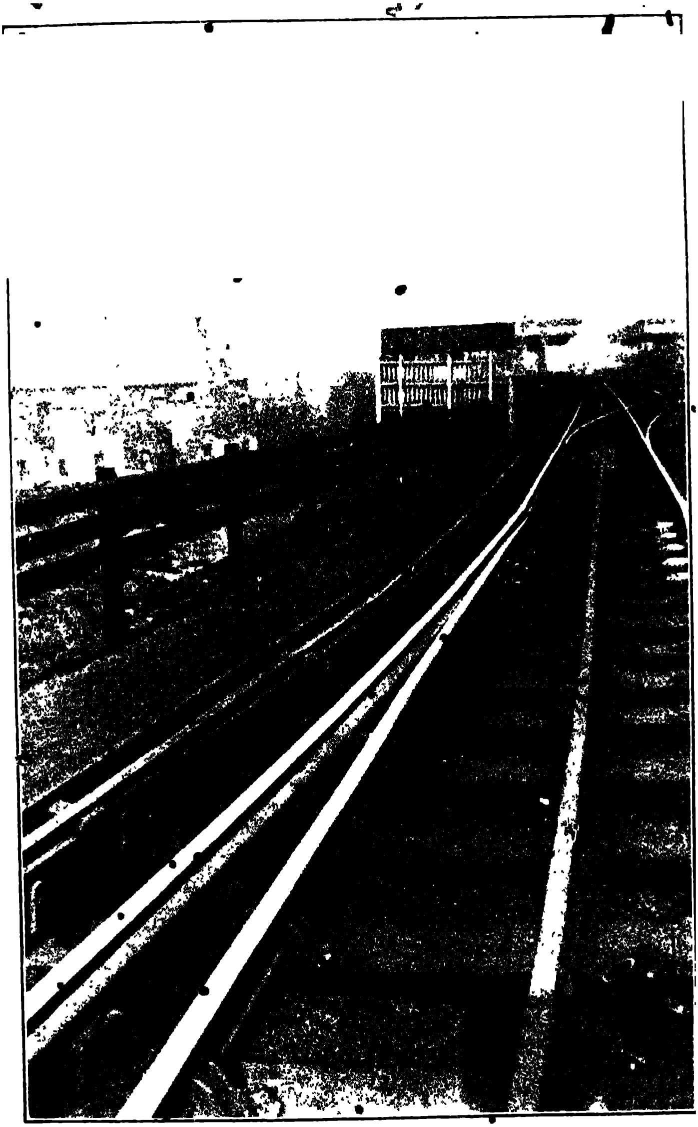
The diagram shews the 95R section rail, full size.

The dotted lines show the cross section of the fishplate for 85, 90, 95 and 100lbs. rails.

Extracted by permission of the British Engineering Standards Association from their Report No. 9, 1922, British Standard Specification and Sections for Bull Head Railway Rails; official copies of which can be obtained from the Secretary of the Association, 28, Victoria Street, Westminster, S.W.1; price 1s. 2d. post free.

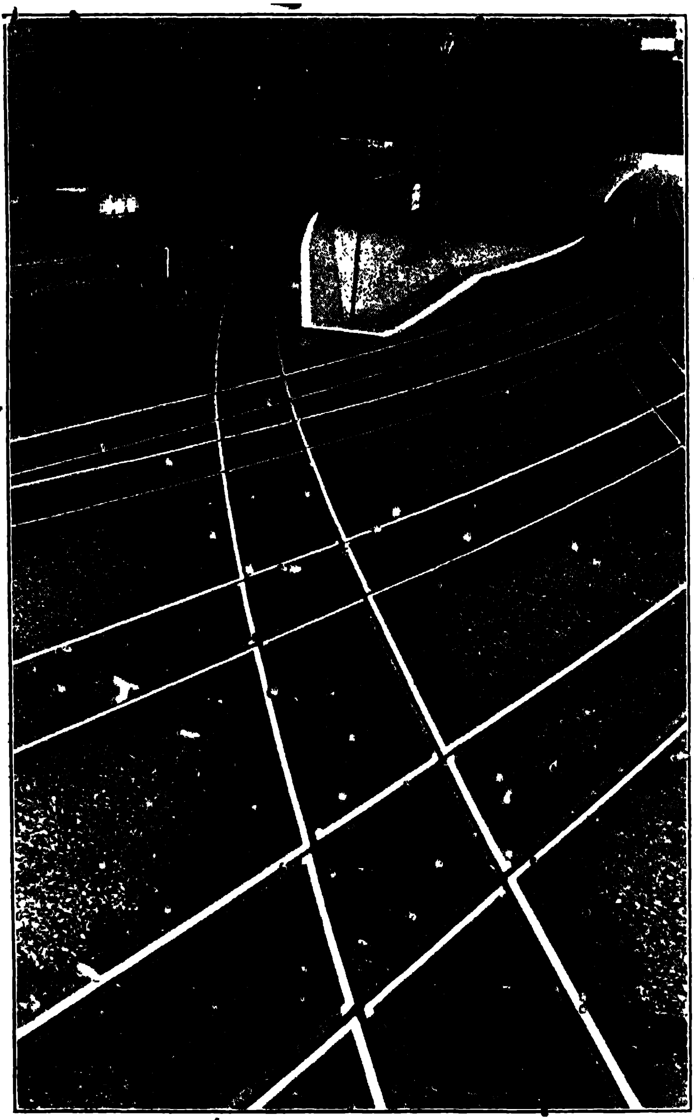
TABLE 49.—*continued.*





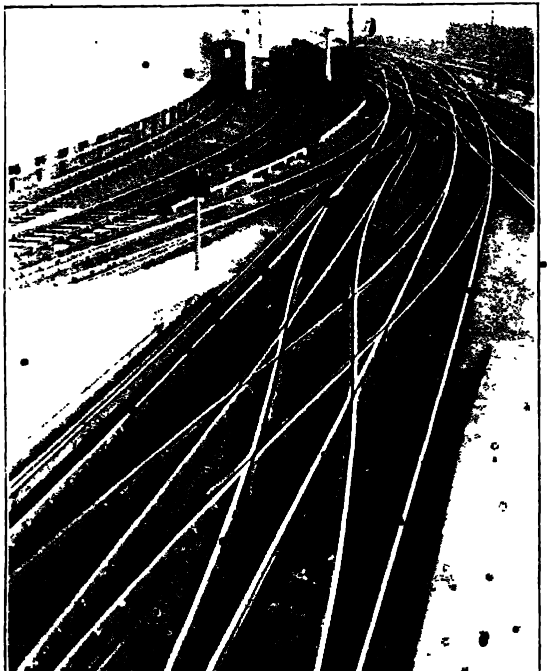
CHECKING OF CURVE AT SWITCHES. *Photo No.* [illegible]

(Chapter IX.)



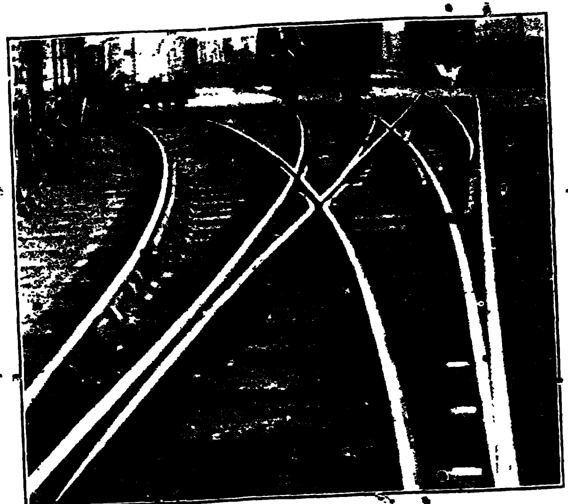
DIAMONDS OF WIDE ANGLE Photo No.

(Chapter XX)



Type I. SCISSORS CROSSOVER ROADS (Chapter XXI)

SLIP ROADS (Chapter XXII)



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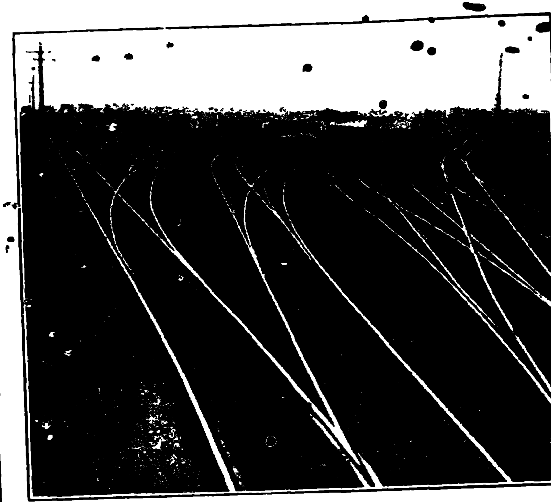

Photo No.

SIDING GROUPS

Chapter XXIII

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*In the case of items marked thus * the problems etc., dealt with will be found under the head of "Contents," page vii. "T" indicates "Table."

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